The robust flight level assignment problem

Olivier Klopfenstein(1,2)
(1) France Telecom $R\&D$
38-40 rue du Général-Leclerc
92794 Issy-les-Moulineaux Cedex 9, France
olivier.klopfenstein@orange-ftgroup.com

Dritan Nace(2)
(2) Laboratoire Heudiasyc UMR CNRS 6599
Université de Technologie de Compiègne
60205 Compiègne Cedex, France
nace@utc.fr

***Abstract*—This paper studies the robust flight-level assignment problem. Our goal is reducing the cost (and more specifically the delay) induced by airspace congestion through an appropriated flight level assignment (FLA) taking account of uncertainties. We investigate a robust optimization framework inspired by Bertsimas and Sim work for linear programs and propose appropriate models for the robust flight level assignment problem.**

Keywords: Flight-level assignment, Robustness, Linear programming.

I. INTRODUCTION

Alleviating delays caused by airspace congestion is, and will continue to be, critical to the operation of the European air traffic control system. Two kinds of congestion can be identified corresponding to two different areas of airspace: terminal congestion (around airports) and en-route congestion (between airports). We will focus on congestion in the airspace rather than at airports, and we are interested here in a specific direction involving flight-level optimization with respect to a given traffic demand and given routes. In other words, our goal is to reduce the additional cost (including delay) induced by conflict resolution procedures through a better assignment of flight levels. Indeed, in case of en-route conflicts, some aircraft has to be rerouted, which produces some delay. This delay can be reduced through increasing the speed of aircraft, which yields additional energetic cost. We typically consider here the energetic cost due to conflict resolution, called simply conflict or energetic cost in the remainder of the paper. With respect to the FLA problem, we restrict ourselves to only three possible levels for each flight. Despite this restriction, the problem remains highly combinatory due to the large number of simultaneous flights. The flight level assignment problem is shown to be NP-complete in the strong sense [2], which makes it hard to solve at optimum even for reduced size instances. The problem becomes rather more difficult when involving the uncertainties in ATM. More precisely, an important question that we raise in this paper is how to include the potential en-route conflicts associated with each aircraft in the model and take into account the uncertainties related to it. All this leads to the robust flight-level assignment problem and the associated mathematical model, which is the main focus of this work.

A. Related works

Optimization problems in ATM have been widely studied, and we do not intend to mention all of them. We prefer to focus on some work related someway to the flight-level assignment problem. Let us first cite Bertsimas and Stock [7], [8] who have looked at the Air Traffic Flow Management Rerouting Problem (TFMRP), considering simultaneously the time and the route assignment problem through a deterministic approach. First in [7], they handle the Air Traffic Flow Management Problem (TFMP) with En-route Capacities, and then in [8] they show how to optimally control aircraft by rerouting, delaying, or adjusting the speeds of the aircraft in the ATC system to avoid airspace regions with reduced capacities due to weather conditions. Delahaye and Odoni in [11], study the problem of airspace congestion from the stochastic optimization point of view and propose a genetic algorithm. Barnier and Brisset (see [10]), consider the problem of level assignment while using an ideal sector-less environment. The main idea is to allocate different flight levels to intersecting routes in order to avoid conflicts. A straight line between an origin and destination pair represents the path of a flow of flights between these two airports; in other words, only direct routes are considered. Then, if two flows are in conflict, they must be routed on two different levels. The problem becomes a graph coloring one: given a graph with a set of vertices and a set of edges, the problem is to color the edges such that any two intersecting edges (not at their extreme vertices) have two different colors, and the number of colors used is the lowest possible. Some other research on this problem, also based on the graph coloring problem is presented in Letrouit's thesis (see [16]). The route assignment problem here is handled by several tasks. The first task is minimizing the number of required levels when assigning each route to a level from the beginning to the end of a flight, and the second task is the distribution of routes among N levels in order to minimize the number of intersections between the routes having the same level. More recently, Constans et al. (see [19]) have studied the problem from the angle of aircraft speed modification. They propose minimizing conflict risks by dynamically imposing feasible modifications on the speeds of the aircraft. Doan et al. (see [9]) have presented a deterministic model intended to optimize route and flight-level assignment in a trajectory-based ATM environment. The aim of the latter study is to address the problem of airspace congestion, and in particular to reduce the number of potential en-route conflicts. This work was the starting point for the study presented here.

