Constructing a Passenger Trip Delay Metric
An Aggregate-level Approach

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Abstract—The on-time performance of passenger trips has received a great attention from government agencies in recent years but lacks a systematic metric to measure or trace the impact of flight delay to air travelers. The proposed model considers possible trip types of a passenger, utilizes system-wide flight-based performance metrics, and employs statistical approaches in order to develop an aggregate delay metric from passenger's perspective. Its results can be used to analyze historical passenger schedule reliability and can also be used to predict passenger experience for future aviation system.

Keywords—delay, passenger trip, performance metric, air travel

I. INTRODUCTION

The on-time performance of flights is a key concern of carriers and administrative agencies of aviation worldwide. It can be easily quantified for the U.S. National Airspace System (NAS) because all flight arrival and departure information is well recorded and disclosed by the Federal Aviation Administration (FAA). For example, the FAA’s Aviation System Performance Metrics database (ASPM) provides individual flight information from all participating carriers at 75 major U.S airports. Arrival delay of flights can thus be calculated by comparing scheduled and actual arrival time [10]. With suitable aggregation methods, delay metrics at airports or at the NAS-wide level can easily be constructed.

While flight delay statistics are well-recorded and well-publicized, they are not necessarily an accurate measure of a passenger’s level of satisfaction. In particular, a passenger’s average trip delay can vary substantially from average flight delay due to trip disruptions due to cancelled flights or missed connections. Bratu and Barnhart [1] analyzed proprietary airline data and indicated that the average time penalty on passenger trip time due to flight cancellations and missed connections is 303 minutes, while the average delay for non-disrupted passengers was only 16 minutes. However, acknowledging that passenger delay is also an important factor of system performance, it is not easily measurable from any publicly accessible data. Since ticket information is not released by airlines nor collected by the government due to privacy concerns, the delay of multiple-leg passenger trips can be traced only with great difficulty. Even through proper sampling and survey techniques, passenger delay can only be observed during the selected survey period. Considering a long term objective of quality assurance of air travel, it would appear that there exists a need for defining a passenger oriented metric to be used as a quantitative measure of system-wide flight delay impact on passenger trips.

There is limited research that models passenger delay most likely because of the relative unavailability of individual passenger trip information. The essential challenge is to quantify the impact of flight delay on passenger trip disruption. Wang [3] treated passenger delay by its causes: delay due to delayed flights and due to cancelled flights. To estimate the passenger delay from cancelled flights, an algorithm was proposed that processed single-segment flight data. The underlying idea was to assign cancelled seats to the temporally closest available flights. Intuitively, this approach should work well in cases where only direct flights are being considered or under the assumption that on multi-leg passenger trips, the passenger always maintains the same intermediate stopping point. It should also be noted that this research does not model the possibility of missed connections on multi-leg flights.

The common characteristics of our paper and Wang [3] are: 1) both develop passenger-based performance metrics, and 2) both quantify the impact of flight cancellations on passenger delays. However, while the Wang model is a detailed “microscopic” model that estimates delays at a flight level, our model is macroscopic scope, attempting to directly estimate overall averages. The FAA’s NAS Strategy Simulator (NSS) is a high-level policy analysis tool that predicts the impacts of future demand growth, policy changes, increasing fuel price, etc. [7]. Our research was specifically aimed at producing a performance module for the NSS. In the NSS context, all input
and output data are maintained at an aggregate level and so it is assumed that flight-level data are not available. Likewise, the required output should be NAS-wide average flight delay and cancellation rates rather than similar flight-specific metrics.

This paper is organized as follows. In Section 2, the concepts of the proposed model are discussed, and statistical methods are performed to estimate the probability of missing a connection flight. Numerical examples are constructed to illustrate the model, and the trend of passenger delay since 2000 is presented in Section 3. In Section 4, sensitivity analysis is conducted by analyzing the impact of key parameters on passenger delay. In Section 5, the potential usages and limitation of the proposed model are discussed.

