Trajectory Clustering of Inbound Aircraft based on Feature Representation and Selection

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***Abstract*—Trajectory clustering is an efficient way to identify the prevailing inbound patterns, which could help to improve both individual flight and system-level efficiency. Most of the existing trajectory clustering methods mainly relied on the framework of partition-and-group or the hierarchical clustering strategy. In this paper, we proposed a new trajectory clustering method that focused on the feature representation and selection for the inbound trajectories. Firstly, the representative features of the inbound trajectory were extracted. Secondly, the irrelevant and redundant features were eliminated based on Laplacian Scores and Spearman's correlation coefficients. Thirdly, since each trajectory can be represented as a sample with the same size of features, various standard clustering algorithms could be applied for identifying the prevailing patterns. Furthermore, we carried out case studies by using the trajectories of inbound flight landing on Shanghai Pudong International Airport. The results indicated that the proposed method could not only distinguish the important features for inbound trajectories but also identify the prevailing inbound patterns effectively and efficiently.**

Keywords-trajectory clustering; air traffic management; prevailing patterns; feature representation; feature selection

I. INTRODUCTION

The application of Performance-Based Navigation (PBN) has facilitated the air traffic operation, especially around the Terminal Area (TMA). However, due to the restricted airspace and air traffic volume around TMA in China, the air traffic controllers tend to instruct the inbound aircraft by changing speed or heading for landing sequence establishment and landing sequence maintenance. The actual trajectories inevitably deviate from the published procedure, which leads to inefficient operation of inbound aircraft. Therefore, how to identify the prevailing inbound patterns has received growing attention [1], which could in turn improve the individual aircraft and system-level efficiency by procedure redesign or airspace optimization.

To find the representative or common patterns shared by different aircraft, we usually need to group similar trajectories into clusters. Thereupon, trajectory clustering [2] has been extensively applied for identifying the common patterns in the aviation domain by using aircraft tracking data.

Some work mainly focused on the en route phase. Marzuoli et al. [3] developed a new framework based on a data-driven approach by using historical data. Such new framework could enhance the ability for en route traffic flow management and airspace health monitoring. Bombelli et al. [4] adopted the Fréchet distance-based hierarchical clustering technique to develop high-fidelity models, which could provide recommendation for traffic flow management optimization. Andrienko et al. [5] proposed an analytical workflow in which interactive filtering tools were used to attach relevance flags to elements of trajectories, clustering was done using a distance function that ignored irrelevant elements.

As far as TMA was concerned, Rehm [6] relied on hierarchical clustering to identify arrival traffic patterns landing on Frankfurt Airport. Gariel et al. [7] clustered the inbound trajectories by identifying and grouping the turning points. Mahboubi and Kochenderfer [8] found that “the turning points method” performed well on simulated data, but due to its reliance on noisy heading rates, it had difficulty with real-world data. So they used Bayesian inference techniques to learn the parameters of the traffic pattern. Olive and Morio [9] presented a new hierarchical clustering method to cluster aircraft trajectories before landing by identifying the sequencing significant points. These efforts either relied on the framework of partition-and-group [10], which treated the inbound trajectories as several sub-trajectories [7], [8], [9], or depended on the strategy of hierarchical clustering [6], [9]. Therefore, the trajectory similarity measurement played a great role, which might be a time-consuming job.

This work was supported by the Joint Fund of National NSFC and CAAC (Grant No. U1933117) and Foundation of Graduate Innovation Center in NUAA (kfjj20180713).

In this paper, we proposed a new trajectory clustering method based on feature representation and selection. Firstly, we extracted the representative features of the inbound trajectory (Section II). Secondly, the feature selection process was carried out to eliminate those irrelevant and redundant features (Section III). Lastly, after representing each trajectory as a sample with the same size of features, all kinds of standard clustering methods could be implemented, including partition-based, hierarchy-based, density-based, grid-based, and model-based clustering method [11]. In Section IV, k-means algorithm was used for the clustering. Consequently, we presented a new different approach to identify the prevailing patterns of inbound traffic with two objectives: distinguishing the important features for inbound trajectories, then putting forward an easy to understand and implement trajectory clustering method.

