Propagation of Airspace Congestion. An Exploratory Correlation Analysis

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***Abstract*—We analyze how large gaps between the planned and realized number of aircraft into flight sectors propagate through the European- and the Japanese Airspace. For this we analyze the sample cross-correlation matrix of the most congested part of the networks. Because of the motion of aircraft, gaps propagate to neighboring sectors, expecting positive correlation coefficients. The question in the analysis is whether there are unexpected coefficients. Such coefficients would be caused by traffic controllers or flow managers who compensate for strong gaps by re-routings or speed adjustments. Such strategies would often lead to negative correlation coefficients. Our results show that meaningful correlations appear on two levels: (i) locally, that is between a sector and its direct neighbors and (ii) globally on 'traffic highways', that is between sectors that are connected through a flight route with high traffic densities. This is true for both, the European- and the Japanese Airspace. Moreover, all correlations are positive and their time-lags correspond to the average travel times. No unexpected correlations have been found. We conclude that no systematic strategies to compensate strong delays are applied by controllers. The results are useful to justify predictive congestion models for future flow planning. They also give a first insight into how controllers deal with their workload, although a more detailed analysis is required to explore this topic.**

***Index Terms*—Flow analysis, correlation analysis**

I. Introduction

Airspace is divided into geographical regions, called sectors. A flight plan is a sequence $(S_1, t_1), ..., (S_n, t_n)$ of sectors S_i and entry times t_i in the sector. Due to uncertainties (weather conditions, congestion etc.), aircraft can deviate from their flight plans. [BLHM05] classify the major sources of uncertainty as

- Demand uncertainty: flights fail to meet planned departure, arrival or en-route travel times. Contributing factors are mechanical problems, boarding passengers or weather conditions.
- Capacity uncertainty: airport and airspace throughput levels vary. Contributing factors are weather conditions and changes in flight sequences that disturb scheduled departure or arrival spacing.
- Flow control uncertainty: actions are taken by the traffic controllers in response to demand and capacity uncertainty. Examples are re-routing, re-sectorization and temporary capacity limitations. The human element of decision making adds another layer of uncertainty to the whole system.

Deviations from flight plans lead to gaps between the planned and the real number of aircraft entering flight sectors. For example in the year 2004, 17.7 % of European flights departed- and 18.5 % arrived more than 15 min behind their schedule [EUR06].

Obviously, a gap between planned and real number of entries in a sector S in time slot t propagates to its neighboring sectors in slot $t+1$, because aircraft cannot stand still. On the other hand, pilots and air traffic controllers can compensate gaps by re-routing or speed adjustments of all aircraft.

In this article we analyze past flight data to see how such gaps propagate in reality through the airspace. Are there strategies of controllers to compensate the gaps successfully? We will look at (i) local propagation, that is propagation between a sector and its direct neighborhood and (ii) global propagation, i.e. between a sector and any other sector in the system. Based on such knowledge flow planning can be improved, because systematic gaps can be controlled, once their mechanisms are understood.

The article is divided into two parts and a conclusion: in the first part we explain the method and give some examples from literature. In the second part we report our results. We conclude with a critical comment and motivate future work.

II. Method and Related Work

We consider $Z_t = [Z_{1t}, Z_{2t}, ..., Z_{mt}]', t, Z_{it} \in \mathbf{Z}$ as a random process where Z_{it} represents the gaps between planned and realized number of aircraft entering sector i in time slot t. Our aim is to study the correlation structure of the process. Positive correlation between two sectors i and j in time slots t_1 and t_2 has the meaning that gaps above average in sector i and slot t_1 are associated with gaps above average in sector j and slot t_2. As mentioned above, we expect such correlations between neighboring sectors. But we are more interested in unexpected correlations in the real traffic data. For example, take a sector with a

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crossing of two routes. When a traffic peak on the first route is predicted to arrive at the sector some time ahead, the controllers could coordinate with sectors on the second route to re-route aircraft for compensation. Such a strategy would cause negative correlation between the sectors along the two routes. Likewise, vanishing correlation between two sectors, when conditioned on the value of other sectors, might help reveal network effects.

