Optimising the Predictability and Flexibility of Dynamic System: Case of 4D Aircraft Trajectory of Air Traffic Management

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***Abstract*—The 4D trajectory is envisioned as the kernel of the future Air Traffic Flow Management system. In this research, we propose an approach dealing with the predictability and flexibility of the system using 4D trajectory. A mix integer programming problem is proposed to minimize the deviation from actual flown 4D trajectory in relationship to the reference trajectory.**

Key words: Air Traffic Flow Management, 4D Trajectory, mix integer programming.

I. INTRODUCTION

In Air Traffic Management (ATM), punctuality is essential to the smooth operations for the safety of flights knowing that most flights are subjected to operational uncertainties due to quality of weather forecast and/or technical and logistics issues. Traditional ATFM systems are flight-based, i.e. the schedule are established with discrete events and determinist approach. Because of operational uncertainties, there exist gaps between scheduled and executed traffic [3]. Removal of these gaps can lead to a better use of airport resources and improve the punctuality of the system.

The SESAR (Single European Sky ATM Research) documents identified the sources of uncertainties and defined the future system based on the notion of 4D trajectory in order to reduce the uncertainties, increase the flight punctuality and satefy of flight,.... A 4D contract is a set of couple space-time $(O_1, t_1), ..., (O_n, t_n)$ where $O_1, .., O_n$ is the space coordinate of waypoints and t_i is the estimated time for arrival (ETA) at which aircraft must reach the waypoint O_i. The time constraint increases the predictability of the system and but degrades the flexibility of the system.

In SESAR, the Key Performance Areas called Predictability and Flexibility are defined as follows:

- Predictability is the ability of the ATM system to ensure a reliable and consistent of 4D Trajectory performance.i.e the ability to control the variability of the deviation between the actually flown 4D trajectory of aircraft in relationship to the Reference Business Trajectory
- Flexibility is the ability of the ATM system and airports to respond to "sudden" changes in demand and capacity: rapid change in traffic patterns, last minute notifications or cancellations of flights, change to the Reference Business Trajectory, late aircraft substitutions, sudden airport capacity changes, late airspace segregation request, weather, crisis situation, etc.

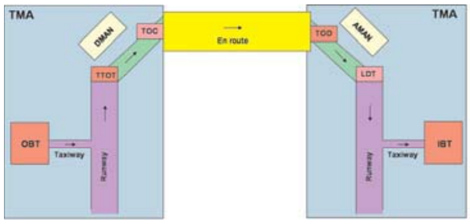

Fig. 1. Flight scheme

Typically a flight time is decomposed of three stages:

- Taxi-out time ($TOT - OBT$), the time elapsed between departure from the origin airport gate (Pushback time: OBT) and wheels off (Takoff Time: TOT). This stage is assisted by a Departure Manager which aimed at optimizing the sequence of traffic in the terminal area including the minimization of flight times by increasing the accuracy of estimated time of departure.
- Taxi-in time ($IBT - LDT$), time time elapsed between wheels down (Landing Time: LDT) and arrival at destination airport gate (In Block Time: IBT). This stage is assisted by an Arrival Manager which aimed at optimizing the sequence of arrival traffic based on available runway and preferred aircraft arrival time data.
- In-flight time ($LDT - TOT$), the total time an aircraft is in the air between an origin-destination airport pair, i.e from the wheels-off at the origin airport to wheels-down airport at the destination.

Amongst the three stages of flight, the predictability at the first and last stages are not good enough: Taxi out time and taxi-in time. Therefore improving the predictability at the first and the last stage of the flight are important. For the better use of airport resources (gates and runways) and aircraft, it is very important in ATFM to be able to predict the takeoff time from pushback time and top of descent from takeoff time for each flight. In other words, it is very important to be able to control the uncertainties at takeoff time and top of descent to optimise the available of runways and to schedule the next flight. Currently, the time window at pushback time is between

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fifteen minutes ahead of planned time and fifteen minutes after the planned pushback time. Similarly the time window at takeoff time is between five minutes prior to and ten minutes after the planned takeoff time. One may suggest that the large margin at pushback time (and takeoff time respectively) are the sources of the large uncertainties at the takeoff time (and Top of Descent time respectively). Therefore a reduction of time window at pushback time and departure time may have an positive impact on the uncertainties of the system.