II. FEATURE REPRESENTATION OF TRAJECTORY

A. Features of Trajectory

Trajectories are often represented by point sequences [12], which could be obtained from Radar Surveillance Data, including the recorded time, aircraft position, speed, heading, and data related to the flight plan. For m aircraft trajectories, we define them as a set TR, where $TR = \{tr_1; tr_2; \cdots; tr_i; \cdots; tr_m\}$. Each trajectory $tr_i\left(i \in \{1, \cdots, m\}\right)$ consists of a series of points: $tr_i = \left(p_1^i, p_2^i, \cdots, p_{L(\boldsymbol{a})}^i\right)$, where $L(\boldsymbol{a})$ represents the number of points of the i^{th} trajectory. Each point p_j^i is a tuple: $p_j^i = t, \boldsymbol{l}, \{\boldsymbol{a}\}$, which describes the position information (***l****, location*) and attribute information (***a****, attributes*) of the aircraft at a specific time (***t***), the position information is longitude, latitude, and altitude, the attribute information may include the heading, speed, distance to go (d) and the like.

Good feature representation is central to achieving high performance in any machine learning task. In this paper, we will construct the trajectory features of inbound aircraft from such series of points: $tr_i = \left(p_1^i, p_2^i, \cdots, p_{L(\boldsymbol{a})}^i\right)$. Firstly, we focused on the attributes of the inbound aircraft when passing the Entry Fix. Secondly, we took the statistical values of the trajectory's attributes into consideration, like the average and standard deviation values of the inbound aircraft's longitude, latitude, and heading. Thirdly, we separated the inbound trajectories into arrival phase and approach phase based on the given threshold of the distance to go (d), and paid more attention to the latter phase. It is due to the fact that the air traffic controllers tend to give the instructions in the approach phase for establishing landing sequences and ensuring safe separations. We extracted two features, ψ_{FAF}^i and Ψ_{app}^i, from the approaching phase, as shown in Fig. 1. The red circles denote the extracted informative points during the approaching phase. ψ_{FAF}^i is defined as the angle between such point of i^{th} trajectory with FAF, and Ψ_{app}^i is the heading of this point.

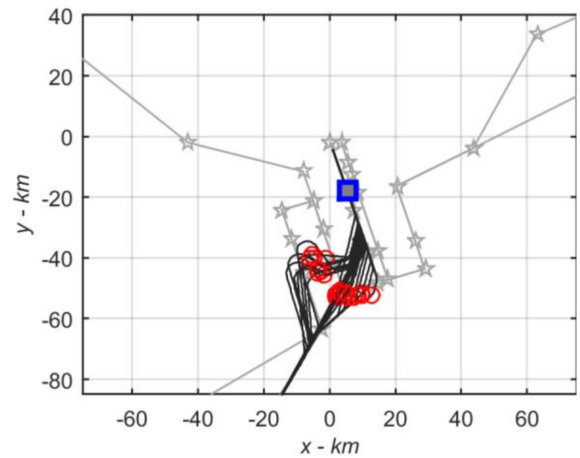


Figure 1. Informative Points Extraction during Approach

TABLE I. FEATURE REPRESENTATION OF TRAJECTORY

Areas	Notations	Explanations
Entry fix	ε_{fix}^i	*x*-coordinates when passing the fix
	η_{fix}^i	*y*-coordinates when passing the fix
	Ψ_{fix}^i	Heading when passing the fix
	d_{fix}^i	Dis. to go when passing the fix
TMA	Ψ_{max}^i	Maximum heading within TMA
	Ψ_{min}^i	Minimum heading within TMA
	$\bar{\varepsilon}^i$	Average *x*-coord within TMA
	$\bar{\eta}^i$	Average *y*-coord within TMA
	$\bar{\Psi}^i$	Average heading within TMA
	σ_{ε^i}	Std. of *x*-coord within TMA
	σ_{η^i}	Std. of *y*-coord within TMA
	σ_{Ψ^i}	Std. of heading within TMA
Approaching	ψ_{FAF}^i	Angle of extracted point and FAF
	Ψ_{app}^i	Heading of the extracted point

