Macroscopic Fundamental Diagram for Air Traffic: Preliminary Theoretic Results and Simulation Findings

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***Abstract*—In this paper we derive a model of air traffic system performance that describes the relationship between the occupancy of aircraft in the airspace and the effective output of the system – the Macroscopic Fundamental Diagram (MFD) model. We define the necessary variables, analytically derive the functional form of the MFD and validate the model using BlueSky open-source air traffic simulator.**

Keywords-air traffic control; air traffic management; airspace performance metrics; MFD; demand; capacity; simulation.

I. Introduction

Today's air traffic management system, designed decades ago, is a highly centralized apparatus geared initially towards serving a relatively low volume of air traffic. In such a system, airspace is divided into hierarchical sections, which significantly lowers the cost of coordination and communication between aircraft and human controllers, compared to the system in which individual pilots coordinate with each other. On the other hand, the research community has amassed a large body of knowledge and technology that, in principle, could enable a decentralized system, where communication and separation between the aircraft could be done entirely automatically, or with less supervision on the part of human controllers. Some of the benefits of decentralization include reduced burden imposed upon human control, flight time, and delay, and increased airspace capacity. However, operating such systems gives rise to the need for a different approach to air traffic management, and new understanding of the dynamics of en-route air traffic.

Capacity – broadly speaking, an upper limit on the amount of traffic that can be safely and efficiently accommodated by given physical and control infrastructure – is one of the most important characteristics of transportation systems. The existing definition of en route air traffic capacity was criticized by researchers for being too simplistic and not representing the real capabilities of the system, with proposals of alternative definitions and models to characterize future automated, or free-flight, systems (e.g., [7]). As an example, Sunil et al. ([1]) have developed an alternative capacity model, in which airspace performance is defined through the number of airborne conflicts and intrusions. In their model, larger occupancy of aircraft in an airspace sector leads to an increase in the number of conflicts, up to a certain level where the number of conflicts grows exponentially. This level of occupancy is defined as capacity.

In this paper, we aim to achieve a similar but more ambitious goal. We define, for the first time in the air traffic management literature, a set of airspace performance metrics which are consistent with state variables in conventional traffic flow theory for road traffic: aircraft occupancy, flow of air traffic, effective flow of air traffic, space-mean ground speed, and effective speed of air traffic. Based on these state variables, a theoretical model that describes how the traffic flow variables scale with the respect to each other and compatible with any existing autonomous separation assurance system (ASAS) algorithm is developed. For example, we would like to describe how the average effective speed of flights changes as we increase the number of flights in an airspace sector.

To achieve this goal, we derive the relationship between two fundamental variables that characterize traffic conditions—flow and occupancy—called the Macroscopic Fundamental Diagram (MFD). MFD is a type of aggregated traffic model that was extensively described by Daganzo [2], but for the case of urban street networks. In the existing MFD model for road traffic, city streets are divided into a set of small relatively homogenous regions (sectors), each defined by the function that relates trip output and vehicle occupancy – the MFD function. The existence of the relationship was shown analytically and empirically for cities such as Yokohama and San Francisco. The MFD function is determined by the specifications of control tools within each region, such as timing of traffic lights and offsets between them. In the last several years the approach was applied to the control of airport surface operations [3] and modelling of crowd behavior [4].

Compared to road traffic, air traffic differs in significant ways –airspace is 2- or 3-dimensional; aircraft are allowed to avoid collisions using rerouting and changing altitude if necessary, with no traffic lights in the literal sense of the word. Despite these differences, we argue that in the airspace the effective flow (output) of an air traffic system and the average speed of flights in the system can be described by a concave unimodal function of the number of flights present during a time period and in an airspace area that corresponds to the system, which we designate as the air traffic MFD function. To show this, we first define the output of an air traffic system and the average speed of flights in the system with variables that are appropriate for flight operations and somewhat different from those used in the context of surface transportation. Then, by making simplifying assumptions we derive an analytical approximation to the MFD function. Finally, we validate the model using an agent-based simulation of en route air traffic. As this research is still on-going, we conclude this paper by describing possible directions for subsequent explorations.