The aim of this research is twofold:

- Study the influence of the time window at pushback time on the takeoff time and the impact of time window at takeoff time on top of descent time. Then we can reasonably modify the time window to reduce the uncertainties of the takeoff time and at the top of descent time.
- Build the predictible model for the top of descent. The predictible model has two important properties: (1) Predictible property, i.e. It will provides the top of descent time and (2) Flexible property, i.e. it must take into account the operational uncertainties

A. Related work

Previous work related to this research topic includes result obtained by [3]. The authors proved that systematically there are gaps between planned and executed traffic. This result means that it is impossible to eliminate the uncertainties in scheduling of flight. Concerning to the prediction takeoff time, the most common approach is use the statistical model to establish the probability distribution of departure delay then to deduce the taxi-out time. This approach used by [5] in their work as they used the queuing theory to estimate the taxi-out time. The authors identifed the takeoff queue size as an important factor affecting the takeoff time and validated the model by calculating the taxi-out time for each runway configuration while [10] focused on the distribution of departure time to deduce the model for the estimation takeoff time.

The research carried out by [8] used the Markov decision process to determine the optimal trajectory of multi-aircraft under uncertainty.

None of these studies had investigated the impact of the margin of pushback time on takeoff time and margin of takeoff time on the top of descent.

Our research is not to provide only the takeoff time predictibe model but also aimes at incorporating the "sudden change" i.e. we investigate a trade-off between the predictability and flexibility.

II. Modeling

A. Assumptions and Notations

To model this problem, our assumptions are as follows:

- Flight Plan: 4D Reference Trajectory including:
  - Origin airport, destination airport.
  - Scheduled departure time (TakeOff Time: TOT), Top of Descent Time (TOD).
  - 4D trajectory defined as a set of consecutives space-time, denote (O_1, t_1), ..., (O_n, t_n) where O is the space coordonates, and t is estimated time that the flight will reach O at t.
- The aircraft flies along the line between two waypoints with constant speed.

Notations:

- F is set of flights.
- For each flight f:
  - Q_f is the set of all possible routes.
  - For $r_f \in Q_f$, $(O_1^{r_f}, t_1^{r_f})$, ..., $(O_{n_{r_f}}^{r_f}, t_{n_{r_f}}^{r_f})$ is the set of consecutive waypoints. In our problem, t_1^f is the Takeoff time and $t_f^{f_n}$ is top of descent time, we can extend our model for further flight path but for the moment we will focus on this segment of flight. So n_f is the number of waypoints on the f's contract Trajectory.
- $v_i^{r_f}$ is the aircraft speed between two waypoints $O_i^{r_f}$ and $O_{i+1}^{r_f}$. $v_{max}^{r_f}$, $v_{min}^{r_f}$ are the boundaries of v_f. These values can be obtained from database of Aircraft performance(BADA) of EUROCONTROL Experimental Centre.
- $m_f(x_t, y_t, z_t, t)$ is the position of aircraft f at t.

From these notations, we have:

1) The flight f takes off at $t_{r_f}^1$ will reach the waypoints $O_2^{r_f}$, ..., $O_{n_{r_f}}^{r_f}$ at $\frac{d(O_1^{r_f}, O_2^{r_f})}{v_1^{r_f}}$, $\frac{d(O_{n_{r_f}-1}^{r_f}, O_{n_{r_f}}^{r_f})}{v_{n_{r_f}-1}^{r_f}}$.
2) The time elapsed from take off time to top of descent is $\sum_{i=1,..,n_{r_f}, r_f\in Q_f} \frac{d(O_i^{r_f}, O_{i+1}^{r_f})}{v_i^{r_f}}$. Then the delay time is $d_{r_f} = \sum_{i=1,..,n_{r_f}, r_f\in Q_f} \frac{d(O_i^{r_f}, O_{i+1}^{r_f})}{v_i^{r_f}} - (t_{n_f}^f - t_1^f)$.
3) The total delay for all flight is:

$$TD = \sum_{r_f\in Q_f, f\in F} d_{r_f} \quad (1)$$

$$= \sum_{r_f\in Q_f, f\in F} \left(\sum_{i=1,..,n_{r_f}, r_f\in Q_f} \frac{d(O_i^{r_f}, O_{i+1}^{r_f})}{v_i^{r_f}} - (t_{n_f}^f - t_1^f) \right). \quad (2)$$

B. Constraints

- Separation minimum: Two any aircrafts must be separated by a minimum distance.i.e $d_{f,g}(t) = ||m_f(x_t, y_t, z_t, t) - m_g(x_t, y_t, z_t, t)||_2 \geq r$, with r is minimum separation and $f, g \in F$. The real number r depends on the aircraft types: heavy, medium or light.
- Sector capacity: The number of aircrafts in the sector at any moment should not exceed the sector capacity. We define the indicator function $I_{fj}(t)$, whose value is 1 if the aircraft is in the sector j at t and 0 otherwise. This constraint can be represented by: $\sum_{f\in F} I_{fj}(t) \leq C_j(t)$, $\forall j$ and t

The goal is to minimize the total delay (TD).