Let us recall that our goal is reducing the cost induced by

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potential en-route conflicts. An important question is then how modeling the induced cost given conflict probabilities. Clearly, the cost induced by an en-route conflict greatly depends on the conflict resolution methods and for a majority of cases, the delay is asymmetrically distributed to the involved aircraft. Due to uncertainties, it is not possible to determine in advance if some conflict will occur and which aircraft will be delayed. All this justifies the need for a robust flight level assignment as depicted below.

B. Paper organization

This paper is organized as follows. After this introduction, in Section II we recall briefly works on robust linear programming (LP). In Section III, we focus on the FLA problem. We present binary linear programming models and in Section IV we discuss its robust versions. Section V is devoted to some numerical results, including remarks on implementation and data estimation. Some concluding notes are provided in Section VI.

II. The robust binary Linear Programming problems

Robust optimization is one of the common approaches to take account of uncertainties in optimization problems. We refer to [18] for a survey in the context of combinatorial optimization. The usual goal of robust optimization is to find the best solution which remains feasible for a whole set of possible events. The main criticism for robust models is the so-called over-conservatism: the obtained solution will be feasible for all the possible events, regardless their occurrence probability. In practice, the worst case may impose a large cost, while being highly improbable. To remedy this disadvantage, some works have proposed to relax this worst case condition [4], [6]. As a result, the solution computed may be feasible for most of the events, but not all of them.

This is the spirit of the robust model proposed by Bertsimas and Sim [6], where the feasibility degree of a solution can be controlled. An important advantage of this model is to be easily used also with integer variables (see e.g. [5]). Indeed, the initial integer linear program (without uncertainty) is transformed into another robust integer linear program. A similar model has been proposed in [15]. The main interest of this latter approach lies in the existence of an efficient solution heuristic. Hence, it can be used on large integer linear problems.

III. The flight level assignment problem

In this section we present LP based approaches for both deterministic and robust variants of the FLA problem.

A. An LP model for the FLA problem

Notation:

- L denotes the set of possible flight-levels l. We denote with L_i the set of preferred flight levels associated with flight i.
- The set of flights is noted with F. F^l groups all flights allowed to flight to level l.
- x_i^l: binary variable $(0, 1)$, takes value 1 when the flight i, fly on level l and 0 otherwise.
- b_i^l: gives the profit associated with flight i when flying on level l.
- p_{ij}: gives the cost penalty associated with aircraft i when resolving a potential conflict with aircraft j. When dealing with the robust variant, it will denote a random variable associated with the additional cost that an aircraft can have due to some potential conflict.
- P_i^l: gives the admissible cumulated cost for a given flight i and level l.
- S_i^l: gives the set of flights j having a potential conflict with flight i at level l.

Given the above notation, an LP model associated with the FLA problem denoted with P is as follows:

$$\max \sum_{i \in F, l \in L} b_i^l x_i^l \tag{1}$$

$$\sum_{j \in S_i^l} p_{ij} x_j^l \leq M_i^l (1 - x_i^l) + P_i^l, \quad i \in F, l \in L_i, \tag{2}$$

$$\sum_{l \in L_i} x_i^l = 1, \qquad i \in F, \tag{3}$$

$$x_i^l \in \{0, 1\} \quad i \in F, l \in L_i, \tag{4}$$

where M_i^l gives a sufficient large value, (for instance $M_i^l = \sum_{j \in S_i^l} p_{ij}$). Without loss of generality, we assume that for the given P_i^l values there exists a feasible solution for problem P.