Related work from the ATM domain can be found in the analysis of flight data on a sector level: [WCGM03] and [WSZ$^+$05] analyze uncertainties in sector demand. One of their observations is that flow control actions against congestion are visible in the data, in cases that the predicted peak counts are greater than some alert value displayed in the Enhanced Traffic Management System (ETMS). [RSWB06] analyze radar data to identify traffic flows in the U.S. airspace. They define a flow as a cluster of aircraft with similar trajectory properties. A trajectory is a high-dimensional vector of geographical components. They apply several clustering techniques to the data. But even after enhancing the data set with additional features (e.g. aircraft type), they conclude that none of the algorithms provides satisfactory results for practical purposes. A correlation analysis of sector data, as proposed in our article, has not been identified in literature review. This might be due to the known difficulties in the interpretation of auto- and cross correlation coefficients [Ken89], [Dig90]. On the other hand, correlation analysis is the first step in an analysis of multiple time series, as for example applied to highway traffic prediction in [KP03]. In what follows, we analyze the risk of misleading coefficients in our data before visualizing the most interesting correlation patterns. This is exploratory work with the aim to generate new hypotheses about the phenomenon.

A. Inference for cross-correlation matrices

In this part we define the sample correlation matrix function between multiple time series and derive bounds for the variability of its coefficients.

1) Estimation: We use the standard estimators of lag-k crosscorrelation

$$\hat{\rho}_{ij}(k) = \frac{\hat{\gamma}_{ij}(k)}{[\hat{\gamma}_{ii}(0)\hat{\gamma}_{jj}(0)]^{1/2}}$$

with sample crosscovariance elements

$$\hat{\gamma}_{ij}(k) = \frac{1}{n-k}\sum_{t=1}^{n-k}(Z_{it}-\bar{Z}_i)(Z_{j,t+k}-\bar{Z}_j)$$

where

$$\bar{Z}_i = \frac{1}{n}\sum_{t=1}^{n} Z_{it}$$

are the component-wise sample means of an observation consisting of n time slots.

These estimators are asymptotically normally distributed [Ken89]. Modifications exist to address issues of bias and high-dimensionality [Ken89], [LW04] and [SS05]. A disadvantage of the latter approaches is that the sample properties of their estimators are not known. Note also that the matrices do not have to be invertible for our study.

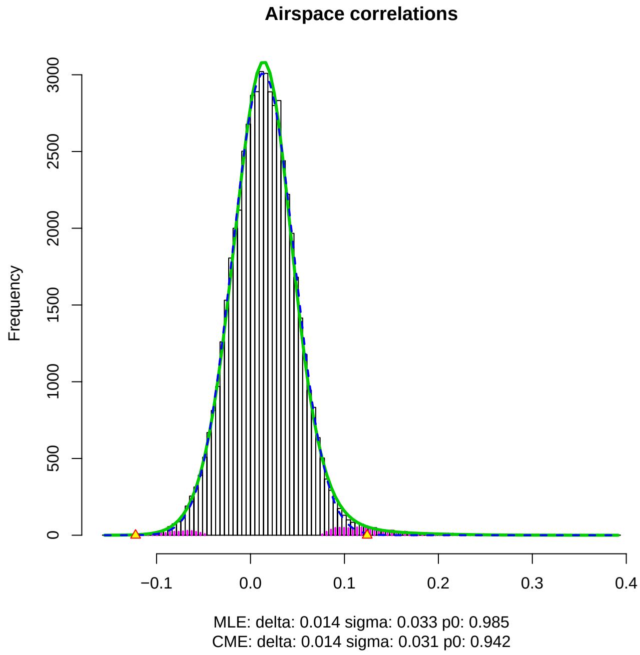

Figure 1. Histogram of estimated correlation coefficients. Bold: empirical distribution. Blue: empirical null distribution. Triangles: local false discovery rate. Data: Japanese Airspace.