The main setting of the paper is simple. We start by defining an airspace sector of high-altitude airspace with through traffic that enters and exits via the boundaries of the sector. We consider that the distribution of origin-destination pairs is such that the density of aircraft in the sector is on average the same everywhere. The aircraft that are scattered throughout the airspace approach each other resulting in airborne conflicts. To avoid dangerous intrusions, conflicting aircraft take collision avoidance maneuvers. Such maneuvers cause aircraft to deviate from their desired paths, causing en route inefficiency. The more aircraft in the airspace, the higher the rate of conflicts per aircraft. The higher the rate of conflict, the more time aircraft spend in the airspace sector. The more time the aircraft spend in the airspace, the higher the average occupancy, and the larger number of conflicts. Our task in this paper is to develop an analytical model that approximates the relationship between the number of conflicts, aircraft occupancy, and traffic flow (flight output) for an air traffic system.

II. Describing Airspace Performance

We begin by introducing a set of definitions that are used to describe the aggregate state of air traffic within given space and time. These definitions are necessary in the derivation of the MFD function.

Definition 1. A *sector* S is part of the airspace that is defined either as an area (in 2D case) or a volume (in 3D case).

The area or volume of S is denoted as A. In this paper, we focus on flights, whose trajectories traverse the sector; the rest of the flights are excluded. The airspace performance metrics that we define below are, as a result, aggregates over sector S.

Definition 2. An *analysis time window* t is the period of analysis specified by the researcher.

An analysis time period (e.g., one day, week, or year) is divided into several subperiods (e.g., hours, minutes, quarter-hours) of equal lengths T. Traffic occupancy, density, flow and speed are calculated across this time window. For example, a typical time window in air traffic management is 15 minutes long. This mean that the 24-hour day is divided into 96 subperiods. For each 15-minute time window we measure airspace performance metrics that are defined below.

Definition 3. *Air traffic occupancy* of sector S over analysis time window t, denoted as n_t, is the total amount of time the incumbent flights spent in S during t divided by the length of t (denoted by T).

$$n_t = \frac{\sum_j t^j}{T} \tag{1}$$

where t^j is the time of flight j spent in sector S within time window t. J is the set of incumbent flights whose coordinates were measured to be located within S at least once during t. Obviously, the amount of time spent by flight j in S during t is bounded by T:

$$0 \le t^j \le T, \forall\, j \in J \tag{2}$$

Under this definition, traffic occupancy is equivalent to counting the average number of flights that had presence in a sector over a time unit. Thus, computing n_t requires knowing the time of entry and exit of each flight to/from sector S. Figure 1 illustrates the computation of traffic occupancy during a time window of $T = 10$ min with three flights. In the example, each line corresponding to a flight indicates the entry and exit times of the flight with respect to the sector of interest. The difference among the three flights in the amount of time spent in the sector is due to the differences in the points of entry and exit, flying speed, and route geometry. Applying Eq. (2), the traffic occupancy can be calculated as:

$$n_1 = \frac{6+3+2}{10} = 1.1 \text{ aircraft} \quad (3)$$

Definition 4. *Traffic density* of sector S over analysis time window t, denoted as k_t, is the ratio of the corresponding occupancy of flights to the volume/area of the sector A:

$$k_t = \frac{n_t}{A} = \frac{\sum_i t_i}{TA} \quad (4)$$

In other words, traffic density is the average traffic occupancy per unit area (2D case) or volume (3D case) of the analyzed airspace. For example, if the airspace of interest is a cube with side length of L, then traffic density will be $n(t)/L^3$.

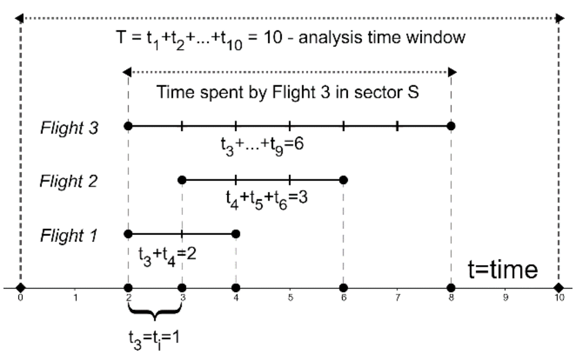

Figure 1. Illustration for calculating traffic occupancy in a sector over a time window of 10 min.

