

# A Percolation Theory Based Approach for Identification of Bottleneck Links in an Airway Network

Chunyao Ma\*, Qing Cai<sup>†</sup>, Sameer Alam<sup>‡</sup>, Vu N. Duong<sup>§</sup>

Air Traffic Management Research Institute, School of Mechanical and Aerospace Engineering  
Nanyang Technological University, Singapore

Email: \*M180146@e.ntu.edu.sg, {<sup>†</sup>qcai | <sup>‡</sup>sameeralam | <sup>§</sup>vu.duong}@ntu.edu.sg

**Abstract**—The ever increasing demand for air travel is likely to induce air traffic congestion which will elicit great economic losses. As air traffic congestion usually originate and propagate from a small region in an airway network, it is becoming important to identify the bottleneck links of an airway network. In this paper, we characterize the organization of air traffic flow as a percolation process. From a percolation process, it can be observed that the global air traffic is dynamically formed by clusters of local air traffic flows which are connected by the bottleneck links. We developed a data driven method to identify such bottleneck links in an airway network based on percolation theory. This method aims to identify links, at the percolation threshold, whose malfunction potentially disintegrate the global air traffic flow into large isolated local flows. These links are identified as bottleneck links since they reduce the efficiency of air traffic flow in the airway network and induce air traffic congestion. With the proposed method, we have carried out a case study on Singapore airway network using one month ADS-B data. Results show there appears to be a presence of airway links that may be bottlenecks in Singapore airway network. When the bottleneck links are dysfunctional, large-scale local traffic flows are unable to exchange freely which can lead to global traffic congestion. This provides an approach to manage air traffic congestion with minor adjustments such as improving the flight efficiency on the bottleneck links.

**Index Terms**—air traffic congestion, airway network, bottleneck links, percolation theory

## I. INTRODUCTION

Air transportation is generally growing physically and operationally [1], [2]. The International Air Transport Association (IATA) estimates that by 2036 the number of air passengers will double to 7.8 billion [3].

As present day air traffic network is reaching its operational capacity, accommodating future air traffic growth will be challenging for air navigation service providers (ANSPs) [4]. The imbalance between the airway network capacity and the increasing air traffic demand is leading to significant congestion as well as flight delays [5]–[7], which will cause huge economic losses to ANSPs, airlines and customers. It is reported that the annual total cost of air transportation delays was over \$30 billion in the United States [8].

In literature, many researches have been done to manage air traffic congestion induced by the increasing traffic demand, among which the main approaches are air traffic flow man-

agement (ATFM), dynamic sectorization and airway network management [9].

Mitigating air traffic congestion by ATFM is usually be achieved by flight scheduling [10] and flight trajectory optimization [11]. In [12], a scheduling algorithms was developed to calculate the optimal departure rates and departure slots for streams of flights that are destined for an airspace experiencing reduced capacity where traffic congestion can easily occur. In [13], a network based dynamic air traffic flow model for en route airspace system traffic flow optimization was proposed to maintain the balance between demand and capacity. In [14], a collaborative flight route planning method was demonstrated to reduce en-route airspace congestion by amending flight plans to avoid congested sectors. In recent years, researchers have started to focus on 4D flight trajectory planning which have combined flight scheduling and trajectory optimization together [15], [16]. For instance, a 4D flight trajectories planning method, which allocates trajectories to flights via rerouting, time controlling, and flight level changing through multi-objective optimization, was illustrated in [17] to avoid conflict and congestion in the airway network.

ATFM reduced the effects of air traffic congestion problems by flight control. However, it is not able to increase the limited airspace capacity which is the source of congestion. Its effectiveness is therefore limited by the capacity of the airspace. Moreover, its good results subject to a good control of uncertainties such weather and traffic demand.

Another frequently-used air traffic congestion management strategy was increasing the airspace capacity by improving the sectorization schemes [18]. Dynamic airspace sectorization is an approach to restructuring airspace to achieve capacity-demand balance, by managing air traffic controllers' workload to ensuring an orderly flow of traffic [19], [20]. However, they almost had reached the structural limits of the system and may induce safety and efficiency problem.

We know that flights have to follow predefined airways from the origin to the destination in an airway network, which consist of waypoints (nodes) and airways (edges). Therefore it is also possible to reduce congestion by improving the structure of airway networks [21]. In [22], a multi-objective particle swarm optimization algorithm was introduce to optimize the crossing waypoints location of an airway network

with the objective of maximizing the flight efficiency as well as the airspace capacity simultaneously. However, this method does not consider actual location restrict and operating limits of the waypoints. In [23], an airway network optimization model was developed to minimize the total operational cost with airspace restriction and air route network capacity being considered as the major constraints. The disadvantage of this work is that it targets at the fragmented airspaces instead of the general airspaces. Essentially, at present, most of the solution algorithms for route network optimization model, such as GA, PSO algorithm [24] and Differential Evolution (DE) algorithm, are both complex and easy to converge to the local optimal because of limited exploration ability of the new space. The stability of the results is also a challenge. More importantly, these efforts often end up with complete new airway structures for a given airspace. The challenge is that, due to operational constraints, complete redesign of airway networks, so as to obtain the global optimal airway structures (for a given airspace/FIR), is impractical. In [25], the authors proposed to close some links in the potentially overdesigned airway networks based on the theory of Braess's Paradox. Using this method, the flight duration of flights can be reduced by doing minor changes to the airway network. However, because of the many approximations and assumptions in the paper, there is still way to go before application.