II. MODEL CONSTRUCTION AND ESTIMATION

Passenger delays can be “inherited” directly from delayed flights but also can result from cancelled flights. Further, on multi-leg passenger trips, long flight delays on the initial leg can result in missed connections and induced delays not equal to, or even proportional to the original flight delay. In fact, cancellations and missed connections very often result in the most severe passenger delays. With these effects in mind, it can be seen that passenger delays depend on:

- Distribution of flight delays
- Flight cancellation rate
- Average load factor
- Percentage of passengers with 2 or more flight legs in their itinerary

In order to accurately address the actual delay experienced by passengers, models and statistical analysis are required that transform statistics related to these factors to passenger delay measures.

A. Scenario Tree Model of Passenger Delay

Our passenger delay model employs in a fundamental way, the concept of a disrupted passenger, which was introduced in Bratu and Barnhart [1]. A disrupted passenger is a customer who must use a flight other than the one on which the customer was originally scheduled due to a missed connection or flight cancellation. Disrupted passengers incur delays not related in a direct way to the delays on any of the flights in their original itinerary. Such passengers might be able to recover quickly, e.g. by taking the “next” flight scheduled to the missed destination or might incur a very long delay, e.g. requiring an unplanned overnight stay.

In order to model passenger delays, we create a scenario tree that represents all possible outcomes of a passenger’s trip. The database of Airline Origin and Destination Survey (DB1BMarket) contains directional market characteristics of each domestic itinerary of the quarterly Origin and Destination Survey [11]. The trip leg information of domestic markets from 2000 to 2007 is summarized in Figure 1, indicating that on average over 97% of the passengers chose direct or two-leg flights. Thus, because of the relative infrequency of three or more leg trips in the U.S., we will represent itineraries as consisting of either one or two flight-legs.

Figure 1. BTS Survey Results on Passenger Trip Leg Information

Our scenario tree is given in Figure 2. It represents the various events that can occur on a passenger itinerary, where for a 1-leg trip, the flight is denoted by \( f_1 \) and for a 2-leg trip the first flight is \( f_1 \) and the second is \( f_2 \). Each leaf of the scenario tree represents a different outcome of a passenger trip and leads to a different “type” of passenger delay.

Expected passenger delay could be computed by computing the expected passenger delay at each leaf node in this tree and the probability of reaching each leaf node. The sum of the product of the leaf node probabilities times their expected delays would give the expected passenger delay. This would accurately compute expected passenger delay given the restriction to one and two leg trips. This is the approach we take; however, we must make several approximations in order to estimate the various probabilities and expectations. We hope that over time some of these approximations can be improved.

In computing our estimate of passenger delay, we use the following quantities:

- **P\_DIRECT**: the fraction of passenger itineraries that are direct flights
• P_CANCEL: the fraction of scheduled flights that are canceled
• F_DELAY: Average flight delay
• DISRUPT: Average delay of disrupted passengers
• P_MISS: An estimate of the probability that a passenger misses connecting flight (the method for computing this estimate is discussed in the next section)

We now list all leaf nodes in the scenario tree, give our approximations of the expected passenger delay at that node and the probability of reaching that node, and discuss the accuracy of these approximations.

The various possibilities that can arise are:

1) Direct Trip, \( f_1 \) canceled:

Probability estimate: \( P_{DIRECT}*P_{CANCEL} \)

Delay estimate: DISRUPT

Discussion: The probability estimate is fairly accurate; however, \( P_{CANCEL} \) is actually a surrogate for the probability that a passenger is booked on a canceled flight. To the extent that there is a greater propensity for airlines to cancel flights with fewer passengers, a more accurate estimate could be obtained by doing a calculation that weights flights by the number of passengers (or seats). In fact there is not source of accurate statistics on the delay of disrupted passengers so the value we use for DISRUPT is a very rough estimate. Further, models could take into account whether a passenger is disrupted by a cancellation or a missed connection. DISRUPT also would be impacted by changes in airline policies and flight characteristics, such as load factor, so these could be used in improving estimates.