Definition 5. *Traffic flow* in sector S over analysis time window t, denoted as Q_t, is the total distance flown by all flights within the sector per unit time during the period covered by t. More specifically, Q_t is the total distance flown divided by the length of time window:

$$Q_t = \frac{\sum_j d^j}{T} \quad (5)$$

where d^j represents the travel distance within the sector and time window of interest.

The above definition of traffic flow draws directly upon traffic flow theory related to surface transportation. In surface transportation, traffic flow on a road link is equal to total distance driven on that link in time window *t* divided by the length of the link. Figure 2 provides an example that shows how Q_t is computed. Suppose the time window length is $T = 5$ min during which one flight is observed in the sector of interest. For this flight, four points along its trajectory are recorded: O and D are entry and exit points of the flight with respect to the sector. C and B are two intermediate points. For each point, the longitude, latitude, and time information is provided. Distances are measured in km. Since there is only flight, Q_t is computed as:

$$Q_t = \frac{2+1+1.5}{5} = 0.9 \text{ aircraft-km/min} \quad (6)$$

Note that Q_t is not equal to the speed of the aircraft, since the aircraft might have taken less time to traverse S than the length of the analysis time window t, for example 2 minutes instead of 5 minutes.

In surface transportation, traffic flow is not the primary output metric. Rather, the number of completed trips per unit time is of more interest, which can have a different distribution from traffic flow due to the differences in demand distribution and trip lengths for vehicles traversing a sector.

Besides the above difference, another distinction between surface and air transportation is that surface traffic is tied to a road grid. The primary method to avoid conflicts are stopping and waiting, and slowing down. In air traffic, separation assurance is of ultimate importance. Avoiding potential conflicts in the air is primarily done through rerouting: a flight takes evasive maneuvers horizontally and/or vertically. The 2- or 3-D airspace as opposed to 1-D space for a road link suggests much more flexibility for aircraft maneuvering. Because of this, unlike in surface traffic, air traffic flow could appear invariant to the number of flights present in the air: as occupancy increases, the total flow increases proportionally to the space-average speed of the aircraft in a sector. This follows from the fact that total flow is equal to the product of aircraft occupancy and average ground speed: $Q_t = \bar{v} \cdot n_t$. If $\bar{v}$ is constant, then total flow increases proportionally to occupancy.

To explicitly account for the flight maneuvering for conflict avoidance, a new term, effective traffic flow, is introduced.

Definition 6. *Effective traffic flow* in sector S over analysis time window t, denoted as Q_t^E, is the total *achieved* distance flown over all flights within the sector per unit time during the period covered by t. Achieved distance for a flight during a flight leg observed during time period t is the difference between the great circle distance from the position of the flight to the destination at the start and end of the flight leg:

$$Q_t^E = \frac{\sum_j (d_s^j - d_e^j)}{T} \quad (7)$$

where d_s^j is the great circle distance of flight $j \in J$ to its destination at the beginning of flight leg t. d_e^j is the great circle distance of flight $j \in J$ to its destination at the end of the flight leg. As a result, Q_t^E captures the useful distance that all incumbent flights have flown towards their destinations, after netting out aircraft rerouting and airborne holding.

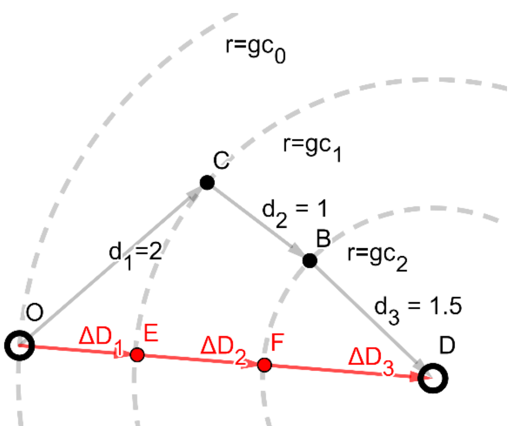

Figure 2. Illustration of computation of flow variables.