2) Sample Variability: Our objective is to decide whether the coefficients of the correlation matrix differ significantly from 0. For this, the variance of the sample correlations has to be known. For a large number n of independent observations the variance of a single sample correlation coefficient under the hypothesis that the true correlation is 0 is $\frac{1}{n-1}$ [Sap06]. There are two reasons why this result cannot be used directly in our analysis: (i) our observations are not independent and (ii) there is a large number of hypotheses to be evaluated.

a) Bartlett: When observations are dependent, a result from Bartlett gives insight into the problem [KSO83]. It shows that when the stationary series $Z_i(t), Z_j(t)$ are uncorrelated and estimated from a single realization

$$V[\hat{\rho}_{ij}(k)] = \frac{1}{n-k}\sum_{s=-\infty}^{\infty}\rho_{ii}(s)\rho_{jj}(s) \tag{1}$$

This means that even for large n, the variance of the sample correlations depends on all correlations of the original processes, which are generally unknown. Consequences are (i) a risk of 'spurious' correlations and (ii) that it is impossible to estimate these quantities directly from a finite sample. In practice, approximations are often used, for example by assuming that the individual series correspond to white noise (for example after pre-whitening).

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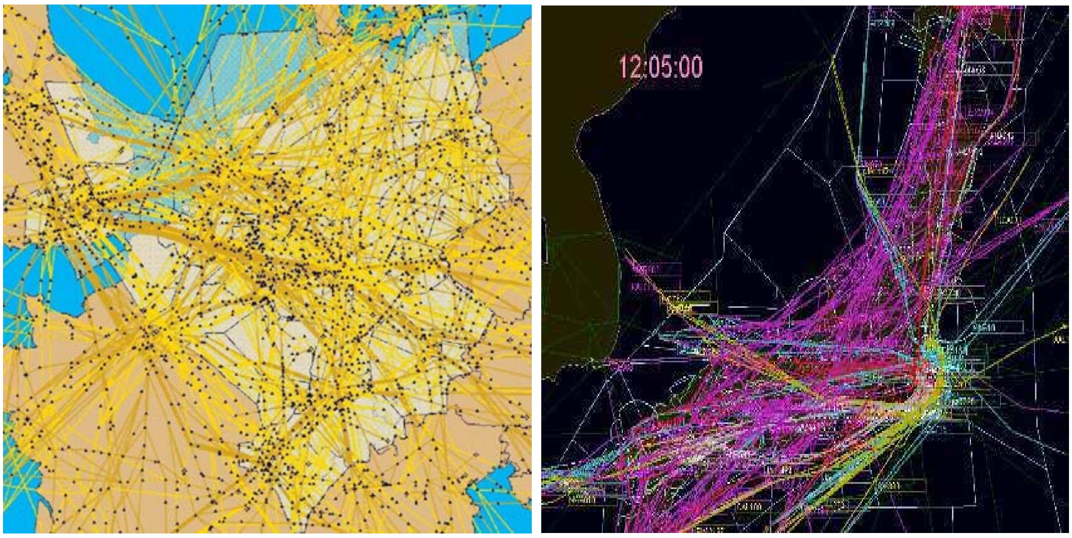

Figure 2. Airspaces. Left: European Central Airspace. Right: Japanese Airspace.

l_{max} \ ρ_{max}	0.01	0.05	0.1	0.2
10	1.002	1.05	1.20	1.8
75	1.015	1.375	2.50	7.0
100	1.02	1.50	3.00	9.0
150	1.03	1.75	4.00	13.0

Table I
VARIANCE INFLATION AS A FUNCTION OF SERIAL CORRELATION

To characterize the risk of spurious correlation in our instance, we calculate the variance inflation for several dependency structures compared to independent observations due to Bartlett's formula. Looking ahead to Figure 4 we used scenarios with a low amount of constant dependency ρ_{max} up to time-slot l_{max} in order to obtain upper bounds for the inflation. The dependency structures are the following:

$$\rho_{ii}(s) = \rho_{jj}(s) = \begin{cases} \rho_{max} & s < l_{max} \\ 0 & else \end{cases}$$

Under this structure, equation 1 becomes

$$V(\hat{\rho}_{ij}(k)) < \frac{1}{n-k}\sum_{s=-\infty}^{\infty} \rho_{ii}^2(s) = \frac{1}{n-k}(1 + 2l_{max}\rho_{max}^2)$$

Table I shows $nV(\hat{\rho}_{ij}(1))$ for different values of ρ_{max} and l_{max}: For example, for a correlation of $\rho_{max} = 0.1$ up to lag $l_{max} = 10$, an inflation of 20% would occur. For stronger correlations, an explosion of the variance can be seen (bottom right part of the table). Again, looking ahead to Figure 4, we expect weak correlations in our series. We can expect 30 - 70 % increase of variance with respect to independent realizations.