As mentioned earlier, air traffic has long been artificially concentrated on airway networks. Note that traffic congestion usually emerges on small parts of the airway network and propagates to the proximities [26], which then largely inhibits traffic flows in the airspace and exacerbates the traffic congestion. In the presence of limited airspace capacity as well as the saturated airway network, it is important to discover the bottleneck links of an airway network. In this paper, the bottleneck links are defined as, from the network perspective, the airway links whose malfunction potentially disintegrate the global air traffic flow into large isolated local flows. These links primarily initiate the traffic congestion whereby hinders the airway network from serving more traffic. Knowledge of these bottleneck links can provide opportunities to improve significantly the global air traffic flow efficiency in the airway network with minor cost (e.g., improving a single airway link).

It should be pointed out that decreased flight duration/increase flight speed can reduce buffer time for airborne congestion as well as the risk of delay propagation through the entire airway network [27], and the congestion can be mitigated consequently. Note that a flight can be re-directed other than change flight speed when facing congestion. Instead of simply taking the average of the real-time speed of flights, here the average speed on a link is determined in this way: the link length divided by the average flight time spent by flights on the link. Hence, the average speed here can take into consideration of the extra flight time caused to flight by re-direction or change of flight levels, which better reflects the flight efficiency on a link. Moreover, higher average flight speed on a link suggests flights can travel smoothly from one region to another through this link while low average flight

speed can block the free traffic from one region to another through the link. Therefore in this paper we use the average flight speed on airway links as one indicator of bottleneck links. Thus one major feature of the bottleneck links is the low average flight speed on them. However, this is not sufficient to make a link the bottleneck. The contribution of different low average speed links on the traffic congestion are also different. Some trivial links can only block air traffic from small regions while some fatal links block traffic between large regions in the airway network. Therefore, the bottleneck links we aim to identify in this paper have two features: 1) the average flight speed on the links are low; 2) the links are at important positions that can block traffic between large regions. Note that the two features of airway links can change overtime because, at different time of a day, the traffic demand and traffic structures are different. Therefore in this paper, we propose a data driven method based on network percolation theory to identify the bottleneck links of an airway network for a given time period with real air traffic data.

To identify the bottleneck links in an airway network for a given time period, for each link we first determine the average flight speed during this period which is then normalized using the maximum speed on this link of the day. Then the airway network is modelled as a planar network with the nodes being the waypoints, links the airways and weights the normalized average flight speed. Then percolation theory is applied onto the graph to determine the links with low weights and whose malfunction can block the connection between large local flows in the network. These links will be identified as the bottleneck links of the airway network.

In order to verify the efficacy of the proposed method, this paper carries out a case study on Singapore Airway Network (referred to as SAN hereafter), using ADS-B data recorded over one month during the calendar year 2018. The results show that different bottleneck links exist under different time periods. Some of the critical links possess a high frequency and one of them is distinctly higher than the rest which requires special attention of air traffic controllers.

## II. PERCOLATION THEORY

Complex networks in reality suffer from manifold perturbations and as a consequence, the components of a network may break down and potential risk is likely to happen. In order to better design the structure of a network so as to make it be robust to perturbations, it is pertinent to analyze the dynamics of a complex network subject to perturbations. Percolation theory has proven as an effective instrument for analyzing the capability of a complex network in face of perturbations [28].

Suppose that  $1 - p$  fraction of components of a network are disconnected to the rest of the focal network due to external/internal perturbations. The disconnection of those failed network components can fragment the focal network into pieces and amongst which there exists the largest connected component (LCC) [29]. The LCC of a network is an important indicator for capturing the network's capability in response to perturbations. When  $p = 0$ , the LCC of the network disappear,

simulating the scenario that the focal network is entirely down due to perturbations. For  $p = 1$ , it corresponds to the situation that the network is not suffering from perturbations and the LCC keeps its original state. When  $p$  increases from 0 to 1, the size of the LCC changes with  $p$ . When  $p$  reaches a certain value, the size of the LCC shows a unique change such as a sharp decline or becomes extremely small or even zero. Such a value of  $p$  is generally termed as the percolation threshold denoted by  $p_c$ .

The percolation threshold  $p_c$  reflects the stability of a complex network. If  $p_c$  is large for a network, then the network has a very stable structure and perturbations will not significantly affect the network's connectivity. On the contrary, a network with a small value of  $p_c$  is fragile to perturbations as its connectivity will be largely affected.

Due to its important implications to a network's stability, percolation theory has been widely applied to investigate the structure properties of diverse complex networks and networked systems [30]–[32]. Percolation in air traffic systems was firstly introduced in [33], where a model was proposed to pursue a systemic approach by means of a theoretical model taking into account the phase transition phenomenon and leading to a more realistic simulation of the air traffic control system. Recently, percolation, at an empirical investigation level, has been proposed in [34] as an approach to detect bottlenecks in urban traffic networks. Percolation theory makes use of statistical physics principles and graph theory to analyze changes in the structure of a complex network. In the percolation theory, the failure of a node/link in a network is modeled as removal/closure. As the closure of more nodes/links, the network undergoes a transition from the phase of connectivity (normal functioning network) to the phase of dis-connectivity (non-normal functioning network). The percolation threshold  $p_c$  signifying this phase transition can be found theoretically or computed numerically by percolation theory.

### III. PROBLEM DESCRIPTION

Percolation actually describes a phase transition process of the failure of a weighted network, whose percolation threshold distinguishes the weighted network from a connected phase to a disconnected phase because of the closure of some links in this network. The closure of links during percolation process depends on the weights of the links. Thus, percolation theory can help to understand the macroscopic failure behavior of networks in relation to the microscopic states of the network components, i.e., the weights and the positions of the network links. It can address questions of practical interest “The failure of which links will break down the whole network, according to their weights and positions in the network?”.