2) Direct Trip, \( f_1 \) not canceled:

Probability estimate: \( P_{DIRECT}*(1-P_{CANCEL}) \)

Delay estimate: F_DELAY

Discussion: Subject to the caveats related to \( P_{CANCEL} \) mentioned above, both the probability estimate and the delay estimate should be highly accurate in this case.

3) Two-leg Trip, \( f_1 \) canceled:

Probability estimate: \( (1-P_{DIRECT})*P_{CANCEL} \)

Delay estimate: DISRUPT

Discussion: See discussion for previous two cases.

4) Two-leg Trip, \( f_1 \) not canceled, \( f_2 \) canceled:

Probability estimate: \( (1-P_{DIRECT})*(1-P_{CANCEL})\) * P_CANCEL

Delay estimate: DISRUPT

Discussion: See discussion for previous two cases.

5) Two-leg Trip, \( f_1 \) not canceled, \( f_2 \) not canceled, connection made:

Probability estimate: \( (1-P_{DIRECT})*(1-P_{CANCEL})*(1-P_{CANCEL}) \)

Delay estimate: F_DELAY

Discussion: As will be discussed later, estimating \( P_{MISS} \) can be very challenging. Our approach is to estimate the probability that flight delay exceeds a certain (constant) threshold. Clearly the required connection time varies substantially by flight so in reality the required threshold itself is a random variable. Further, it can be the case that both \( f_1 \) and \( f_2 \) are delayed so that even with a large delay on \( f_1 \) the connection can be made. Assuming the connection is made the passenger delay equals the delay on \( f_2 \) so that \( F_{DELAY} \) is a good estimate of passenger delay in this case.

6) Two-leg Trip, \( f_1 \) not canceled, \( f_2 \) not canceled, connection missed:

Probability estimate: \( (1-P_{DIRECT})*(1-P_{CANCEL})*(1-P_{CANCEL})*P_{MISS} \)

Delay estimate: DISRUPT

Discussion: See discussion in previous case regarding \( P_{MISS} \). As discussed earlier it is certainly the case that the expected delay experienced by a disrupted passenger could vary depending on whether a canceled flight or missed connection was involved.

Based on this scenario tree and the preceding analysis, our estimate of average passenger delay, Pax_DELAY can be computed as:

\[
Pax\_DELAY = (P_{DIRECT})*(P_{CANCEL})*DISRUPT + (P_{DIRECT})*(1-P_{CANCEL})*F\_DELAY + (1-P_{DIRECT})*(P_{CANCEL})*DISRUPT + (1-P_{DIRECT})*(1-P_{CANCEL})*(1-P_{MISS})*F\_DELAY + (1-P_{DIRECT})*(1-P_{CANCEL})*(1-P_{CANCEL})*P_{MISS}*DISRUPT
\]

B. Probability of Passenger Missing Connection

Three of the inputs in the Pax_DELAY equation, i.e. F_DELAY, P_CANCEL and P_DIRECT, can be easily obtained from historical NAS performance statistics. For example, the monthly flight arrival delay and cancellation rate for the NAS can be calculated from ASPM individual flight data; the percentage of direct trips can be estimated from the quarterly market survey provided by the Bureau of Transportation Statistics, as shown in Figure 1. However, two inputs, i.e. DISRUPT and P_MISS, require reasonable approximation or further modeling efforts since they are not readily available in any data sources or previous research.

In order to provide a reliable estimate of P_MISS, we conduct a statistical analysis on the composition of P_MISS. If we denote by \( D_f \) the random flight delay, then we define our estimate of the probability that a connection is missed because of a delayed flight by:
\[ \text{P_MISS} = \text{Prob} \{ \text{Df} > \text{Threshold} \} \]

where \( \text{Threshold} = \text{LAY} - \text{CONNECT} \), \( \text{LAY} \) is a nominal flight layover time for connecting flights, and \( \text{CONNECT} \) is an estimated minimum time required to connect between two flights.