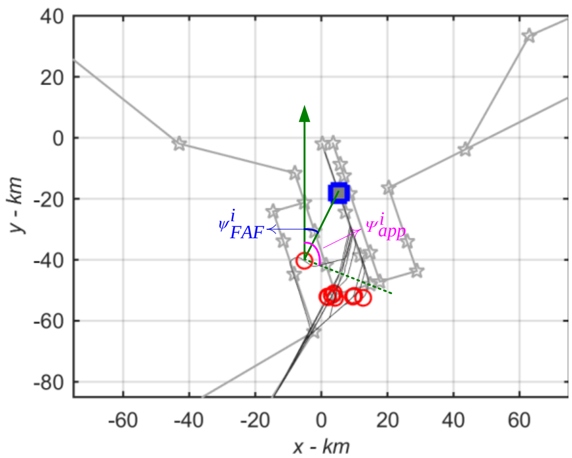


Figure 2. Illustration of the feature ψ^i_{FAF} and Ψ^i_{app}

Accordingly, the feature representation of inbound trajectories is provided in Tab. 1. Fig. 2 elaborates on what ψ^i_{FAF} and Ψ^i_{app} stand for. It should be noted that the geographic features (Longitude and Latitude) have been projected into Cartesian coordinates.

B. Preprocessing of Heading

As shown in Tab. 1, heading is a crucial attributes, which represents the flight procedures and controller instructions. However, the heading varies from 000° to 360°, which is not a monotonous value. Sometimes, the heading value cannot reflect the actual flight situation when it changes nearby 360°. Therefore, the heading values of each trajectory need to be adjusted as follows:

1) Find the difference in heading between two consecutive points:

$$\Delta\psi^i_j = \psi^i_j - \psi^i_{j+1}, \quad j = 1, \cdots, L(\boldsymbol{a}) - 1 \tag{1}$$

where j is the particular point the i^{th} trajectory, and $L(\boldsymbol{a})$ represents the total number of points of the i^{th} trajectory.

2) Modify $\Delta\psi$ according to the left or right turning:

$$\begin{cases} \Delta\psi^{i'}_j = \Delta\psi^i_j - 360^\circ, & \Delta\psi^i_j > 180^\circ \\ \Delta\psi^{i'}_j = \Delta\psi^i_j + 360^\circ, & \Delta\psi^i_j < -180^\circ \\ \Delta\psi^{i'}_j = \Delta\psi^i_j & \text{others} \end{cases} \tag{2}$$

where $\Delta\psi^{i'}_j$ is the modified value of $\Delta\psi^i_j$.

3) Calculate the adjusted heading:

$$\begin{cases} \psi^i_j = \psi^i_j & j = L(\boldsymbol{a}) \\ \psi^i_j = \psi^i_{j+1} + \Delta\psi^{i'}_j & j = L(\boldsymbol{a}) - 1, \cdots, 2, 1 \end{cases} \tag{3}$$

In Fig. 3, the red line indicates the trend of original heading with “Distance To Go”. There is one leap around 40 km. The adjusted one, represented by the blue line, is able to smooth the heading value, which could better describe the trajectory.