As mentioned in Section I, average flight speed on a link will influence the flight duration and consequently the air traffic congestion. Therefore in this paper, with the traffic data during a given time period, we model the airway network as a weighted network according to the flight speed, and we aim to identify links in this weighted network whose closure will break down the whole network based on percolation theory.

More specifically, we will determine the links which are closed at the percolation threshold and consequently disintegrate the connected network into several disconnected clusters.

## IV. METHODOLOGY

### A. Methodology Overview

This paper presents a frame for airway network bottleneck links identification based upon percolation theory. Fig. 1 presents a concept diagram of the proposed method. The proposed approach encompasses three key steps: network modelling, network percolation and bottleneck links identification.

In Fig. 1, we take SAN as an example to illustrate the proposed method. In the network modelling step, given the air traffic data during a specific time period, the airway network configuration is first extracted based on the flight fixes and flight paths of flights. Besides, the average flight speed on each link in the airway network is determined and is then normalized by the daily maximum speed on this link. Then the airway network is modelled as a planar network with the nodes being the waypoints, links the airways and weights the normalized average flight speed. In the network percolation stage, the weighted network is gradually broken into multiple clusters by incrementally closing the links with low weights. By observing the percolation threshold where the closure of some links disintegrates the present largest cluster when the second largest cluster reaches its maximum. The links connect the clusters, constituting the disintegrated largest cluster, are identified as the bottleneck links during this specific time period. In what follows, we delineate in detail how each step works.

### B. Network Modelling

The purpose of the network modelling is to prepare a weighted network for the network percolation process. As mentioned earlier, we use the normalized average flight speed on each link as its weight. Therefore, network modelling consists of two steps, namely, extract the airway network configuration and determine the weight on each link from the air traffic data. The traffic data required for network modelling includes the flight paths of each flight, i.e., the flight fixes of the flight trajectory, and the time when the flight is reported to be at these fixes.

From the flight path information, we will be able to construct the airway network configuration by setting the flight fixes as the nodes and determining the connections between node, i.e., links, from the path of each flights. If there are flights whose paths pass the link between two nodes, the two nodes will be regarded as connected and consequently there will be a link connecting the two nodes on the resulted airway network. In this manner, the airway network will be constructed completely from the traffic data.

With the time information of flights reaching the fixes, the average speed of a flight  $A_i$  on the link  $E_j$  of its path can be computed by averaging the length  $L_j$  of  $E_j$  over the flight duration  $t_{ij}$  on  $E_j$ :

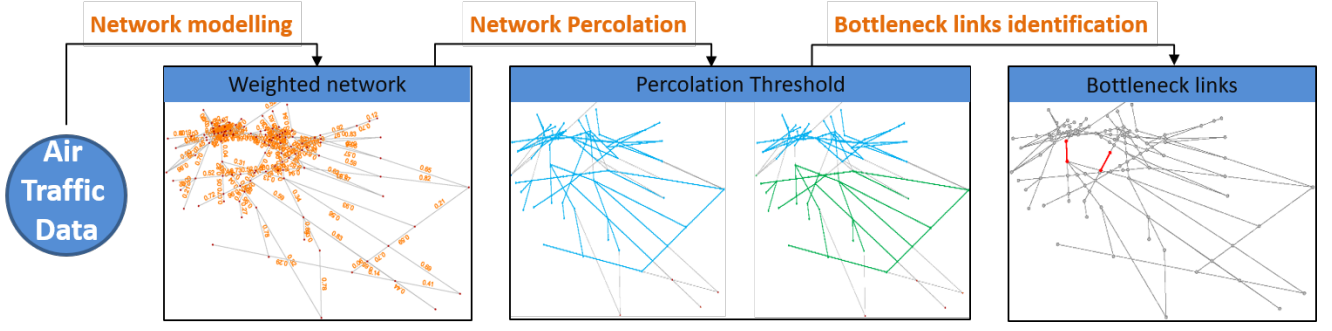


Fig. 1. The conceptual diagram of the proposed method for airway network bottleneck links identification. The air traffic data is converted into a weighted network through network modelling. During network percolation, we aim to identify the percolation threshold where the closure of some links will break the connected network into large local clusters. And these links are identified as the bottleneck links.

$$s_{ij} = L_j / t_{ij} \quad (1)$$

Note that we are identifying bottleneck links for a given time period  $T_k$  (from  $T_k^0$  to  $T_k^1$ ), therefore the required weight  $w_j$  on link  $E_j$  is the normalized average flight speed of all flights passing  $E_j$  during  $T_k$  instead of simply the normalized average flight speed of all flights passing  $E_j$ . This means that for flight  $A_i$ , only the part  $l_{ij}^k$  on  $E_j$ , that has been flown during  $T_k$ , will be considered, and we can not simply take the average of the average flight speed  $s_{ij}$  of all flights on link  $E_j$ . From the traffic data available, we are not able to obtain the time information of when a flight  $A_i$  is exactly on  $E_j$  during  $T_k$ . Therefore to reduce the bias, we assume that the flight  $A_i$  is flying on  $E_j$  with the constant speed  $s_{ij}$ . Then given the entry time  $O_{ij}$  and exit time  $D_{ij}$  of  $A_i$  on  $E_j$ , the flight duration  $t_{ij}^k$  of  $A_i$  on  $E_j$  during  $T_k$  can be estimated:

$$t_{ij}^k = \min\{D_{ij}, T_k^1\} - \max\{O_{ij}, T_k^0\} \quad (2)$$

Then  $l_{ij}^k$  can be determined as  $L_j$  weighted by the proportion of  $t_{ij}^k$  to  $T_k$ :

$$l_{ij}^k = L_j \times t_{ij}^k / T_k \quad (3)$$

The average speed  $s_{ij}^k$  on link  $E_j$  during  $T_k$  can be calculated by averaging the sum of flight distances of all the flights on  $E_j$  over the according sum of flight duration:

$$s_{ij}^k = \sum_{i=1}^n l_{ij}^k / \sum_{i=1}^n t_{ij}^k \quad (4)$$

where  $n$  is the total number of flights passing link  $E_j$  during  $T_k$ .