We assume that schedules are created so that if a flight arrives “on-time” then it makes its connection. Here on-time is defined relative to the U.S. Department of Transportation standard so that a flight is not classified as delayed if it is no more than 15 minutes late. Thus, if \( \text{Df} \) is less than or equal to 15 minutes, then we assume the passenger makes the connection successfully to the second flight leg. The probability of passenger missing connecting flight can thus be modeled as a conditional probability. Specifically, the probability that the connection is missed “given that” the flight is delayed (more than 15 minutes) is represented as:

\[
\frac{\text{Prob} \{ \text{Df} > \text{Threshold} | \text{Flight being Delayed} \}}{\text{Prob} (\text{Df} > 15)} = \frac{\text{P_MISS}}{\text{P_DELAY}}
\]

where \( \text{P_DELAY} \) = the probability that a flight’s delay > 15 = \( \text{Prob}(\text{Df} > 15) \). The probability of missing a connecting flight can thus be represented as:

\[ \text{P_MISS} = \text{P_DELAY} \times \text{Prob} \{ \text{Df} > \text{Threshold} | \text{Df} > 15 \} \]

The first term is the probability that a flight is delayed more than 15 minutes. The second term is a conditional probability. \( \text{P_DELAY} \) can be estimated directly from flight delay data for the purposes of computing a metric. We also provide a way of estimating it using only an estimate of \( \text{F_DELAY} \). This was done in order to derive estimates for future years in the context of the FAA Strategy Simulator. Our approach to estimating the second term for a time period, e.g. one month, will be to estimate the distribution: \( \text{Prob} \{ \text{Df} > \text{D} | \text{Df} > 15 \} \) based on several years of historical data. The parameters of this distribution will be estimated as a function of \( \text{F_DELAY} \) and \( \text{PCancelar} \). The value of \( \text{Threshold} \) and these flight performance statistics for the time period in question will be plugged into the distribution function to determine the estimate of the second term.

### C. Probability of a Flight Being Delayed

As discussed above, \( \text{P_DELAY} \) can be computed directly from historical data. However we also provide a way of estimating it from flight delay statistic. From the ASPM database [10], for each month from January 2000 to December 2004, we computed the monthly values of \( \text{F_DELAY} \) and \( \text{PCancelar} \). Due to the obvious non-linearity in distribution functions, we postulated a quadratic relationship between \( \text{F_DELAY} \) and \( \text{PCancelar} \). A simple regression produced the following model with an \( R^2 \) of 0.9628.

\[
\text{P_DELAY} = [ -0.0206 \times \text{F_DELAY} \times \text{F_DELAY} + 2.0431 \times \text{F_DELAY} ] / 100
\]

### D. Estimating Conditional Distribution of Flight Delays

In this section we describe our approach to estimating the conditional distribution function: \( \text{Prob} \{ \text{Df} > \text{D} | \text{Df} > 15 \} \). Individual flight information stored in APSM database was used to compute the arrival delay of flights, which is defined as the difference between actual and scheduled arrival time. For each of month, an empirical distribution of flight delays > 15 was created. Specifically, each flight delayed over 15 minutes was placed into a 15 minute bin (15-30, 30-45, etc.) based on its delay value.

Empirical flight delay distributions were obtained in this way for each month from January 2000 to December 2004. These distributions were then fitted with the Bi-Weibull distribution. The Bi-Weibull, which is a combination of two Weibull distributions, is widely used in reliability applications. The Bi-Weibull distribution assumes a different form based on its shape parameters, which are:

- \( x_0 \): the point at which the parameters change, and
- \( (\alpha_1, \beta_1) \) and \( (\alpha_2, \beta_2) \): the parameters of the two Weibull distributions.

The parameter \( \beta_2 \) is a function of the other parameters, so there are four parameters in total to be estimated.