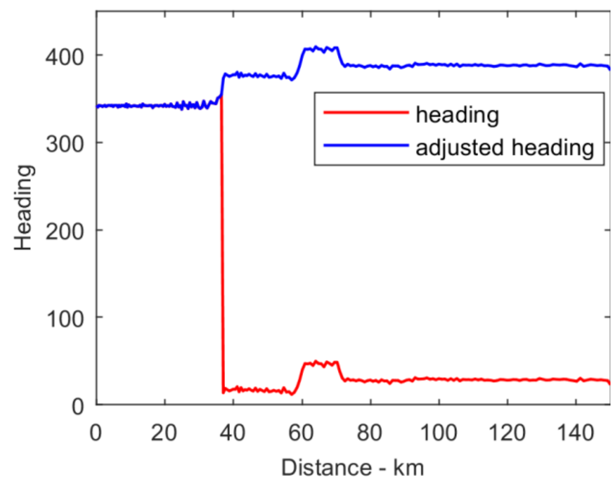


Figure 3. Illustration of the Heading Preprocessing

III. Feature Selection of Trajectory

We have constructed fourteen features for the trajectory clustering of inbound aircraft which is presented in Section II. In this section, the feature selection methods will be applied to tackle the issues of feature irrelevance and feature interaction. In result, we are expecting that the trajectory clustering performance will be improved after reducing the irrelevant and interactive features.

Feature selection for trajectory clustering belongs to the unsupervised feature selection field, which means that [13], [14], given all the feature dimensions of an unlabeled data set, selecting the ideal feature subset among them to make this subset have maximum relevance and minimum redundancy. Laplacian Scores and Spearman’s correlation coefficient are two potent candidates for the feature selection of unsupervised machine learning tasks.

A. Laplacian Score

Laplacian Score (LS) is based on two assumptions [15]: the first one is two data points which are close to each other are likely to belong to the same category; the second one is the local structure of the data space is more efficient than the global structure. Therefore, for any completely unlabeled sample, LS can evaluate the capability of locality preserving and screen out the features which best represent the local structure of the data. The general steps of LS-based feature selection for trajectory clustering are as follows:

1) Construct the nearest neighbor graph G: If two trajectories tr_i and tr_j are "close", they are connected by an edge. The *k*-nearest neighbors of tr_i can be employed to establish the nearest neighbor graph *G*.

2) Determine the weight matrix *S*: If two trajectories tr_i and tr_j are connected by an edge, the weight is obtained by (4). Otherwise, let $s_{ij}=0$.

$$s_{ij} = \exp\left(-\left\| tr_i - tr_j \right\|^2 \big/ \delta \right) \tag{4}$$

where δ is an appropriate constant. Since there are m trajectories, the weight matrix $S = \left[s_{ij} \right]_{m\times m}$.

3) Calculate the graph Laplacian: For the r^{th} feature, $\boldsymbol{f}_r = \left[f_r^1, f_r^2, \cdots, f_r^i, \cdots, f_r^m \right]^{\mathrm{T}}$, where $\boldsymbol{f}_r$ refers to r^{th} feature of the i^{th} trajectory, in which m is the total number of trajectories. Calculate the degree matrix $D = diag\left(S\mathbf{1}\right)$, where $diag\left(\cdot\right)$ is a diagonal matrix, and $\mathbf{1} = \left[1,1,\cdots,1\right]^{\mathrm{T}}$, then define $L = D - S$.

4) Calculate the Laplacian Score of the r^{th} feature:

$$\tilde{\boldsymbol{f}}_r = \boldsymbol{f}_r - \frac{\boldsymbol{f}_r^{\mathrm{T}} D \boldsymbol{f}_r}{1^{\mathrm{T}} D 1} 1 \tag{5}$$

$$L_r = \frac{\tilde{\boldsymbol{f}}_r^{\mathrm{T}} L \tilde{\boldsymbol{f}}_r}{\tilde{\boldsymbol{f}}_r^{\mathrm{T}} D \tilde{\boldsymbol{f}}_r} \tag{6}$$

The smaller the Laplace Score, the smaller the intra-class distance and the greater the inter-class distance of such a feature. As a result, the corresponding feature is more important.

B. Spearman's Correlation Coefficient

Spearman's correlation coefficient is a nonparametric measure of rank correlation. In this paper, we used it to measure the correlation between different features for feature selection.