Finally, the weight  $w_j$  on link  $E_j$  during the given time period  $T_k$  is determined by normalizing the average speed  $s_{ij}^k$  by the daily maximum speed  $s_{max}$  on link  $E_j$ :

$$w_j = s_{ij}^k / s_{max} \quad (5)$$

In this way, the weight on link  $E_j$  is determined, which will be a number between 0 and 1.

### C. Network Percolation

The objective of the network percolation step is to identify the percolation threshold through the percolation process of an airway network. More specifically, by incrementally closing the low weight links in the weighted airway network. Lower weight of a link means that the current travelling speed on the link is more degenerated comparing to the best daily flight speed on this link. Therefore, these lower weight links can be regarded as malfunction links which can potentially slow down the flights and induce congestion. By incrementally closing these low weight links, we will be able to observe the links whose closure, i.e., malfunction, will lead to a transition of the airway network from the phase of connected to the phase of disconnected.

Note that an airway link  $E_j$  is characterised by the weight  $w_j$ . Therefore for a given threshold  $q$ , the link  $E_j$  can be classified into two categories: functional when  $w_j \geq q$  and dysfunctional when  $w_j < q$ . This can be represented as:

$$E_j = \begin{cases} 1, & w_j \geq q \\ 0, & w_j < q \end{cases} \quad (6)$$

As the value of  $q$  increases, more low weight links are closed, which makes the network becomes more sparse. Note that the weight on a link refers to the normalized average flight speed of flights on this links. It indicates that As  $q$  increases, links on which the flight speed is low are closed and links with higher flight speed remain active. In this way, a functional airway network can be constructed for a given  $q$  value according to the traffic dynamics of the original airway network.

As  $q$  increases, the original network will be disintegrated into several isolated clusters because of the closure of some low speed links. Therefore, the size of the largest cluster decreases, and the second-largest cluster reaches a maximum at the percolation threshold  $q_c$  which is the transition to the disconnected phase from the connected phase of the airway network, as shown in Fig 2. The vertical axis refers to the fraction of the size of the largest cluster (LCC1)/second largest cluster (LCC2) comparing to the size of the original network, which is a value between 1 and 0 and calculated as the fraction

of the number of nodes in the target cluster in the total number of nodes in the original network.

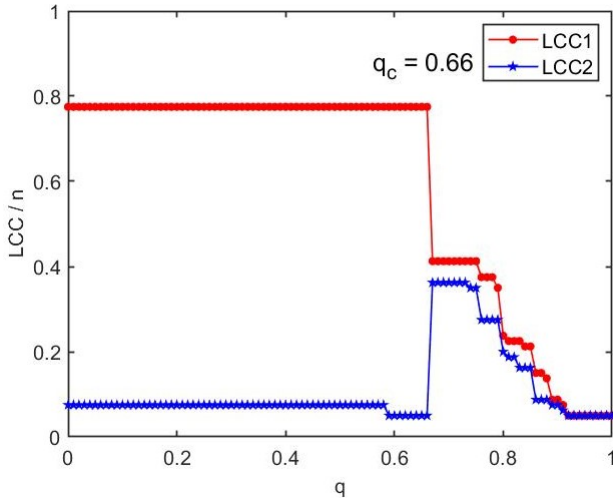


Fig. 2. Size of the largest cluster (LCC1) and the second-largest cluster (LCC2) of airway networks as a function of  $q$ . The sudden decrease of the size of LCC1 and the increase of the size of LCC2 indicates that, at the percolation threshold  $q_c$ , LCC1 is broken into several isolated clusters because the closure of some links. Therefore the network transits from a largely connected phase to a disconnected phase at  $q_c$ .

As an indicator of the robustness characteristics of network connectivity [35], the percolation threshold  $q_c$  in this percolation process quantifies the organization efficiency of real air traffic. Flights are able to travel to most nodes in the airway network (largest connected component of airway network) with normalized speed below  $q_c$ , while flights will be trapped in small isolated clusters when they fly with normalized speed above  $q_c$ . Hence,  $q_c$  measures effectively the maximum normalized speed with which flights can travel over a large part of the airway network, which reflects the global efficiency of air traffic from a network perspective.

#### D. Bottleneck Links Identification

Based on our observation that air traffic can be viewed as a percolation process, it is suggested that high velocity links tend to form clusters, which are bridged by low-velocity roads. These bottleneck links determine the global connectivity of the airway network at each instant. At the critical threshold  $q_c$ , several links will be closed when we tune the  $q$  value slightly higher. While some links are closed by chance, some links play a critical role in connecting different local traffic clusters in airway network. Only these links, whose closure at the critical threshold  $q_c$  disintegrate the overall connected network into local clusters, are considered as bottleneck links. Closure of these bottleneck links will result in disintegration of a globally connected airway network into several disconnected clusters.