The above model is a binary integer LP problem involving a large number of constraints and variables, which makes it hard to be solved by exact methods. From mathematical point of view, it can be seen as a specific case of a multi-dimensional multiple-choice knapsack problem. We will provide in the following the framework of an approximated method for the FLA problem. The main idea behind this method is considering the assignment problem separately for each level. There are two main bricks: the first one, **Step 0**, is devoted to maximize the number of flights assigned to their preferred level. Thus, we solve a reduced problem involving only flights with their preferred level and next, fix all assigned flights. The second brick is concerned with the remaining flights. This problem, called P^l, is slightly different to P_l' as we use as constants (i.e;, $x_i = 1$) for all already assigned flights, and we add all other flights concerned with this level that are not yet assigned. Both these problems are different to P as we do not need to force any flight to be assigned to some level, as described below. It provides the main block to construct the solution approach outlined below:

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Approximated flight level assignment (ApproxFLA)	
Step 0:	Proceed with robust flight assignment separately for each level (Solve problem P^l); Fix the level for flights already assigned.
Step 1:	Proceed with robust flight assignment separately for each level (Solve problem P'^l); Fix the level for flights already assigned.
Step 2:	If all flights are assigned, STOP. Else, increase the admissible cost for each non assigned flight and return to **Step 1**.

The key element of the method is the procedure of flight assignment (P^l and P'^l) associated with a given level of **Step 0** and **Step 1**. We will focus only on P^l used in **Step 0**. Before detailing the mathematical formulation, let give some precision on the notation. As there is no need to distinguish flight levels, the binary variable x_i^l is now replaced by x_i, and as before it takes value 1 when the flight i flies on level l and 0 otherwise. Respectively b_i^l and P_i^l are now replaced by b_i and P_i. For sake of simplicity we will allow ourselves to use the same notation for F^l as in P, but here it groups only flights having l as their preferred level. Notice also that the order of level examination would have an impact on the obtained solution. We propose to start with the most loaded levels. Problem P^l follows:

$$\max \sum_{i \in F^l} b_i x_i \tag{5}$$

$$\sum_{j \in S_i^l} p_{ij} x_j + M_i x_i \leq M_i + P_i, \quad i \in F^l, \tag{6}$$

$$x_i \in \{0,1\} \quad i \in F^l. \tag{7}$$

The above model has an interesting structure as it corresponds to a *simple* multi-dimensional knapsack problem. The problem P'^l can be written in a similar way except that some constraints of type $x_i = 1$ are added and F^l groups all concerned flights.

IV. Modeling and solving the Robust FLA problem

Assuming separate probability conditions, the robust version of the FLA can be formulated with probability constraints as follows:

$$\max \sum_{i \in F^l, l \in L} b_i x_i$$

$$Pr(\sum_{j \in S_i^l} p_{ij} x_j + M_i x_i \leq M_i + P_i) \geq 1 - \epsilon, \quad \forall i \in F^l, l \in L.$$

Following the Bertsimas and Sim work, we can deduce the robust variant of the above ILP problem. This yields still another ILP problem, which is at least as difficult as the standard problem. All this justifies heading to approximated methods to deal with it: we will make use of the framework approximated method given above for the FLA problem, except that we consider the robust variant of the P^l problem (called RP^l), instead.

The key element of the method is the procedure of robust flight assignment RP^l associated with a given level of **Step 0**. Let us deduce first its robust variant.