The fitted distributions gave 60 sets of observations of \( (x_0_0, \alpha_1, \beta_1, \alpha_2) \). A regression was performed on each of these parameters, respectively, by using independent variables \( \text{F_DELAY} \) and \( \text{PCancelar} \). The results from the regression are as follows:

- \( x_0 = 11.1081 + 0.014 \times \text{F_DELAY} \times \text{F_DELAY} + 741.87 \times \text{PCancelar} \) \( R^2 = .93 \)
- \( \alpha_1 = 0.37 + 0.00083 \times \text{F_DELAY} \times \text{F_DELAY} + 3.2 \times \text{PCancelar} \times \text{PCancelar} + 0.0032 \times \text{F_DELAY} \) \( R^2 = .87 \)
- \( \beta_1 = 11 + 2.83 \times \text{F_DELAY} + 112.12 \times \text{PCancelar} \) \( R^2 = .901 \)
- \( \alpha_2 = 0.1143 + 0.0013 \times \text{F_DELAY} \times \text{F.Delay} + 0.87 \times \text{PCancelar} \times \text{PCancelar} \) \( R^2 = .82 \)

Thus, the distribution \( \text{Prob} \{ \text{Df} > \text{D} | \text{Df} > 15 \} \) was estimated as a Bi-Weibull distribution whose parameters are given as functions of \( \text{F_DELAY} \) and \( \text{PCancelar} \).

### III. Model Application and Data Analysis

The passenger delay model takes into account several major factors that impact passenger delay. Some model inputs are the results of aforementioned statistical models; some are available from reliable data source or analysis.

As the market survey results on trip leg information from 2000 to 2007 shown in Figure 1, it is observed that on average two-thirds of the passengers take direct flights. Hence, for model application purposes, we set \( \text{P_DIRECT} = 66\% \).

Disrupted passengers might be re-assigned to a later flight and often experience overnight stays. There are no publicly
available data about average delay of disrupted passengers. The research results of Bratu and Barnhart [1] based on a combination of proprietary data and simulation provide an estimate of 303 minutes as the average delay of disrupted passengers. Hence we set

\[
\text{DISRUPT} = 303 \text{ minutes.}
\]

The delay threshold of not missing a connection flight is the difference between the average flight layover time and minimum required connection time. Calculating average layover time experienced by a passenger requires detailed analysis on either passenger itinerary information or flight schedule along with seat information, which are not publicly accessible. The minimum required connection time can differ among individual airlines or even airports. Therefore, we take a conservative estimate on these two inputs based on empirical experience and assume that \( LAY = 45 \) minutes and \( CONNECT = 15 \) minutes. Thus, the delay threshold of not missing connecting flight is:

\[
\text{Threshold} = LAY – CONNECT = 30 \text{ minutes.}
\]

We now have provided models, estimation methods or approximation to obtain all required inputs for our metric. We use a simple example summarized in Table 1 to show how the passenger delay metric is computed. Given that Monthly NAS delay is 13.62 minutes and cancellation rate is 3.08\%, the probability of a flight being delayed as well as the parameters of flight delay distribution is determined. The probability of flight delay more than connection threshold is computed by using the fitted distribution. As a result, the probability of missing a connection flight is 0.113, and the estimated monthly average passenger trip delay is 35.95 minutes. The relation among major model components is shown in Figure 3.

**TABLE I. A NUMERICAL EXAMPLE OF PASSENGER DELAY MODEL**

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg Monthly NAS Delay</td>
<td>13.62 mins.</td>
<td>Historical data or estimated from other models</td>
</tr>
<tr>
<td>Monthly NAS Cancellation Rate</td>
<td>3.08%</td>
<td>Historical data or estimated from other models</td>
</tr>
<tr>
<td>( P_{DIRECT} )</td>
<td>66%</td>
<td>BTS DB1B Database</td>
</tr>
<tr>
<td>DISRUPT</td>
<td>303 mins.</td>
<td>Result from Bratu’s study</td>
</tr>
<tr>
<td>Threshold</td>
<td>30 mins.</td>
<td>Assumed</td>
</tr>
<tr>
<td>( P_{DELAY} )</td>
<td>24%</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>( x_0 )</td>
<td>36.55</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>0.57</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.35</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>49.65</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>( \beta_2 = x_0^2a_1(\alpha_2 – \alpha_1) )</td>
<td>22.96</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>( P_{MISS} )</td>
<td>0.1134</td>
<td>Estimated by this study</td>
</tr>
<tr>
<td>Pax_DELAY</td>
<td>35.95 mins.</td>
<td>Calculated by using scenario tree formula</td>
</tr>
</tbody>
</table>