### V. CASE STUDY

The above Section describes the proposed method for bottleneck links identification for an airway network with given air

traffic data. To verify the efficacy of the proposed method, in this section we carry out a case study on SAN using one month en-route air traffic data. A total number of 44215 flights' trajectory information, including flight fixes as well as the time passing these fixes, has been used in this study. The flight data covers eight sectors of Singapore airspace.

#### A. Network Modelling

We first construct the network structure of SAN using the 1 month air traffic data. Fig. 3 displays the airway structure of SAN which consists of 120 nodes and 183 links.

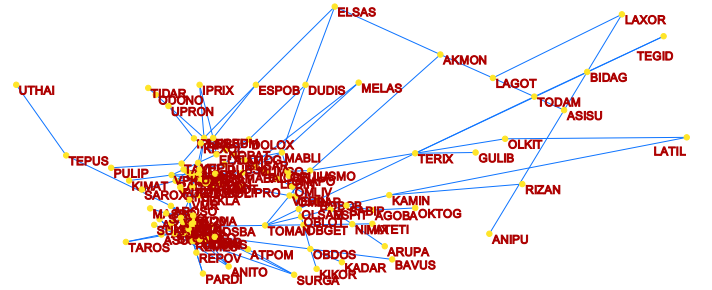


Fig. 3. Airway structure of SAN.

Since the bottleneck links can be different at different time, in this case study, we identify the bottleneck links for every 30 minutes. Therefore, for air traffic data every 30 minutes, we calculate the normalized average flight speed of flights on each link which will be set as the weight of the link describing the traffic efficiency of the link. In this way a weighted airway network will be constructed for every 30 minutes and Fig. 4 shows an example of the weighted airway network.

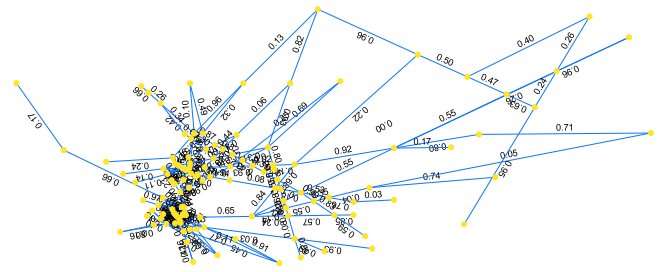


Fig. 4. A weighted airway network representation of SAN. The weight on each link is the normalized average flight speed of flights on this link.

#### B. Network Percolation

Upon constructing the weighted network for every 30 minutes time period, network percolation process is then applied to identify the percolation threshold  $q_c$  of the network.

In this case study, we increase  $q$  by 0.01 for each iteration. As  $q$  increases, the network will gradually break into several clusters as more and more low efficiency links are closed, therefore the size of the largest cluster will decrease consequently. When  $q$  reaches the percolation threshold, a slight increase of  $q$  will disintegrate the largest cluster into two or more large clusters and we will observe a distinct decrease of

the size of the largest cluster and increase of the size of the second largest cluster, as the example shown in Fig. 5.

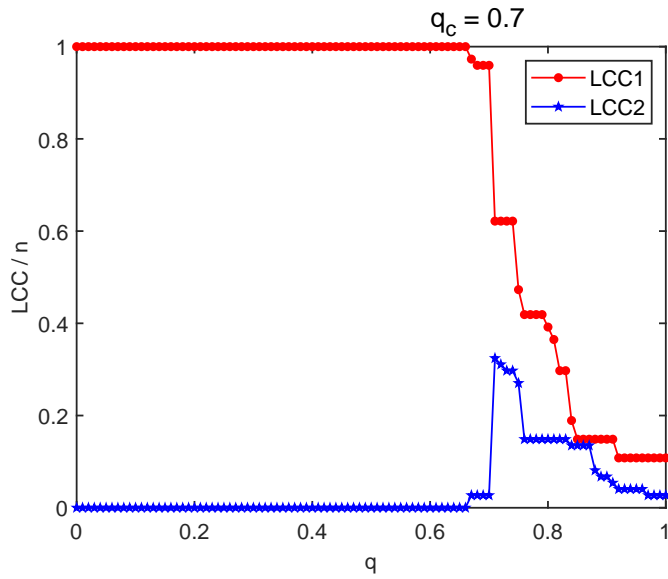


Fig. 5. Size of the largest cluster (LCC1) and the second-largest cluster (LCC2) of airway networks as a function of  $q$ . At the percolation threshold  $q_c$ , LCC1 is broken into several isolated clusters because the closure of some links. Consequently, there is a sudden decrease and sudden increase of LCC1 and LCC2 respectively. The network transits from a largely connected phase to a disconnected phase at  $q_c$ .

Fig. 6 and Fig. 7 shows the disintegration process of the network at and after the percolation threshold, i.e.,  $q$  equals to and slightly exceed  $q_c$  respectively. It can be observed that, at the percolation threshold, because of closure of the airway link from fixes "VISAT" to "MABAL", the original largest cluster (the blue cluster shown in Fig. 6) is broken into two large clusters (the red and green clusters shown in Fig. 7). Flights are able to travel to most nodes in the airway network (the blue cluster shown in Fig. 6) with normalized speed below  $q_c$ , while flights will be trapped in small isolated clusters (the red and green clusters shown in Fig. 7) when they fly with normalized speed above  $q_c$ .