Let $\boldsymbol{f}_r$ and $\boldsymbol{f}_k$ be two feature vectors, in which f_r^i and f_k^i represent the values of r^{th} and k^{th} features in the i^{th} trajectory. Through sorting the feature vectors (either ascending or descending), two sorting vectors, $\boldsymbol{p}_r$ and $\boldsymbol{p}_k$, are obtained, in which p_r^i and p_k^i represent the ranks of f_r^i and f_k^i in the feature vectors, $\boldsymbol{f}_r$ and $\boldsymbol{f}_k$, respectively. The Spearman's correlation coefficient between $\boldsymbol{f}_r$ and $\boldsymbol{f}_k$ can be calculated by:

$$\rho_{rk} = \frac{\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \left(p_r^i - \bar{\boldsymbol{p}}_r \right)\left(p_k^i - \bar{\boldsymbol{p}}_k \right)}{\sqrt{\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \left(p_r^i - \bar{\boldsymbol{p}}_r \right)^2 \sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \left(p_k^i - \bar{\boldsymbol{p}}_k \right)^2}} \tag{7}$$

The larger the absolute value of ρ_{rk} is, the higher dependency the two features possess.

C. Principles of Feature selection

To carry out feature selection, the following principles should be applied:

1) We obtained the importance ranking of the features by the Laplacian Score. If the Laplacian Score is smaller than 0.3, the corresponding feature is retained and should be added to a feature subset. A feature is considered irrelevant if its Laplacian Score is higher than 0.7, and the corresponding feature will be deleted. The rest of features with Laplacian Score between 0.3 and 0.7 will be evaluated lately.

2) Calculate the Spearman's correlation coefficient for features with Laplacian Score between 0.3 and 0.7. If the correlation coefficient of those feature pairs is greater than the predetermined value, the redundant feature is the one with the higher Laplacian Score, and it will be deleted. The remaining features will be added to the feature subset.

3) Suppose *n* features are retained after eliminating the irrelevant and redundant features. Accordingly, each trajectory could be denoted with same dimension. For example, the i^{th} trajectory can be represented by :

$$tr_i = \left\{ f_1^i, f_2^i, \cdots, f_n^i \right\} \tag{8}$$

IV. TRAJECTORY CLUSTERING

The trajectory can be characterized as a vector owing to the feature representation and feature selection. Therefore, various clustering algorithms can be implemented to tackle the trajectory clustering problem. In this paper, we adopted the *k*-means algorithm. The clustering results, representative central trajectories within each cluster, could be extracted to compare with the existing flight procedure. Accordingly, we could find the regular instructions of controller or provide the suggestion for flight procedure optimization and redesigning.

The aim of clustering is to organize data into different groups where the within-group-data similarity is maximized, and the between-group-data dissimilarity is maximized [16]. Davies-Bouldin (DB) index is one of the promising candidates by combining the intra-class compactness and inter-class separability. The DB index is computed in the following way:

$$D_{jk}^{inter} = C_j - C_k \tag{9}$$

$$D_j^{intra} = \frac{1}{\left|C_j\right|} \sum_{tr_i \in C_j} tr_i - C_j \tag{10}$$

$$R_j = \max_{k \neq j} \left(\frac{D_j^{intra} + D_k^{intra}}{D_{jk}^{inter}} \right) \tag{11}$$

$$DB = \frac{1}{n_c} \sum_{j=1}^{n_c} R_j \tag{12}$$

where C_j and C_k are the centers of j^{th} and k^{th} cluster, the total number of the clusters is n_c. D_{jk}^{inter} and D_j^{intra} represent the inter-class and intra-class distance, respectively.