A. Modeling the RP^l problem

In the P^l we have assumed that p_{ij} are some given constants expressing the potential cost induced by some conflict involving aircraft i and j. In the following, we assume that p_{ij} are random variables that take values in an interval data already estimated. This assumption leads us to the robust version of the P^l problem, that is RP^l, and subsequentially to the robust variant of the FLA problem. Naturally, some way to take into consideration the uncertainties is not to allow all en-route potential conflicts to count for the total cost estimation. We have thus a robust version of the FLA problem in the sense that for a given aircraft only a part of potential en-route conflicts are assumed to occur and expected to generate additional costs. First, let us precise the assumptions related to the robust problem RP^l and the ways used to introduce the uncertainty in the model. Following the Bertsimas and Sim works on this area, it seems natural to model the uncertainty by introducing a protection coefficient, which gives the maximum number of conflicts that can occur for a given flight. In our model we do not make use of conflict probabilities in a direct way but consider their consequences, that is the corresponding additional costs. These potential costs are modeled by intervals $[0, \bar{p}_{ij}]$, with $\bar{p}_{ij} > 0$. Resolving a conflict that involves a pair of aircraft, yields delay and hence an additional cost, non necessarily symmetrically distributed among involved aircraft. This statement leads us to the following assumption: any flight i will experience (most probably) a reduced number of potential conflicts during his time flight, (which yields additional costs to the involved aircraft) and this number (Γ_i) varies in $[0, |S_i^l|]$. Then, we are interested in "best" solutions that remain feasible for any scenario with at most Γ_i coefficients taking the worst value $\bar{p}_{ij}$. Such a solution is obtained through the following program:

$$\max \sum_{i \in F^l} b_i x_i$$

$$\sum_{j \in S} \bar{p}_{ij} x_j + M_i x_i \leq M_i + P_i, \quad i \in F^l, S \subseteq S_i^l : |S| = \Gamma_i,$$

$$x_i \in \{0,1\} \qquad i \in F^l.$$

The above program contains a large number of constraints and it is hard to solve at optimum. However, it has been shown by Bertsimas and Sim that it can be modeled through an ILP (Integer Linear Programming). The latter program, provided below, still contains a large number of constraints and variables and remains hard to be solved by exact methods.

Following the work of Bertsimas and Sim, the robust variant of problem P_l with respect to a given vector Γ, denoted $RP_{l\Gamma}$, is as follows:

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$$\max \sum_{i \in F^l} b_i x_i$$

$$\Gamma_i z_i + \sum_{j \in S_i^l} \delta_{ij} y_{ij} + M_i x_i \leq M_i + P_i, \qquad i \in F^l,$$

$$z_i + \delta_{ij} y_{ij} \geq \delta_{ij} x_j \quad i \in F^l, j \in S_i^l$$

$$x_i \in \{0,1\}, z_i \geq 0, y_{ij} \geq 0 \quad i \in F^l, j \in S_i^l.$$

Hence, we have opted to use another (alternative) robust model of above, following the one introduced in [15]. The following model uses a parameter vector $\gamma \in [0,1]^{|F^l|}$ instead of the vector Γ:

$$\max \sum_{i \in F^l} b_i x_i$$

$$M_i x_i + \min \left\{ \sum_{j \in S_i^l} \bar{p}_{ij} x_j, \gamma_i . \sum_{j \in S_i^l} \bar{p}_{ij} \right\} \leq M_i + P_i \quad i \in F^l,$$

$$x_i \in \{0,1\}, \quad i \in F^l.$$

The above model is denoted below $RP_{l\gamma}$. This formulation can be simplified a lot. Let us focus on the robust constraint i. Either we consider the worst case (maximum conflict induced costs), or we have a constraint: $M_i x_i + \gamma_i . \sum_{j \in S_i^l} \bar{p}_{ij} \leq M_i + P_i$. In this latter case, two sub-cases occur: when $\gamma_i . \sum_{j \in S_i^l} \bar{p}_{ij} > P_i$, then $x_i = 0$; when $\gamma_i . \sum_{j \in S_i^l} \bar{p}_{ij} \leq P_i$, we have a dummy constraint which can be ignored.

Hence, this robust model leads to three different configurations:

- either $x_i = 0$: the flight i does not use level l;
- or $x_i = 1$ and no constraint is associated to flight i: this means that flight i uses level l with zero conflict costs;
- or $x_i = 1$ and the worst case is taken into account: the flight i uses level l with maximal conflict costs.

These three cases are in fact summarized in the two following ones:

- either flight i has zero conflict costs;
- or flight i is associated maximal conflict costs.