We further divide Q_t^E by the average achieved distance traveled (which is the average shortest path length through S, or the average distance between the entry point into S and the exit point from S) of all incumbent flights through sector S. This leads to the average flight output per unit of time (Q_t^{OUT}):

$$Q_t^{OUT} = \frac{Q_t^E}{\overline{AD}} \quad (8)$$

where $\overline{AD}$ is the average great circle distance (shortest path) between the entry and exit points of S across all flights. For example, if $Q_t^E = 100\ \frac{\text{flight-km}}{\text{min}}$, and the average shortest path length through S is equal to 100 km, we can say that the average output of the system is equal to $\frac{Q_t^E}{100\ km} = 1\ \frac{\text{flight}}{\text{min}}$.

We can again use Figure 2 to see the differences between physical and achieved distances. The only flight in the figure flies from O to D. The actual trajectory is depicted by the sequence of three grey arrows. The total distance flown is $d_1 + d_2 + d_3$. The beginning of each arrow is associated with a measurement of flight position. The sequence of the three red arrows represents the corresponding achieved distances. Points E and F, which are in the middle of the three consecutive arrows, are constructed by connecting O and D by a line and intersecting the line with two concentric circles that are centered on D with radii of |CD| and |BD|. The achieved distances corresponding to d_1, d_2, and d_3 are ΔD_1, ΔD_2, and ΔD_3. Total achieved distance flown, $\Delta D_1 + \Delta D_2 + \Delta D_3$, is the great circle distance between O and D. It is obvious that $d_i \geq \Delta D_i, i = 1,2,3$ with the equality being true only when flying between two concentric circles is strictly along a radius of the outer concentric circle.

It is possible that due to aircraft performing airborne holding or rerouting, $\Delta D_i < 0$ between two consecutive measurements, i.e., the flight is even farther away from its destination. For the similar reason of airborne holding or rerouting, the sum of multiple ΔD_i's from consecutive measurements can be zero.

Definition 7. The *average ground speed* of air traffic in sector S over analysis time window t, denoted as $\bar{v}_t$, is the ratio of traffic flow in the sector over t to the corresponding traffic occupancy:

$$\bar{v}_t = \frac{Q_t}{n_t} = \frac{\sum_j d^j}{\sum_j t^j} \quad (9)$$

In other words, $\bar{v}_t$ is the ratio of total distance traveled by all aircraft in the sector to the total time spent by the aircraft in the sector.

Definition 8. The *average effective speed* of air traffic in sector S over analysis time window t, denoted as $\bar{v}_t^E$, is the ratio of effective traffic flow the sector over t to the corresponding traffic occupancy:

$$\bar{v}_t^E = \frac{Q_t^E}{n_t} = \frac{\sum_j (d_s^j - d_e^j)}{\sum_j t^j} \quad (10)$$

The definitions of $\bar{v}_t$ and $\bar{v}_t^E$ are analogous to the generalized definitions of average harmonic speed for surface traffic based on the classic traffic flow theory (for example, see [8]). As flight operators often prefer to maintain an optimal speed that minimizes the sum of fuel cost and flight time cost, $\bar{v}_t$ is expected to vary only little. Most air traffic control will be done via changing aircraft heading, which includes rerouting and airborne holding. On the other hand, $\bar{v}_t^E$ is the speed after adjustment for rerouting, and therefore capture only the useful part of speed that brings flights closer to their destinations.

TABLE I. LIST OF VARIABLES AND IMPORTANT RELATIONSHIPS FOR A 2-DIMENSIONAL SECTOR.

Variable	Variable Name	Units
A	Sector Area	L^2
t^j	Time observed between two trajectory points	T
T	Analysis time window length	T
n	Average Occupancy	$veh.$
k	Average Density	$veh \cdot L^{-2}$
Q, Q^E	Flow (Effective Flow)	$veh. \cdot LT^{-1}$
$\bar{v}, \bar{v}^E$	Av. Speed (Effective Speed)	LT^{-1}
d^j	Distance Flown	L
d_s^j, d_e^j	Great Circle Distance from Destination at Start and End of Flight Leg	L
$\overline{AD}$	Average Length of the Shortest Path Through Sector	L
Q^{CONF}	Conflict rate per flight	$conf \cdot T^{-1}$
Q^{LOOK}	inflow of vehicles into lookahead volume, per flight	$veh. \cdot T^{-1}$
δ	Additional distance flown per conflict	$L \cdot conf^{-1}$
C	Conflict Constant	$conf \cdot L \cdot veh^{-1}$