b) False discovery rates: The second problem is that of the large number of coefficients to be evaluated. Classical hypothesis tests would expect a large number of rejections by their very nature [Efr04]. [ETST01] proposes a heuristic method to identify a number of 'interesting' coefficients in large-scale testing contexts. They define the *local false discovery rate*

$$fdr(\hat{\rho}) \equiv f_0(\hat{\rho})/f(\hat{\rho})$$

where $f_0(\hat{\rho})$ is the density of uninteresting coefficients and $f(\hat{\rho})$ the density of all coefficients. fdr is the expected proportion of of null coefficients in a selection of coefficients with value $\hat{\rho}$. Interesting coefficients are those with $fdr(\hat{\rho}) < c$, a threshold value, comparable in meaning with the significance level of classical tests.

Figure 1 shows the histogram of all 21*21*30 = 13230 cross-correlation coefficients in our matrix for the Japanese Airspace (please see below for details on the selection of the 21 sectors). It has been estimated from 11 days of data, each consisting of 288 observation intervals. The bold line (green) is the empirical distribution, fitted by a polynomial of degree 3. The dotted blue line is the empirical null distribution, fitted by Efron's method. It is a normal distribution with unknown variance. Both distributions look almost identical; small differences can be seen at the peak and $\hat{\rho} \sim 0.1$. The triangles mark the interval, outside which the computed $fdr < 0.2$. Finally, the pink bars represent the estimated mass of non-null coefficients. The majority of their mass lies inside the fdr interval. We obtain three results: (i), correlation coefficients above 0.12 can be regarded as interesting, (ii), the standard deviation of the empirical null distribution is 0.033, which is $\approx 86\%$ larger than the null variance for independent observations (for 11 days, each 288 observations). This is in agreement with the results from the previous paragraph. And (iii), a risk that interesting coefficients will be undetected exists.

To summarize, we analyzed how dependent observations

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and a large number of variables affect statistical methods to infer significant correlation coefficients. The first approach showed that a variance inflation has to be expected and the second that there is risk of leaving interesting coefficients undetected. Both methods suggest a rather small critical value for interesting coefficients. At this point we remind that we wish to explore meaningful patterns of correlation rather than single coefficients. Subjective judgement may prove useful in this task.

III. Results

We analyzed correlations in the European and in the Japanese airspace. The left part of Figure 2 shows the most congested part of the European Airspace. It comprises 31 sectors covering London, Zurich and Berlin, belonging to 9 control centers. The daily number of aircraft is about 8000 for this area. The yellow routes are from North to South and the brown ones from South to North. Between London and Frankfurt, one can see a bi-directional high density route. The Japanese airspace can be seen in the right part of Figure 2. Our area of interest contains 21 sectors, covering Fukuoka (south), Tokyo (center) and Sapporo (north). These sectors belong to 3 control centers. For these sectors, more than 85 % of the entry-times of the aircraft could be determined accurately. About 4000 aircraft use this part of the airspace every day. One can see high traffic routes from/to Tokyo (yellow, blue) as well an important number of over-flights (pink).

More formally, we consider the vector of random processes $\mathbf{GAP}_t = \mathbf{PLN}_t - \mathbf{REAL}_t, t \in \mathbf{R}$, where the ith component $GAP_{i,t} = PLN_{i,t} - REAL_{i,t}$ represents the gaps between the planned and realized number of entries in sector i. The process is observed in 5 minutes time intervals, leading to 288 samples per day. For the European Airspace, 91 week-days are available (Mon-Thu) in the summer period May, 13 - Sept. 29. 2004. For the Japanese Airspace, 11 days from August and November in the Year 2006 are available.

c) Time-Plots: Typical time plots of one component process $GAP_{i,t}$ can be seen in Figure 3. The top panel shows a sector from the European Airspace, the bottom shows an example from the Japanese Airspace. In both, the gaps fluctuate around 0, the variance looks constant during the day (7-19h). The marginal distributions of the processes turned out to be symmetric, as expected (not shown). In the following, we assume that the component processes are second-order stationary during the day.