Hence, the analysis of the above robust model leads to a new one, which is very simple. Let $I_c \subseteq F^l$ be a subset of flights:

$$\max \sum_{i \in F^l} b_i x_i$$

$$M_i x_i + \sum_{j \in S_i^l} \bar{p}_{ij} x_j \leq M_i + P_i \quad i \in I_c,$$

$$x_i \in \{0,1\}, \quad i \in F^l.$$

The parameter enabling to tune robustness is the subset I_c, and we denote the problem by $RP^l(I_c)$.

B. Solving the RP^l problem

In the precedent section we have described how an instance of the robust FLA problem can be modeled by ILP. Let recall that we are interested in robust solutions that remain feasible in a large part of scenarios, that is, which has a high enough feasibility probability. Obviously, if we take $\gamma_i = 1$, for all i in F^l (which gives $I_c = F^l$), we obtain feasible solution for all scenarios. One idea is to start with $I_c = \phi$ and to make it grow gradually until a solution with the desired feasibility probability is achieved. Such ideas have already been exploited in [14], [15]. The algorithm can be depicted as follows:

A fast heuristic approach for solving RP^l	
Step 0:	Set $I_c = \phi$.
	Select an index $i \in F^l$ such that:
	$i = \arg\min\{P_i - \mathrm{E}[\sum_{j \in S_i^l} p_{ij}]\}$
	Set $I_c \leftarrow I_c \cup \{i\}$.
Step 1:	Solve $RP^l(I_c)$.
	Let $\bar{x}$ be the solution found.
Step 2:	If feasibility probability of $\bar{x}$ is high enough, STOP.
	Else, select an index $i \in F^l \setminus I_c$ such that:
	$i = \arg\min\{P_i - \mathrm{E}[\sum_{j \in S_i^l} p_{ij} \bar{x}_j]\}$
	Set $I_c \leftarrow I_c \cup \{i\}$; Return to **Step 1:**.

As it can be seen from the algorithm, during **Step 0** we look for a *strongly constrained* constraint to introduce in I_c. The solution will admit all flights in this level except the flight i or a few constraining the selected flight i (it depends on the associated benefits). At this stage, the above solution is most probably not feasible and we need to pursue with other steps in order to further constrain the set of flights to be assigned at this level. An immediate way to accelerate the algorithm is to introduce at step 0 in I_c a larger number of constraints. For more details on the general framework of the algorithm and a deeper study on its theoretical properties, we refer to [15].

Notice also that the above algorithm doesn't ensure the optimality of the obtained solution. An important element of the resolution scheme given above is measuring the probability of the obtained solution. There are two ways to estimate the feasibility probability associated with some solution.

1) First method: The main idea behind the first method is using the Hoeffding's inequality [12], which is a result in probability theory that gives an upper bound on the probability for the sum of random variables to deviate from its expected value. This yields general results but it could be pertinent since variables p_{ij} can be assumed independents in our model. Let us recall first this fundamental result (see [12] for details):

Let $X_1, \ldots, X_n$ be independent random variables. Assume that the X_i are almost surely bounded; that is, assume for $1 \leq i \leq n$ that $Pr(X_i \in [a_i, b_i]) = 1$. Let be $S = \sum_i^n X_i$ and $E[S]$ its expected value. Then, we have the inequality

$$Pr(S - \mathrm{E}[S] \geq nt) \leq \exp\left(-\frac{2\, n^2\, t^2}{\sum_{i=1}^n (b_i - a_i)^2}\right),$$

which is valid for positive values of t.

To apply this result to our problem, we first need to compute the expected value for each random variable. For doing this, let us try to express these variables in a more formal way.