Therefore, the whole process of trajectory clustering of inbound aircraft is presented by the following steps:

Step 1) Decode the raw data, and obtain the flight trajectories based on the standard document for surveillance data exchange (category 062);

Step 2) Carry out the preprocessing for the flight trajectories;

Step 3) Represent the flight trajectories with features according to Tab. 1;

Step 4) Eliminate the irrelevant and redundant features based on Laplacian Score and Spearman's Correlation Coefficient;

Step 5) Conduct the trajectory clustering by using k-means clustering algorithm;

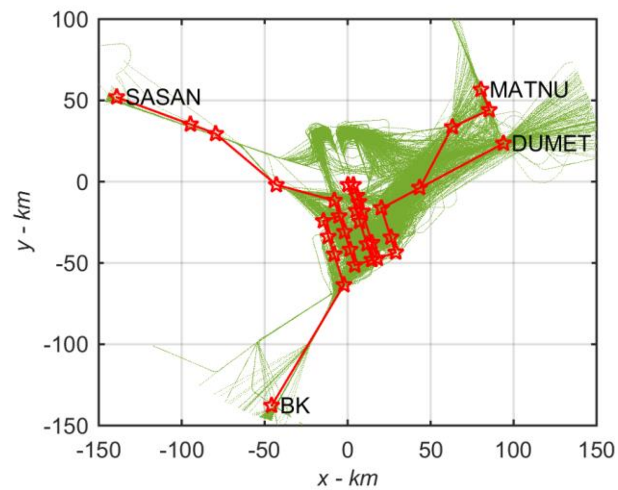
Step 6) Evaluate the clustering results through the Davies-Bouldin index.

V. CASE STUDIES

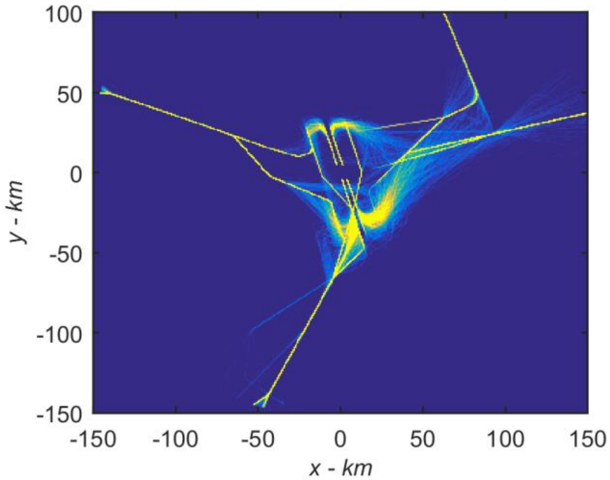
A. Data Preparation

The inbound aircraft of Shanghai Pudong International Airport (ZGPD) were considered in this section.

Fig. 4 (a) and (b) presented the radar trajectories about 4 rush hours during two weeks' operation. Fig. 4 (a) included the entering Fixes (SASAN, BK, MATNU, and DUMET) and the STARs, while Fig. 4 (b) provided the heat map of the inbound trajectories. In the following subsections, only northbound operation via BK fix was taken into account.



(a) Entry Fixes and the STARs



(b) Heat map of Inbound Trajectories

Figure 4. Illustration of the Trajectories of Inbound Aircraft

B. Feature Selection

Firstly, the Laplacian Scores of all the features listed in Tab.1 are provided in Fig. 5. The corresponding features with Laplacian Scores smaller than 0.3 (denoted as green color in Fig. 5) are η^i_{fix}, Ψ^i_{max}, σ_{Ψ^i}, and they were chosen as the candidates. The corresponding features with Laplacian Scores greater than 0.7 (denoted as red color in Fig. 5) are Ψ^i_{fix}, Ψ^i_{min}, and they are eliminated.