d) Cross-correlation plots: We now analyze in more detail cross-correlations between local neighbors (local correlation) and between far lying sectors (non-local correlation). Figure 4 shows typical cross-correlation matrices. In the left panel, the 2x2-matrix from the two neighboring sectors T01 and T27 from the Japanese airspace are shown. The diagonal elements correspond to the autocorrelation functions (acf) up to lag 30, corresponding to 2h30. Both

Airspace	Type	avg	max
Europe	local	0.19	0.34
	non-local	0.16	0.24
Japan	local	0.24	0.28
	non-local	0.23	0.36

Table II
Summary statistics for correlation coefficients. Top: European airspace. Bottom: Japanese Airspace.

Airspace	Type	# coeffs	lag-range
Europe	local	1.29	[-4, 3]
	non-local	1.09	[-6, 6]
Japan	local	4.3	[-4, 2]
	non-local	5.2	[-8, 8]

Table III
Summary statistics for correlated sectors. Top: European Airspace. Bottom: Japanese Airspace.

show no peaks. The off-diagonal elements display the cross-correlations for positive lags in the upper diagonal $\rho(GAP_{it}, GAP_{j,t+k})$ and negative lags in the lower diagonal $\rho(GAP_{it}, GAP_{j,t-k})$. A peak at lag -3 has value 0.26. Its neighbors (lag -2 and -4) show still some higher value than the remaining ones. These three coefficients are the only interesting in the plot.

For more insight into correlation between far lying sectors, we analyze the two sectors EXH and EUY from European Airspace. They are separated by the two sectors EUF and EXE. Their correlation matrix function is plotted in the right panel of Figure 4. A decay of autocorrelation, starting from -0.1, can be seen. A peak in the cross-correlation is found at lag -5.

Table II summarizes the significant correlations of the full cross correlation matrices. In Europe, local correlations are on average 0.19 and have a maximum of 0.34 (columns 2, 3). The non-local correlations are on average 0.16 and have a maximum of 0.24. In Japan, the local correlations are on average 0.24 with a maximum of 0.28. And the non-local correlations are on average 0.23 and have a maximum of 0.36. All correlation coefficients are positive. Table III summarizes how two sectors are correlated. Of interest are the number of significant coefficients (at different lag values) and the time lag of these coefficients. In Europe, for locally correlated sectors, 77 % have exactly one significant coefficient, 19 % have two and 4 % three or four, leading to an average of 1.29 coefficients (column 2). In the Japanese Airspace, the average number is 4.3. For non-locally correlated European sectors, 91 % have exactly one and 9 % have two significant coefficients, averaging 1.09. The Japanese is higher again, with 5.2 significant coefficients per correlated sectors. Local correlations occur between lags -4 and 3, and non-local ones between lags -6 and 6 (column 3) in the European and between lags [-4,2] and [-8,8] in the Japanese Airspace. The