Recall that the random variable p_{ij} corresponds to the cost induced by some resolution conflict procedure. Then, it de-

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pends to two factors: first, if some conflict is occurring, and second, the resolution procedure engaged by the air traffic controllers. Hence, the first event is an en-route conflict[1] modeled below with a random variable c_{ij}, which follows a Bernoulli distribution ($Pr(c_{ij} = 1) = q_{ij}$, and $Pr(c_{ij} = 0) = 1 - q_{ij}$). In case of conflict, the cost induced to the involved aircraft, represented by random variables p'_{ij}, is assumed identically distributed in the interval $[0, \bar{p}_{ij}]$. We have: $p_{ij} = p'_{ij}.c_{ij}$. Then, $\mathrm{E}[p_{ij}] = \mathrm{E}[p'_{ij}.c_{ij}]$. Since the random variables p'_{ij} and c_{ij} are stochastically independent: $\mathrm{E}[p_{ij}] = \mathrm{E}[p'_{ij}].\mathrm{E}[c_{ij}] = \frac{\bar{p}_{ij}}{2} q_{ij}$. Clearly for a given vector x, we obtain $\mathrm{E}[p_{ij}x_j] = \frac{\bar{p}_{ij}}{2} q_{ij}x_j$.

Considering that the FLA problem has separated probability constraints, we need to ensure that for each constraint the following probability condition is satisfied:
$Pr(\sum_{j \in S_i^l} p_{ij}x_j + M_i x_i \leq M_i + P_i) \geq 1 - \epsilon$. As
$Pr(\sum_{j \in S_i^l} p_{ij}x_j + M_i x_i \leq M_i + P_i) \geq$
$Pr(\sum_{j \in S_i^l} p_{ij}x_j \leq P_i)$,
we restrict ourselves in ensuring that
$Pr(\sum_{j \in S_i^l} p_{ij}x_j \leq P_i) \geq 1 - \epsilon$ for all $x_i = 1$.
Applying the Hoeffding's inequality we have:
$Pr(\sum_{j \in S_i^l} p_{ij}x_j \geq P_i) =$
$Pr(\sum_{j \in S_i^l} p_{ij}x_j - \mathrm{E}[\sum_{j \in S_i^l} p_{ij}x_j] \geq P_i - \mathrm{E}[\sum_{j \in S_i^l} p_{ij}x_j]) \leq$

$$\exp\left(-\frac{2\,(P_i - \sum_{j \in S_i^l} \frac{\bar{p}_{ij}}{2} q_{ij}x_j)^2}{\sum_{j \in S_i^l} \bar{p}_{ij}^2 x_j}\right) = \epsilon'_i.$$

Notice that the interval of values for variable $p_{ij}x_j$ is given by $[0, \bar{p}_{ij}]$, which explains the above formula. Clearly, we have reached a feasible robust solution x when for all i in F^l with $x_i = 1$, we have $\epsilon'_i \leq \epsilon$.

The method is attractive and does not need restrictive probability conditions but it could lead to costly solutions as the probability bounds are quite general and could be weak. To remedy this, a natural idea is to use Monte-Carlo simulation.

Remarque. It is possible to formulate an ILP model for computing a robust solution with the desired feasibility probability. For this, we need to combine the search for some robust solution x with some additional conditions that a feasible solution must satisfy. As shown in [15], it yields an ILP model involving additional variables and constraints.

2) Second method: The second way to handle the feasibility probability computation is using Monte-Carlo simulation. The main idea behind is simulating the departures times for all flights, simulating next the most convenient resolution en-route procedure, and estimating the induced cost. Once all coefficients of the model estimated, we check the feasibility of our solution for the given scenario. We repeat this a large number of times, and deduce the feasibility probability associated with the robust solution.

V. NUMERICAL TESTS

Our approach for the robust FLA problem is implemented in C++ using CPLEX 10.0. Let us give some details on the implementation approach: we start by considering levels one by one, from the most loaded to the least one. For each level, we start with a set of a reduced number of flights. More precisely, we initiate the RP^l problem with about 5% of concerned flights. We choose the most constrained ones, that is in increasing order of $\{P_i - \mathrm{E}[\sum_{j \in S_i^l} p_{ij}]\}$ values. Further iterations could be necessary to ensure the probability feasibility of the obtained solution. Hence, for a given solution x, we add in the RP^l problem a few new flights in decreasing order of $\epsilon'_i (> \epsilon)$ values.