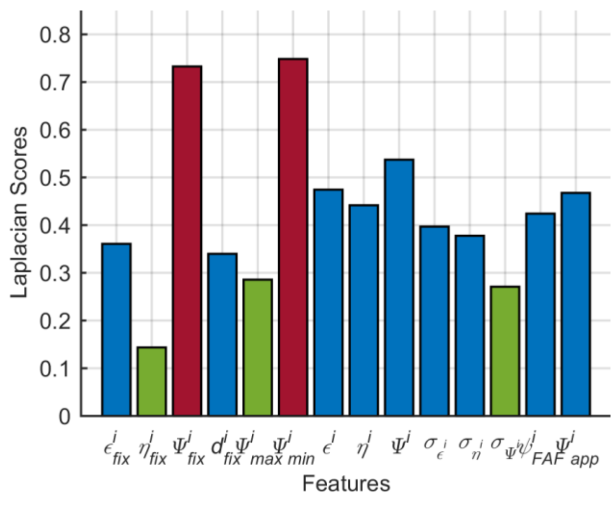


Figure 5. Illustration of Laplacian Scores

Secondly, the Spearman's correlation coefficients of the rest of features (denoted as blue color in Fig. 5) are calculated, which is presented and visualized in Fig. 6. If the correlation coefficient of any feature pairs is greater than the predetermined value (0.7), the corresponding feature with a higher Laplacian Score will be deleted. Therefore, $\bar{\varepsilon}^i$, σ_{ε^i}, σ_{η^i} are eliminated.

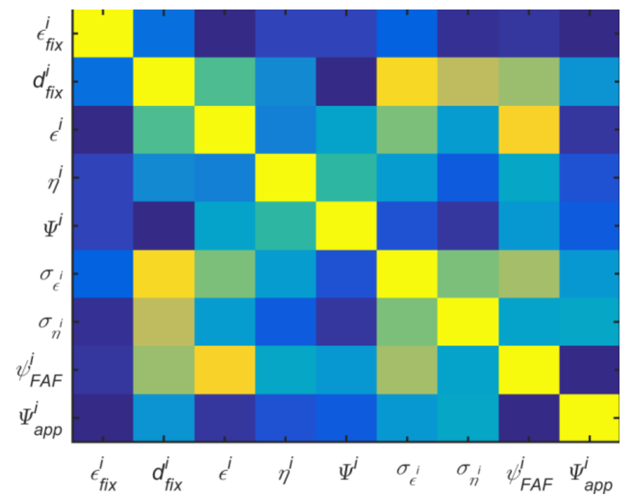


Figure 6. Illustration of Spearman's correlation coefficients

Thirdly, after eliminating the irrelevant and redundant features, all of the 9 features are retained. That means an aircraft trajectory tr_i could be represented as a 1 × 9 vector:

$$tr_i = \left\{ \varepsilon^i_{fix}, \eta^i_{fix}, d^i_{fix}, \Psi^i_{max}, \bar{\eta}^i, \bar{\Psi}^i, \sigma_{\Psi^i}, \psi^i_{FAF}, \Psi^i_{app} \right\} \quad (13)$$

When there are m flight trajectories, the sample matrix $TR = \left[tr_1; tr_2; \cdots; tr_m \right]$ can be formed.

C. Trajectory Clustering

K-means algorithm was implemented for trajectory clustering. The appropriate k parameter (the number of clusters) was determined to be 7 based on the DB index as shown in Fig. 7, which could ensure the maximization of within-group-data similarity and between-group-data dissimilarity, simultaneously.