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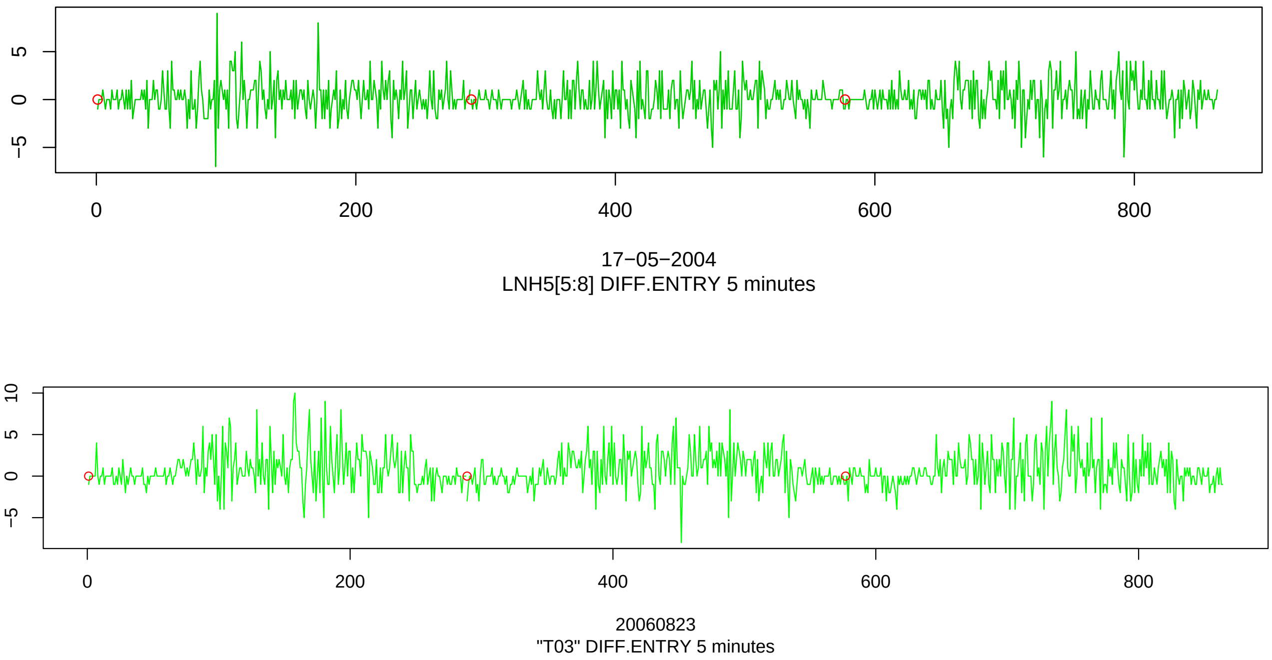

Figure 3. 3 successive week-days of gaps between planned and realized traffic at a sector entry, 5 minutes time-scale. Top: Sector from European Airspace. Bottom: Sector from Japanese Airspace. Daily repeating patterns. Constant mean around 0, constant variance over time (except night hours).

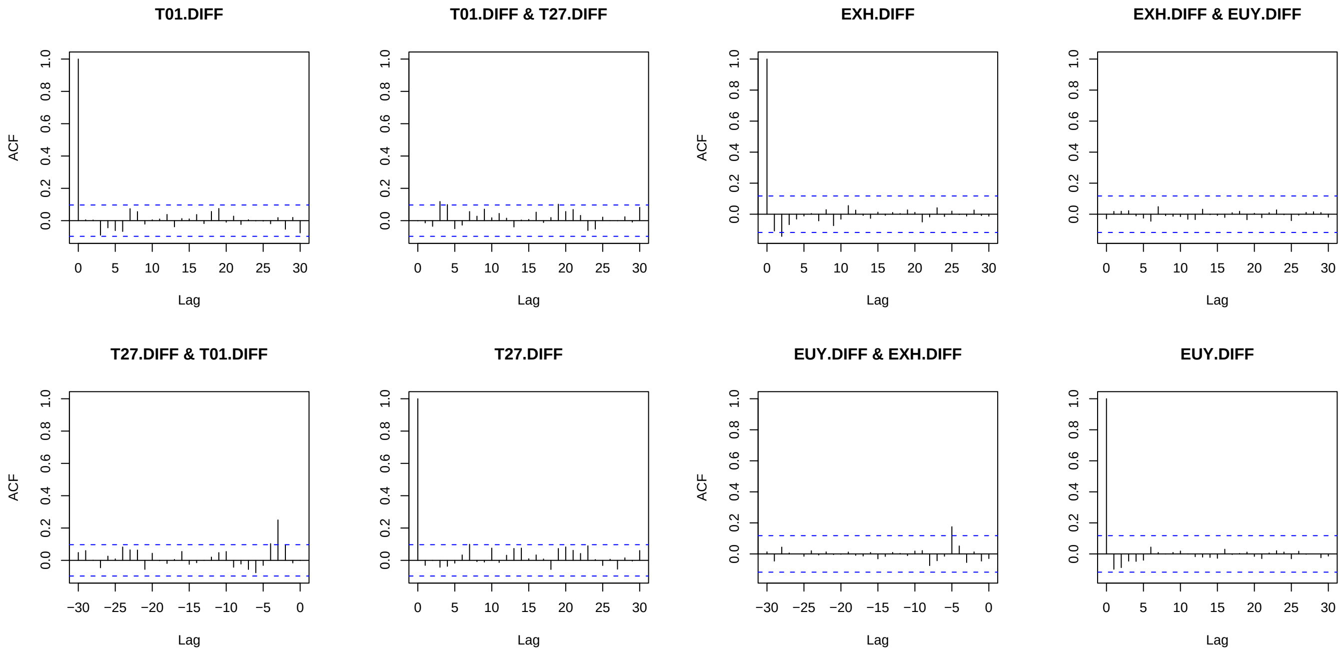

Figure 4. Cross-correlation matrices. Left: local neighbors. Right: non-local sectors