$Q = \bar{v}n;\ Q^E = \bar{v}^E n; n = k \cdot A$

$L - length, T - time, veh. - vehicles, conf. - conlicts.$

III. DERIVATION OF MFD

In the previous section, we introduced a set of definitions that are used to describe the aggregate state of air traffic. Using these definitions along with some simple assumptions, in this section we derive an approximate model that relates the air traffic state variables to each other. As we said before, this model bear resemblance to the MFD for surface transportation network.

Let us start by describing our problem setting. Suppose that each flight in sector S searches for potential conflicts within a certain pre-defined look-ahead volume of the "downstream" airspace. The look-ahead volume could be the same as the volume of the sector itself or smaller. For each flight that entered the look-ahead volume, the ASAS algorithm determines whether this flight and the flight under study will be in conflict. If a conflict is identified, the flight under study will perform a collision avoidance maneuver, causing an en route inefficiency characterized. More specifically, we assume that maneuvering reduces the achieved distance by a length of δ. In the simplest case, the conflict may be triggered if the intruder flight is projected to be within a certain distance of the "avoider" flight. However, the details of conflict avoidance are not important to the model: we simply say that there is an algorithm that determines whether a conflict is present. The key is that the rate of conflicts for a flight is proportional to the *total traffic flow* in its look-ahead volume for a given time window.

To reiterate: the model does not assume decentralized conflict resolution. First, we hypothesize that this, or a similar, model can be applied to any conflict resolution setting. Second, we hypothesize that each conflict resolution method will generate its own unique MFD function that will not be transferable to another conflict resolution method.

The total flow of traffic per unit perimeter length/surface area of the sector boundary is equal to the product of ground speed and density: $Q_T = \bar{v}k$, which is derived based on the fact that $Q = Q/A$, $Q = \bar{v} \cdot n$, and $k = \frac{n}{A}$. As a result, the *total inflow of flights into the look-ahead volume*, Q_T^{look}, is obtained by multiplying the perimeter/surface area of the boundary of the sector and per-unit area total flow:

$$Q^{LOOK} = PQ \tag{11}$$

The unit of Q^{LOOK} is flights per unit time. P is the perimeter/surface area of the look-ahead volume. For example, if $Q = 0.01 \frac{\text{flights}}{\text{minute}\cdot\text{km}^2}$, and $P = 100 \text{ km}^2$, then $Q^{LOOK} = 1 \frac{\text{flight}}{\text{minute}}$.

Next, the *rate of conflicts* Q^{CONF} is considered a fixed proportion of the inflow air traffic into the look-ahead volume:

$$Q^{CONF} = GPQ \tag{12}$$

where G is the proportionality constant, determined by the ASAS algorithm. For example, if $G = 0.1\ conflicts/vehicle$, then on average 10% of all flights entering the look-ahead zone will be in a conflict with the ownship. In principle, G can also be a function of Q^{LOOK}. But for simplicity, here we assume that G is constant. Since both G and P are constant, their product is also a constant, termed as $C = GP$.

The *inefficiency of a flight per unit time per flight*, denoted as I, is set to be the product of the inefficiency caused by one conflict and the rate of conflicts:

$$I = \delta Q^{CONF} = \delta CQ = \delta C\bar{v}k \tag{13}$$

To compute the total inefficiency over all flights in sector S over time window t, I in Eq. (13) needs to be multiplied by the corresponding traffic occupancy.