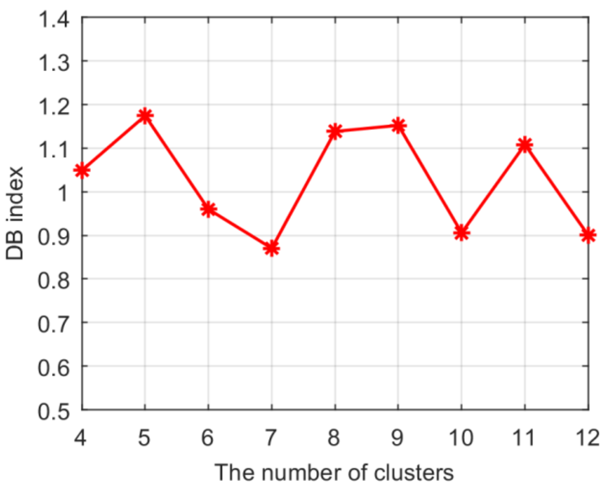


Figure 7. DB Index When Varying the Number of Clusters from 4 to12

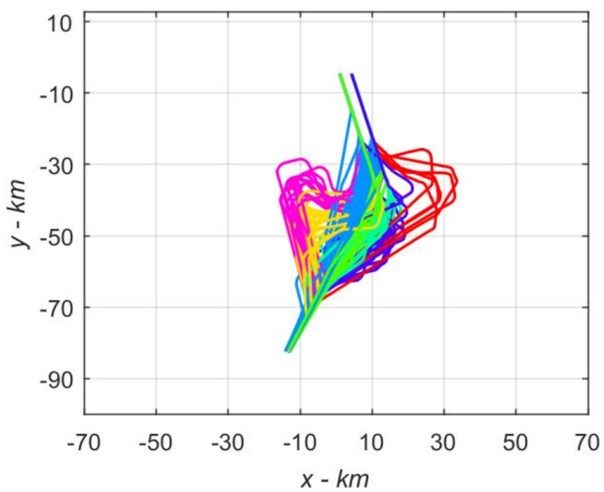


Figure 8. Illustration of trajectories clustering results via BK fix.

Fig. 8 provided the clustering results of inbound trajectories under northbound operation via BK fix. The prevailing inbound patterns, as shown in Fig. 8, were extracted from the clustering results.

From Fig. 9, we could find that each trajectory represented a prevailing inbound pattern. and represented straight-in pattern (“short-cut”) landing on different runways, accounting for 225/482 and 96/482 respectively; , , , and represented the patterns guided by the controllers’ instructions (“dog-leg”), accounting for 115/482 totally; represented a pattern that deviated significantly from the published procedure, accounting for 6/482. Thereupon, and could be treated as another two optional routes, which would improve the individual aircraft and system-level efficiency.

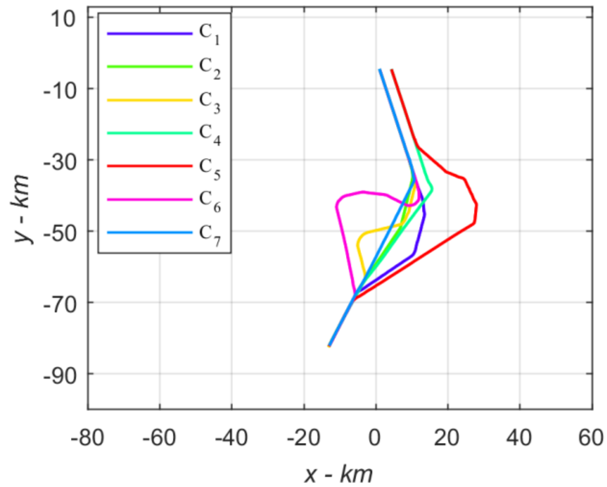


Figure 9. Illustration of the prevailing inbound patterns

VI. Conclusion

We proposed a new method to identify the prevailing patterns of inbound traffic based on trajectory clustering. Unlike the popular partition-and-group strategy, the framework of feature representation and selection was adopted in our work. And the method was properly validated for the inbound traffic of ZSPD airport via BK fix. Firstly, the feature extraction method could guarantee each trajectory represented by the same size of feature vector. Secondly, the feature selection could ensure the irrelevant and redundant features could be eliminated, which improves the performance of trajectory clustering. Thirdly, we could easily identify the prevailing patterns of inbound traffic via BK fix, including “short-cut” and “dog-leg”patterns.

Future works may include extracting other features, making attempts of other standard clustering algorithms (Hierarchy based or Density-based method), and applying into different situations. Besides, how to take the temporal information into the current spatial analysis needs more efforts.

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