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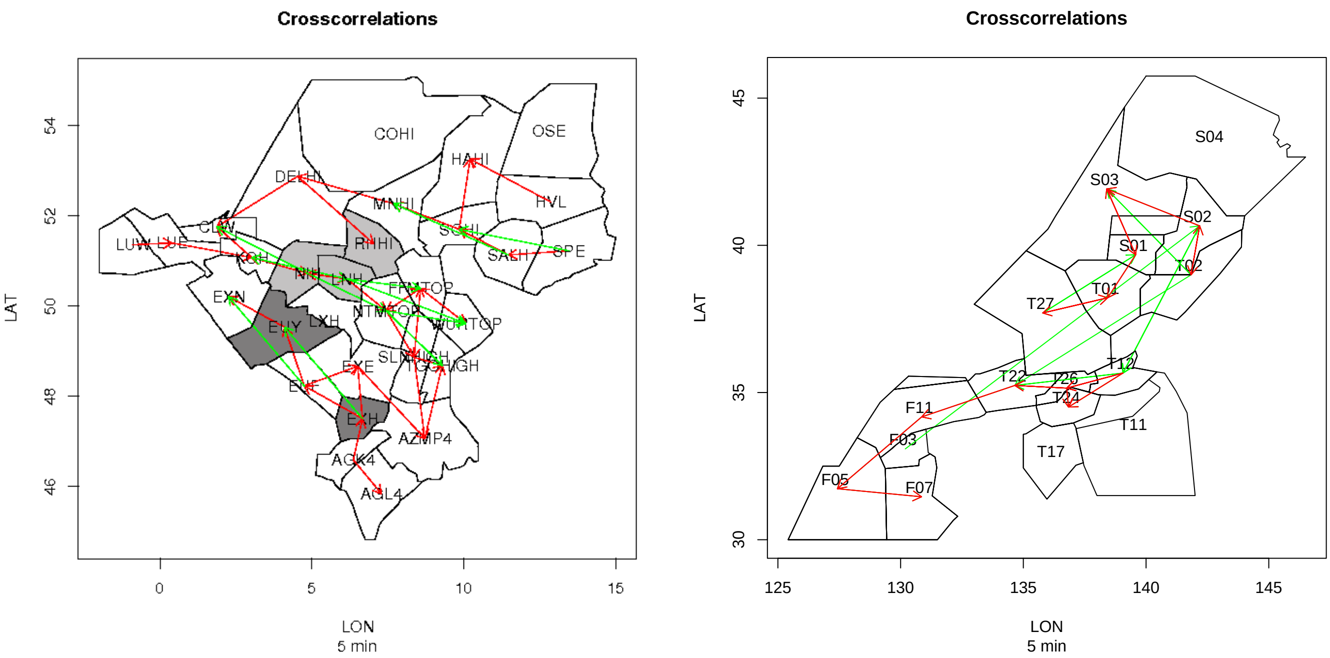

Figure 5. Visualization of the cross-correlation matrix. Left: Europe, Right: Japan.

higher average values in the Japanese Airspace have been analyzed further: there are generally many coefficients close to the critical value. This can be attributed to the higher sample variability as compared to the European data, because of the smaller sample size and because of the quality of the Japanese Airspace data [Gwi08].

The weak autocorrelation of the component processes and the sparse number of peaks in the crosscorrelation matrices suggest that the correlation structure in the system (i) does not contain spurious correlations because the component processes do not imply a severe variance inflation and (ii) has an intuitive explanation: all coefficients lie in the range of expectation since the traversal time for one sector lies between 6 and 10 minutes.

e) Visualization of correlation matrix: The correlation matrix functions for all 31 European and all 21 Japanese sectors were estimated up to lag $k = 30$, corresponding to 2.5 hours.