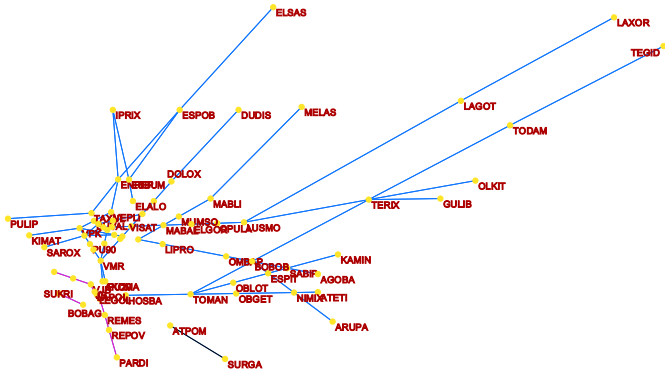


Fig. 6. Airway network at percolation threshold.

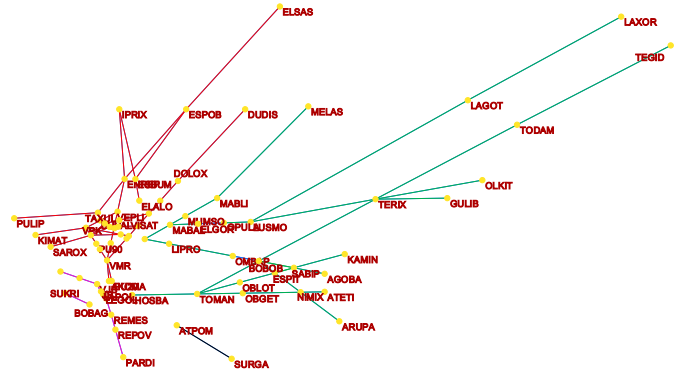


Fig. 7. Airway network after percolation threshold.

### C. Bottleneck Links Identification

Since the closure of links depends on the efficiency of the links and links with lower normalized average flight speed will be closed earlier than other links, the closure of links actually means the malfunction of the links when flights travel with higher speed, which limit the travel efficiency of flight in the airway network. The influence of the malfunction (closure) of links can be different. Many links can be closed without distinctly reducing the global connectivity of the airway network while the links closed at the percolation threshold disintegrate the network into large local clusters which substantially limits the travel efficiency of flights. Therefore links that are closed at the percolation threshold and connecting large local clusters can be recognized as critical links, such as link "VISAT" to "MABAL" in the example case.

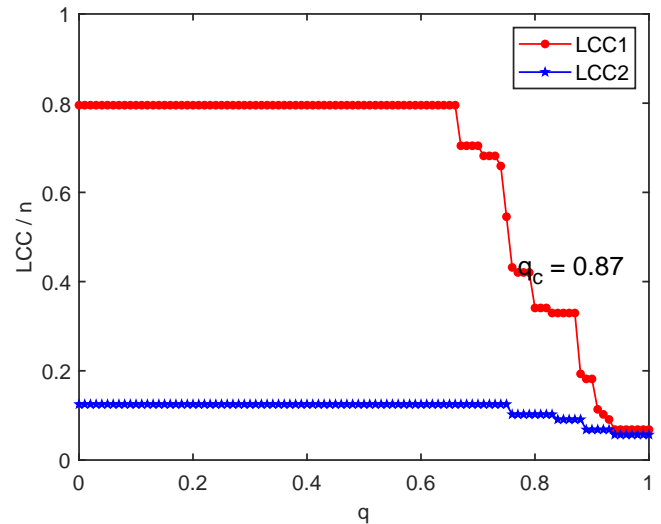


Fig. 8. Percolation curve of cases under which no bottleneck links can be identified. There is no distinct increase of LCC2 and decrease of LCC1. Therefore no evidence shows that the malfunction of links can largely reduce the connectivity of the airway network.

We apply the proposed identification method to the 1 month air traffic data on an every 30 minutes basis to identify the bottleneck links. Note that during some time periods, there

doesn't exist critical links, as shown in Fig. 8. It can be seen that both the size of the largest cluster and second largest cluster decrease slowly as  $q$  increases without distinct decrease or increase. In such cases, there are no links whose malfunction will substantially isolated the global air traffic into local clusters. Therefore no bottleneck links can be identified under such cases. Thus, we only identify bottleneck links for cases whose percolation curve shows distinct increase in  $LCC2$  and decrease in  $LCC1$ , i.e. distinct decrease of the difference between the size of  $LCC1$  and  $LCC2$  at  $q_c$ .

We use  $d$  to represent the decrease in the difference between the size of  $LCC1$  and  $LCC2$  at  $q_c$ . A higher  $d$  indicates a more distinct increase in  $LCC2$  and decrease in  $LCC1$ . When we set a threshold of 0.5 for  $d$ , i.e., the decrease in the difference between the size of  $LCC1$  and  $LCC2$  at  $q_c$  is required to be above 0.5 as a qualification for the existence of bottleneck, the top 10 frequent bottleneck links are shown in Fig. 9. The red links are the bottleneck links identified and the thickness of the links are proportional to their frequency.

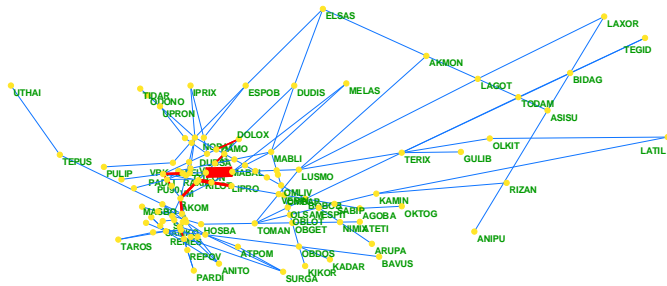


Fig. 9. Top 10 frequency bottleneck links. The red links are the bottleneck links identified and the thickness of the links are proportional to their frequency.