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But we can not solve this problem of optimisation with real decision variables v_t.

We denote v^f_{ref} the reference speed of flight f. This speed depends on the aircraft characteristics like the engine types, the flight level,... This speed is used to establish the Reference Trajectory of flight f. We can rewrite the total delay under the form:

$$TD = \sum_{f\in F}(\sum_{i=1,..,n_{r_f},r_f\in Q_f} \frac{d(O^f_i, O^{r_f}_{i+1})}{v^f_{ref}} \frac{v^f_{ref}}{v^{r_f}_i} - (t^f_{n_f} - t^f_1)). \quad (3)$$

In order to keep the aircraft along the reference trajectory, we use the speed ajustement and alternative route as tools. Theorically, the ratio of real speed to reference speed can vary from 0.001 to infinity but in this framework this ratio is limited from 0.8 to 1.2. Note x^f_i is the integer number, $x^f_i \in 0, ..., 40$ and we can approximate the ratio of speed variation as $\frac{80+x^f_i}{100}$. The problem to solve is:

$$Min \sum_{f\in F}(\sum_{i=1,..,n_{r_f},r_f\in Q_f} \frac{d(O^f_i, O^{r_f}_{i+1})}{v^{r_f}_{ref}} \frac{80 + x^f_i}{100} - (t^f_{n_f} - t^f_1)). \quad (4)$$

under the constraints sector capacity and minimum separation.

To simplify the problem, we can divise the segment into two phases: climbing phase and cruising phase. During the cruising phase, we can suppose the ratio of speed variation within 0.95 to 1.05 and during the climbing phase, we limit this ratio of speed variation within 0.9 to 1.1.

- According to the simplification above, the speed variation during the cruising phase is limited between 0.95 and 1.05. The ratio can ve represented by $\frac{v^f_i}{v^f_{ref}} = \frac{95+\sum_{j=1,..,10} x^f_{ij}}{100}$ for $i = 2, ..., n_{r_f}$.
- This ratio during the climbing phase can be represented by $\frac{v^f_i}{v^f_{ref}} = \frac{90+\sum_{j=1,..,20} x^f_{ij}}{100}$, with $i = 1$.
- x^f_{ij} is the decision variables $0 - 1$. And $x^{r_f}_i = \sum_{j=1,..,10} x^f_{ij}$, with $i = 2, ..., n_{r_f}$ is the level of speed modification during cruising phase of flight f along the route r_f
- $x^{r_f}_i = \sum_{j=1,..,20} x^f_{ij}$, with $i = 1$ is the level of speed modification to minimize the deviation from the reference path

$$MinTD = Min[x + y] \quad (5)$$

where

$$x = \sum_{f\in F}(\sum_{i\geq 2,r_f\in Q_f} \frac{d(O^f_i, O^{r_f}_{i+1})}{v^{r_f}_{ref}} \frac{90 + \sum_{j=1,..,10} x^f_{ij}}{100} - (t^f_{n_f} - t^f_1)) \quad (6)$$

and

$$y = \sum_{f\in F}(\sum_{i=1,r_f\in Q_f} \frac{d(O^f_i, O^{r_f}_{i+1})}{v^{r_f}_{ref}} \frac{90 + \sum_{j=1,..,20} x^f_{ij}}{100} - (t^f_{n_f} - t^f_1)) \quad (7)$$

We have to solve the problem (5) under the constraints of sector capacity and minimum separation.

C. *Problem complexity*

This is a Integer programming problem, the constraints are sector capacity and minimum separation.

- For each flight we have: $\sum_{r_f\in Q_f} n_{r_f}$ decision variables
- Total decision variables: $\sum_{f\in F}(\sum_{r_f\in Q_f} n_{r_f})$
- This sum is majored by $R||F||||N||$ with $N = max_{r_f\in Q_f, f\in F} n_{r_f}$ and R is the number of possible routes that an aircraft can fly along of.

III. FUTURE WORK

The research has been initiated and real data is currently under investigating for inital testing of the model. In the Doctoral symposium the author wishes to be advised on the topics such as:

- Investigation of the current taxi-out time and taxi-in time.
- Simulation with different airports.
- Incorporation of the ”random walk” into model.
- The model will be validated by comparing the results obtained from simulation and real results recorded by radars.

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