Figure 5 visualizes the results. An arrow between two sectors (i, j) represents a significant correlation at least one lag k. Positive and negative lags have opposite arrows. Local correlations are drawn in red. They reproduce almost the route network. For example, in the central flow (Frankfurt-London), they are bi-directional, whereas in the flow from Zurich to London, they are mono-directional. Non-local correlations are plotted in green. They reproduce only routes with high traffic densities. No correlations between two sectors that are not connected by a route are found.

IV. Conclusion and Future work

We analyzed how gaps between planned and realized traffic propagate through the European and the Japanese airspace. For this we did a correlation analysis for the most congested part of the systems. Because of the motion of aircraft, gaps propagate to neighboring sectors, expecting positive correlation coefficients. The question in the analysis was whether there are unexpected coefficients. Such coefficients would be caused by traffic controllers or flow managers who compensate for high gaps by re-routings or speed adjustments. Such strategies would often lead to negative correlation coefficients. We first analyzed the risk of obtaining misleading coefficients in a large correlation matrix. Then, we analyzed data from the European and Japanese airspace.

Our main results were:

- European and Japanese Airspace show similar patterns.
- significant cross-correlations appear on two levels: (i) locally, that is between a sector S and a direct neighbor and (ii) on high density routes, that is between two sectors S_1, S_2 that are connected through a flight route with high traffic densities.
- all correlations are positive.
- their lags correspond to the average traversal times.

No unexpected correlations have been found, and none of the correlations appears to be induced by the autocorrelation structure of a component process.

On the other hand one can argue that systematic re-routings would cause only weak correlations. Also, correlation assumes that the only source of covariation lies in the two variables under study. Indeed, the average strength of correlation was

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0.2 in our data sets. This means that the non-existence for such strategies cannot be concluded; it can only be confirmed that such strategies currently show very weak effects in the counts of aircraft entering flight sectors. Such information is useful for demand prediction based on traffic densities: network-effects from far-lying sectors appear to have negligible effect. In order to get a deeper understanding of how controllers treat high workloads, a more specific model should be built. As a next step, inspiration for the construction of semi-empirical models (of conflict probabilities) can be found in the work of [Jar03]. This work is a step toward the identification of the mechanisms that lead to congestion in air traffic. Based on this, flow planning can be improved by taking into account the traffic predictions.

V. ACKNOWLEDGEMENTS

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VI. VALIDATION

Figure 6 shows 4 scatter plots of variables in the system. The two upper ones are from the Japanese- the two lower ones from European Airspace. In each panel the bold line is the sample mean. It is reasonably linear. No other functional form of dependency is visible, neither. The first and third have significant coefficients of linear correlation. The second and fourth ones have not. Thus, linear correlation as a measure for dependence seems justified, even if the dependency between the variables is visibly weak.

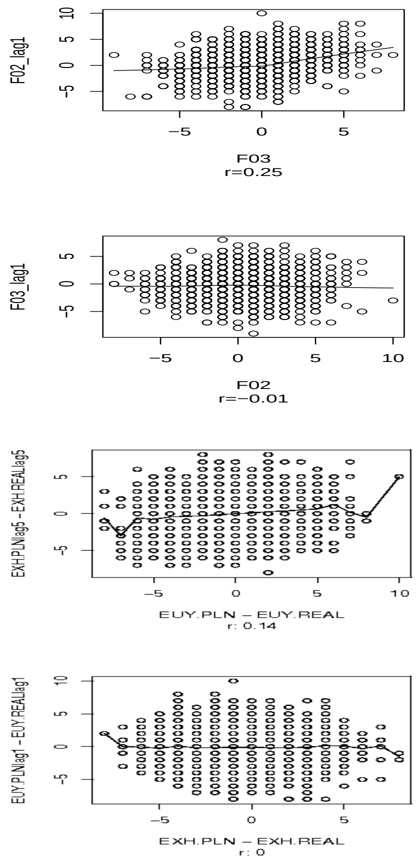

Figure 6. Scatterplots of gaps in one sector against time-lagged gaps in other sectors. First and third panel: significant coefficients (Japan/Europe), second and fourth: non-significant coefficients.

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