Then, by subtracting the total inefficiency from the total traffic flow Q, the effective traffic flow is obtained as shown in Eq. (14), which presents the formula that describes the MFD:

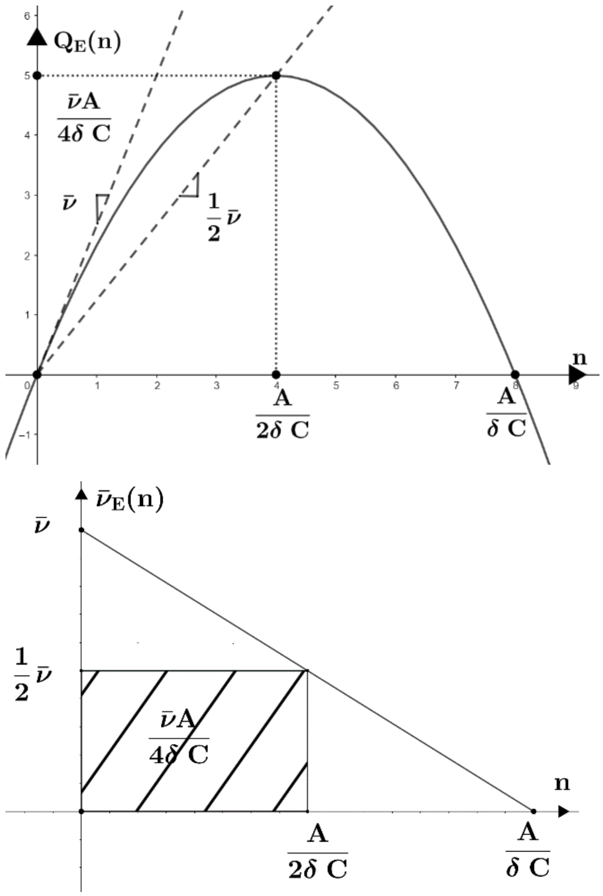

Figure 3. Effective flow-density (top) and effective speed-density (bottom) macroscopic fundamental diagrams. The shaded area is the total area – product of effective speed and density at that point on the diagram, which corresponds to theoretical maximum flow.

$$\begin{aligned} Q^E(n) &= Q - I \cdot n \\ &= \bar{v}n - \delta C \bar{v} k n \\ &= \bar{v}n - \frac{\delta C \bar{v} n^2}{A} \end{aligned} \tag{14}$$

The last equality comes from the fact that traffic density is equal to traffic occupancy divided by the volume of the sector: $k = \frac{n}{A}$.

According to this equation (Eq. (14)), the effective traffic flow of the system – here a system consists of flights which spent some time in the sector of interest over an analysis time window – is a concave quadratic function of the number of traffic occupancy of the sector. This can be easily verified as the second derivative of the right-hand side of Eq. (14) is negative.

Using Eq. (14), we can further obtain the relationship between the average effective speed of the system as a function of the traffic occupancy (Eq. (15)). This is done by dividing both sides of Eq. (14) by traffic occupancy $n(t)$, according to the definition of $\bar{v}_E$:

$$\bar{v}^E(n) = \bar{v} - \frac{\delta C \bar{v} n}{A} \tag{15}$$

Figure 3 illustrates the traffic flow-occupancy (top) and average effective speed-occupancy (bottom) diagrams. According to the quadratic MFD equation, the capacity of the system is equal to the maximum of the flow-occupancy function, i.e., $\frac{\bar{v}A}{4\delta C}$. The corresponding traffic occupancy is $\frac{A}{2\delta C}$. The average effective speed at the capacity is equal to half of the average speed. The flow on the average effective speed-occupancy diagram is determined by finding the product of traffic occupancy (density) and the effective speed (shaded rectangle in Figure 3).

Note that the parameters involved in the above description of air traffic states, including δ, C, $\bar{v}$, and A are not necessarily constants. In practice, these parameters may be stochastic functions of the level of traffic occupancy, demand distribution, and weather. It can be shown that under certain conditions, such as positive non-decreasing δ and $C \geq 0$, the traffic flow—occupancy function is a unimodal concave function of occupancy of air traffic.

Furthermore, recall that the ASAS algorithm used in the analysis determines the rate of conflicts for each flight per unit time, and the inefficiency associated with each conflict. Different models may be developed for different ASAS algorithms (for example, with non-constant $\bar{v}$), but we expect the essence of the model to remain the same – the average effective speed of flights in an air traffic system is a decreasing function of the traffic occupancy.