To explore the influence of different threshold of  $d$  over the frequency and the significance of different bottleneck links, besides 0.5, we also set the threshold of  $d$  as 0.3 and 0.4 respectively. The frequency of the top 10 links are shown in Fig. 10. The green, red and blue bars refer to frequency of critical links identified with thresholds of  $d$  as 0.3, 0.4 and 0.5 respectively. As the threshold of  $d$  decreases, the frequency of bottleneck links increases. It makes sense since under lower threshold of  $d$ , more cases of different time periods will be qualified for bottleneck links identification and as a consequence, the frequency of bottleneck links will increase. It can also be observed that the 10 links and their sequence are all remain unchanged in accordance to different settings of the threshold of  $d$ . The link "VISAT" to "MABAL" has the highest frequency and is distinctly higher than other links, which likely requires the special attention of air traffic controllers.

## VI. CONCLUSION

In this paper, we proposed a data driven method for identification of airway network bottleneck links based on percolation theory. The proposed method comprises three components: network modelling, network percolation and bottleneck links identification. The key idea is to determine the links whose

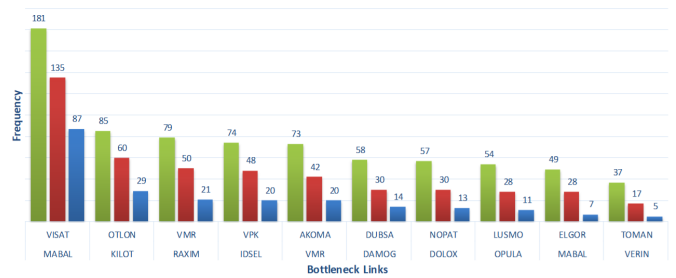


Fig. 10. The frequency of the top 10 bottleneck links identified according to different threshold of  $d$ . The green, red and blue bars refer to frequency of critical links identified with thresholds of  $d$  as 0.3, 0.4 and 0.5 respectively.

malfunction will isolate the global air traffic into local clusters. We then carried out a case study over Singapore airway network using one month air traffic from Dec. 1, 2018 to Dec. 31, 2018. Results show that different bottleneck links exist under different time periods. Some of the critical links possess a high frequency and one of them is distinctly higher than the rest which requires special attention of air traffic controllers. This proposed method enables us to identify the airway links connecting different air traffic clusters of higher flight speed (with respect to the bottleneck). These bottleneck links identified provide opportunities to improve significantly the global air traffic with minor cost (e.g., improving the operation on a single airway link). Understanding the congestion formation and dissipation mechanisms through our proposed method in a network view can serve better prediction and control of air traffic.

## ACKNOWLEDGMENT

This research is supported by the Civil Aviation Authority of Singapore under the Aviation Transformation Programme.

## REFERENCES

- [1] F. Netjasov, "Framework for airspace planning and design based on conflict risk assessment: Part 1: Conflict risk assessment model for airspace strategic planning," *Transportation Research Part C: Emerging Technologies*, vol. 24, pp. 190–212, 2012.
- [2] H. A. Blom, J. Krystul, and G. Bakker, "A particle system for safety verification of free flight in air traffic," in *Proceedings of the 45th IEEE Conference on Decision and Control, San Diego, CA, USA*. IEEE, 13-15 December, 2006, pp. 1574–1579.
- [3] "2036 forecast reveals air passengers will nearly double to 7.8 billion," <https://www.iata.org/pressroom/pr/Pages/2017-10-24-01.aspx>, accessed on January 28, 2019.
- [4] D. Delahaye and S. Puechmorel, "Air traffic complexity based on dynamical systems," in *49th IEEE Conference on Decision and Control (CDC), Atlanta, GA, USA*. IEEE, 15-17 December, 2010, pp. 2069–2074.
- [5] H. Balakrishnan, "Control and optimization algorithms for air transportation systems," *Annual Reviews in Control*, vol. 41, pp. 39–46, 2016.
- [6] "European air traffic control delays loom over summer air travel," <https://www.iata.org/pressroom/pr/Pages/2018-07-18-01.aspx>, accessed on January 28, 2019.
- [7] W.-B. Du, M.-Y. Zhang, Y. Zhang, X.-B. Cao, and J. Zhang, "Delay causality network in air transport systems," *Transportation research part E: logistics and transportation review*, vol. 118, pp. 466–476, 2018.
- [8] M. Ball, C. Barnhart, M. Dresner, M. Hansen, K. Neels, A. Odoni, E. Peterson, L. Sherry, A. Trani, and B. Zou, "Total delay impact study: a comprehensive assessment of the costs and impacts of flight delay in the united states," 2010.