TABLE I
TEST INSTANCE

Network	Number of Flights	Used Airports	Used WayPoints
NET_FR	1377	134	769

For our tests we use collected data on departure and arrival times, aircraft type, velocities, trajectory crossing angle and flight levels for a set of flights. Next, for each flight we will compute the en-route conflict probability following the guidelines given in [3]. The test data corresponds to French air traffic of August 12th 1999. Table I presents the characteristics of test data. All the tests were run on a machine with the following configuration: Windows XP, 1 processors Pentium 4 2.4GHz, 1 Gb of RAM.

At this stage, the first difficulty encountered when implementing the model, is concerned with providing the right parameters $\bar{p}_{ij}$ and P_i. Indeed, the best choice would be to estimate the interval $[0, \bar{p}_{ij}]$ as a function of crossing angle, and type of aircraft, and last, estimate the P_i^l as a few percent of the energetic cost of the flight. In this first series of tests, essentially because of lack of data and time, we have set the same unitary cost for all conflicts. Thus, we have set $P_i = \max \alpha, c * (duration - 1) + \alpha$, where duration gives the flight duration, c and α are both constants. For instance, for any flight with duration less than 1 hour, we have fixed P_i to $\alpha = 3$, while for the others we also take into account the duration of the flight according to the above formula with $c = 0.1$. Our goal is to measure the impact of robustness on the number of flights assigned to their preferred level comparing to these that have to be changed. We have also varied the level of robustness parameter ϵ. To measure the feasibility of the solution, we have used the Hoeffding's formula. In table II are shown some results obtained with the above parameters for three different values of ϵ, which gives the allowed infeasibility probability. The second column, ("Number of changed levels") gives the number of flights not assigned to their preferred levels and accommodated to adjacent levels because of en-route conflicts. The last column ("Gap Robust/Deterministic") gives the percentage of additional flights assigned to their preferred levels thanks to robustness in comparison with the deterministic model. These results show that when using the robust model we can have some increase in the capacity of accommodating flights in their preferred levels with very high probability feasibility comparing to the standard problem when considering the worst case. This latter case is computed by

[1] The conflict probability associated with a pair of aircraft can be computed following the method given in [3].

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TABLE II
NUMERICAL RESULTS

ϵ	Number of changed levels	Gap Robust/Deterministic
0.05	170	6.5%
0.10	163	10.4%
0.15	139	23.6%
0.20	135	25.8%
0.25	126	30.7%

adding all constraints corresponding to flights in the problem. We do not need at all to do any computation on the probability feasibility: the obtained solution is feasible for any case and all constraints are satisfied. For our deterministic problem using the above set of parameters we have obtained 182 flights not assigned to their preferred levels.

In our computations, most of remained flights not assigned to their preferred levels are accommodated to adjacent levels, while for some of them we have needed to increment their cumulated allowed cost as indicated in **Step 2** of the *ApproxFLA* Algorithm described in Section III.

Indeed, we expected to have a larger difference between the deterministic and the probabilistic model. We believe that this is because of using the Hoeffding bound which is somehow weak. To remedy this, two directions need to be followed: first, using a better parameterizing of the model, and next switching to Monte-Carlo simulations, better suited to this kind of problems. This work is in progress.

VI. CONCLUDING REMARKS

In this paper we have provided a mathematical model for the robust FLA problem. We have first discussed the model following the Bertsimas and Sim [5] approach and focus on a second one inspired from [15], for which an approximated tractable iterative approach is available. We have adapted this later work in the context of ATM for solving the robust FLA problem. This work is a first stage to achieve a thorough study on the robust flight level assignment problem. As remarked above, the obtained results rises the problem of how parameterizing the model. Another point, in addition to those shown above, is related with the assumption of considering only en-route conflicts between aircraft flying horizontally in the same level. We are actually thinking in considering air conflicts that involve crossing aircraft flying on different levels, for instance when one of them is climbing or descending. This assumption will also allow a better modeling of the problem and can contribute in avoiding the above limitations of the robust model. Further investigations are needed.

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