IV. SIMULATION EXPERIMENTS

This section presents how we validate the proposed MFD model. A series of computational experiments are designed that use BlueSky air traffic simulation framework [5]. BlueSky simulator is a real-time agent-based simulation that allows us to model self-separating free-flow air traffic. The automatic separation of aircraft is achieved using the so-called modified voltage-potential algorithms, where the aircraft are modeled as charged particles trying to repel from each other (MVP)[6]. MVP separation has several steps. First, the algorithm projects the location of all aircraft in the airspace within several time steps, given that the aircraft maintain their heading and velocity. Then, the model determines pairwise conflicts between the aircraft. If two aircraft are expected to be within some distance of each other that violates the pre-set separation requirements, a conflict is identified. MVP calculates a new vector for each aircraft that allows them to avoid all conflicts.

The focus of this paper is not the details of a particular separation assurance system, but modelling of the aggregate airspace performance given variables such as aircraft ground speed, air traffic occupancy, the amount of available airspace, and air traffic input rate. By decentralized self-separation we mean that aircraft separation is maintained autonomously by pilots, as opposed to the existing systems in which air traffic controllers possess authority to safely separate the aircraft. The BlueSky MVP provides us with realistic aircraft performance and separation models, which makes it ideal for validation of the MFD model.

To demonstrate how the model describes air traffic performance as airspace demand increases, we set up four simulation scenarios with identical distribution of airspace demand – the origin destination pairs and flight schedules are the same in all simulations. The analysis area is a two-by-two degree square located at FL300. The duration of each simulation is five hours. For each hour, we set the aircraft inflow rate, which increases in steps over time: 50, 200, 400, 500, 700 flights per hour. All origin and destination pairs are randomly generated on the boundary of analysis region, which conforms with the assumption that air traffic density is uniform everywhere. The MVP auto-separation algorithm uses several input parameters that determine the behavior of individual aircraft: horizontal separation, vertical separation, look-ahead time, and others.

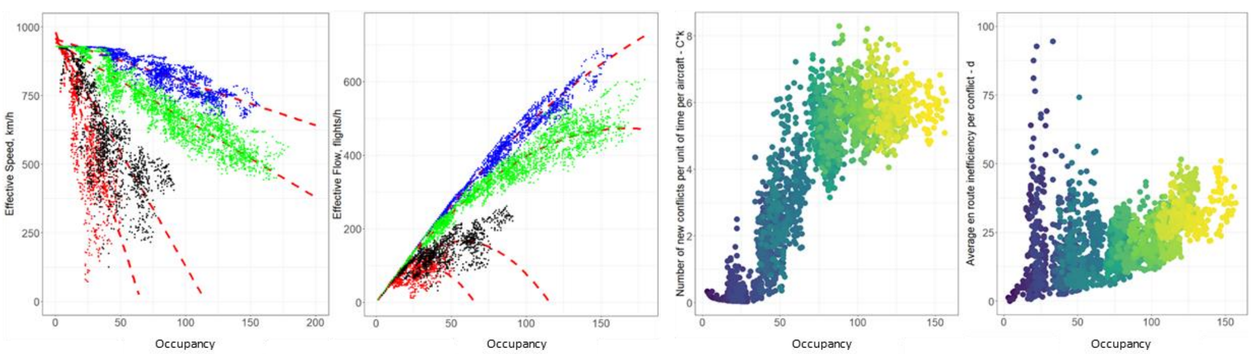

Figure 4. Effective speed (left) and effective flow (center-left) as a function of aircraft occupancy in sector. Colors indicate different horizontal separation minima. Red color – 10 nm, black – 7.5 nm, green – 5 nm, blue – 3.5 nm. Red dashed lines – estimated linear MFD functions. Number of conflicts per unit of time per aircraft $Ck(t)$ (center-right), en route inefficiency per conflict δ (right). The color of the dots refers to the aircraft input rate. Yellow- highest input rate, dark blue – lowest input rate.

For this demonstration, we utilize the simplified version of MVP algorithm. First, conflict avoidance is performed only by changing aircraft heading, with no consideration of changing aircraft velocity. Second, vertical conflict resolution is turned off – all flights remain at 30,000 feet, traveling at the speed of ~480 knots (900 km/hr). Third, in each simulation we vary only the horizontal separation requirements to demonstrate its effect on the MFD: we set the horizontal separation to 3.5, 5, 7, and 10 nm for the four simulations respectively.