- [9] A. S. A. Alochukwu and M. O. Olusanya, "On modeling and optimisation of air traffic flow management problem with en-route capacities." Ph.D. dissertation, 2016.
- [10] J. Nosedal, M. A. Piera, S. Ruiz, and A. Nosedal, "An efficient algorithm for smoothing airspace congestion by fine-tuning take-off times," *Transportation Research Part C: Emerging Technologies*, vol. 44, pp. 171–184, 2014.
- [11] P. Dutta, S. G. Park, and P. Menon, "In-flight synthesis of optimal en-route trajectory option sets," in *17th AIAA Aviation Technology, Integration, and Operations Conference, Denver, Colorado, 5-9 June 2017*, p. 3433.
- [12] S. Grabbe and B. Sridhar, "Congestion management with an aggregate flow model," in *AIAA Guidance, Navigation, and Control Conference and Exhibit, San Francisco, California, 15-18 August 2005*, p. 6277.
- [13] D. Chen, M. Hu, H. Zhang, J. Yin, and K. Han, "A network based dynamic air traffic flow model for en route airspace system traffic flow optimization," *Transportation Research Part E: Logistics and Transportation Review*, vol. 106, pp. 1–19, 2017.
- [14] C. Hurter, Y. Brenier, J. Ducas, and E. Le Guilcher, "Cap: Collaborative advanced planning, trade-off between airspace management and optimized flight performance: Demonstration of en-route reduced airspace congestion through collaborative flight planning," in *2016 IEEE/AIAA 35th Digital Avionics Systems Conference (DASC), Sacramento, CA, USA*. IEEE, 25-29 Sept. 2016, pp. 1–9.
- [15] D. Bertsimas, G. Lulli, and A. Odoni, "An integer optimization approach to large-scale air traffic flow management," *Operations research*, vol. 59, no. 1, pp. 211–227, 2011.
- [16] L. Huaxian, Z. Yanbo, C. Kaiquan, and P. Qingge, "Route network flow assignment in the new generation of aviation by cooperative co-evolution," in *2011 IEEE 5th international conference on cybernetics and intelligent systems (CIS), Qingdao, China*. IEEE, 17-19 Sept. 2011, pp. 175–180.
- [17] Y. Su and C. Kaiquan, "A multi-objective multi-memetic algorithm for network-wide conflict-free 4d flight trajectories planning," *Chinese Journal of Aeronautics*, vol. 30, no. 3, pp. 1161–1173, 2017.
- [18] M. Schultz, I. Gerdes, T. Standfuß, and A. Temme, "Future airspace design by dynamic sectorization," in *ENRI International Workshop on ATM/CNS, Nakano, Japan*. Springer, 14-16 November, 2017, pp. 19–34.
- [19] P. Kopardekar, K. Bilimoria, and B. Sridhar, "Initial concepts for dynamic airspace configuration," in *7th AIAA ATIO Conf, 2nd CEIAT Int'l Conf on Innov and Integr in Aero Sciences, 17th LTA Systems Tech Conf; followed by 2nd TEOS Forum, Belfast, Northern Ireland, 18-20 September 2007*, p. 7763.
- [20] J. Tang, S. Alam, C. Lokan, and H. A. Abbass, "A multi-objective approach for dynamic airspace sectorization using agent based and geometric models," *Transportation research part C: Emerging technologies*, vol. 21, no. 1, pp. 89–121, 2012.
- [21] X. Sun and S. Wandelt, "Network similarity analysis of air navigation route systems," *Transportation Research Part E: Logistics and Transportation Review*, vol. 70, pp. 416–434, 2014.
- [22] C.-I. Chen and W.-b. Du, "A multi-objective crossing waypoints location optimization in air route network," in *2011 3rd International Workshop on Intelligent Systems and Applications, Wuhan, China*. IEEE, 28-29 May 2011, pp. 1–4.
- [23] W. Shijin, C. Xi, L. Haiyun, L. Qingyun, H. Xu, and W. Yanjun, "Air route network optimization in fragmented airspace based on cellular automata," *Chinese Journal of Aeronautics*, vol. 30, no. 3, pp. 1184–1195, 2017.
- [24] K.-q. Cai, J. Zhang, and C. Zhou, "Optimization of the crossing waypoints in air route network," in *29th Digital Avionics Systems Conference, Salt Lake City, UT, USA*. IEEE, 3-7 Oct. 2010, pp. 2–E.
- [25] C. Ma, Q. Cai, S. Alam, B. Sridhar, and V. N. Duong, "Airway network management using braess's paradox," *Transportation Research Part C: Emerging Technologies*, vol. 105, pp. 565–579, 2019.
- [26] A. Churchill, D. Lovell, and M. Ball, "Flight delay propagation impact on strategic air traffic flow management," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2177, no. 13, pp. 105–113, 2010.
- [27] P. Baumgarten, R. Malina, and A. Lange, "The impact of hubbing concentration on flight delays within airline networks: An empirical analysis of the us domestic market," *Transportation Research Part E: Logistics and Transportation Review*, vol. 66, pp. 103–114, 2014.
- [28] R. Zallen, *The physics of amorphous solids*. John Wiley & Sons, 2008.
- [29] Q. Cai, M. Pratama, S. Alam, C. Ma, and J. Liu, "Breakup of directed multipartite networks," *IEEE Transactions on Network Science and Engineering*, 2019, DOI: 10.1109/TNSE.2019.2894142.
- [30] R. Albert and A.-L. Barabási, "Statistical mechanics of complex networks," *Reviews of modern physics*, vol. 74, no. 1, p. 47, 2002.
- [31] F. Morone and H. A. Makse, "Influence maximization in complex networks through optimal percolation," *Nature*, vol. 524, no. 7563, pp. 65–68, 2015.
- [32] S. Boccaletti, J. Almendral, S. Guan, I. Leyva, Z. Liu, I. Sendiña-Nadal, Z. Wang, and Y. Zou, "Explosive transitions in complex networks' structure and dynamics: Percolation and synchronization," *Physics Reports*, vol. 660, pp. 1–94, 2016.
- [33] S. B. Amor, T. D. Huy, and M. Bui, "A percolation based model for ac simulation," in *RIVF*, 2006, pp. 17–22.
- [34] D. Li, B. Fu, Y. Wang, G. Lu, Y. Berezin, H. E. Stanley, and S. Havlin, "Percolation transition in dynamical traffic network with evolving critical bottlenecks," *Proceedings of the National Academy of Sciences*, vol. 112, no. 3, pp. 669–672, 2015.
- [35] M. Barthélemy, "Spatial networks," *Physics Reports*, vol. 499, no. 1-3, pp. 1–101, 2011.