Given the application procedures in Figure 3, monthly passenger delay metrics from January 2000 to May 2007 are computed by using ASPM flight delay and cancellation data. Figure 4 shows the time series of monthly passenger delay against flight delay and cancellation rate. Most of the spikes of passenger delay trend are due to high cancellation rates in those months as more passengers are disrupted. This suggests that there will be large penalty for passengers in terms of delay-minutes whenever a flight is cancelled, and also provides an explanation for why passenger experience varies from year to year as the overall cancellation rates change.

![Figure 4. Time Series of Passenger Delays](image-url)

The comparisons of modeled passenger delay against cancellation rate and average flight delay in the NAS are plotted in Figures 5 and 6, respectively. It can be also seen that as flight delay increases the passenger delay increases in more than a linear fashion. This validates our claim that as flight delays increase, more passengers are disrupted and the impact on passenger delays is much worse than actual flight delays.
IV. SENSITIVITY ANALYSIS

As one of the modules in a high-level policy analysis tool, our model is designed to use flight performance statistics and to evaluate the passenger trip experience in response to changes in aviation system. The creditability of our model relies on proper inputs of parameters, either processed from historical data or calibrated from other modeling efforts. To better understand how passenger delays correspond to average flight delay, sensitivity analysis is conducted by varying the values of several key parameters. The parameters of our base scenario are summarized in Table 2.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_CANCEL</td>
<td>2%</td>
</tr>
<tr>
<td>P_DIRECT</td>
<td>66%</td>
</tr>
<tr>
<td>DISRUPT</td>
<td>300 mins.</td>
</tr>
<tr>
<td>Threshold</td>
<td>35 mins.</td>
</tr>
</tbody>
</table>

Figure 7 illustrates the relation between flight delay and passenger delay with increasing values of DISRUPT, which is the average delay of disrupted passengers. Certainly, DISRUPT is the most difficult to estimate input parameter. We see that P_DELAY increases with DISRUPT but that the sensitivity is fairly modest and the functional relationship between F_DELAY and P_DELAY generally retains its structure as DISRUPT changes. Figure 8 provides a similar sensitivity analysis for P_DIRECT. Note that the nonlinear structure of the curves in Figures 7 and 8 results from the fact that the probability of missing connections increases more than linearly with average flight delay.
The multiplier effect of reducing Threshold on the probability of missing connection becomes more significant as flight delay increases. When system performance is getting worse, stringent connection times will increase the chances of missing connections and aggravate passenger trip delay. At 15 minutes of flight delay, the probability of missing connections with Threshold=40 is about 170% of that with Threshold=100. At 25 minutes of flight delay, the probability of missing connections with Threshold=40 is more than 220% of that with Threshold=100.

The results can not be validated because of the unavailability of comprehensive passenger trip records.

The proposed model uses NAS-wide performance metrics, i.e. average flight delay and cancellations, in order to measure passenger delay from a strategic perspective. The inputs of the passenger delay metric are obtained from historical data analysis, statistical models, and reasonable approximation. Its intention is to provide an efficient but dependable estimate of passenger schedule reliability without much effort on analyzing detailed flight activities. Using models that forecast NAS-wide performance metrics, e.g. the flight delay models in Wieland [8] and Subramanian [9] and the cancellation rate model in Subramanian [9], the results of this research can also be used to predict passenger experience of future aviation system.

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REFERENCES