The simulation results are presented in Figure 4, which plots the average effective speed (left) and effective traffic flow (right) as a function of traffic occupancy. To further demonstrate the overall trends of the relationships, separate linear regression models are estimated one for each horizontal separation value. For the average effective speed-traffic occupancy diagram, the regression lines show the overall trends as linear relationships. For the effective flow-traffic occupancy diagram, instead of estimating the ordinary least squares regression, parameters are estimated based on quantile regression, due to the presence of significant heteroskedasticity. Quantile regression allows us to estimate the median model, as well as models based on other quantiles. In Figure 4, the regression models are fit to the median. The estimated parameters of the quantile regression can be interpreted as parameters of the MFD function. The intercept of the line is the ground speed estimate $\hat{\bar{v}}$, the slope of the regression line is the product of three parameters: $\widehat{\delta C \bar{v}}$. Table 1 reports the model estimates of the linear regression lines (dashed red lines on the left of Figure 4). Additionally, we show the estimates of capacity ($Q_{\max}$) and traffic occupancy at capacity ($n^{Q_{\max}}$) for each simulation.

TABLE II. SUMMARY OF MFD MODEL PARAMETER ESTIMATES FOR THE FOUR SIMULATIONS WITH DIFFERENT HORIZONTAL SEPARATION MINIMA.

Horizontal Separation	$\hat{\bar{v}}$	$\widehat{\delta C \bar{v}}$	$n^{Q_{\max}}$	Q^{max}
10 nm	980.8	-14.9	32.8	96.6
7.5 nm	958.7	-8.3	57.7	166.0
5 nm	955.7	-2.8	169.0	484.7
3.5 nm	944.5	-1.6	300.7	851.9

Note: Q^{max} is measured in units of the number of flights per hour.

$Q_{\max}$ is in the number of flights per hour, assuming average flight length of 50 km, $\bar{v}$ is in km/hour.

As can be observed in Figure 4, the variance of the state estimates is high and grows as traffic occupancy increases. Nonetheless, it is clear that the average effective speed is a decreasing function of traffic occupancy. Similarly, the average effective flow is a concave unimodal function of occupancy. The peak of this function corresponds to the theoretical capacity of the system.

Figure 5 demonstrates the behavior of parameters that we used in the MFD model: the number of new conflicts per unit time per aircraft, the average en-route inefficiency per conflict, and the average inefficiency per unit time per flight. In the previous section, it was assumed that the rate of conflict is a linear function of traffic density, and the inefficiency is constant. However, the simulation results suggest that this is not the case – the aerodynamic limitations of the aircraft and the MVP algorithm settings are such that the number of conflicts per aircraft reaches a peak and stops growing. The inefficiency per conflict keeps growing but slowly. The product of the two, i.e., $\delta Ck(t)$, is growing approximately linearly, which is in accordance with our initial assumptions.

V. FUTURE WORK AND CONCLUSION

In this paper, a new set of definitions for traffic flow, occupancy, and speed is introduced to characterize the aggregate air traffic states which are consistent with conventional traffic flow theory. These definitions are used to derive a simple model of airspace performance, which relates the output of the system or the effective speed with the amount of aircraft in the airspace sector.

Such models can be used in a variety of application – airspace capacity estimation, air route assignment, traffic management initiative (TMI) planning in case of adverse weather events, as well as TMIs in the context of urban air mobility with thousands of aircraft sharing the same airspace. For example, MFD-derived capacity can be used as acceptance rate in TMIs such as AFP and CTOP. Effective speed and effective pace functions can be used to approximately estimate trajectory costs, such as airborne delay and excess fuel consumption. In UAM context, the model can be used to design input control methods similar to those used in urban street networks.

Our on-going work focuses on further development of the model, which introducing other definition of airspace performance measures, relaxing model assumptions, and testing how other ASAS algorithms affect the results. It should be emphasized that the model presented in this paper is not limited to self-separating air traffic systems – in the existing centralized system the air traffic behavior is consistent with the MFD model can be observed empirically. As a result, the model can be instrumental in the existing ATM decision-making process that relies on centralized air traffic separation.

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