The Relationship Between Traffic Stability and Capacity for Decentralized Airspace

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***Abstract*—The work that is presented in this paper is part of an ongoing study on the relationship between structure and capacity of decentralized airspace concepts. In this paper, the effect of traffic stability, which considers the occurrence of conflict chain reactions as a result of conflict resolution maneuvers, on capacity is examined closely. Using the domino effect parameter as a measure of traffic stability, a model relating stability and capacity is derived. Although the derivation of this model is not complete, its current form shows that traffic stability, and therefore capacity, is also affected by the safety and efficiency characteristics of decentralized concepts. This suggests that the capacity measurement of decentralized concepts must consider the variation of intrinsic system-wide properties with density, using a minimum of safety, efficiency and stability metrics. Future work will continue the development of the model, and its validation using large-scale simulation experiments.**

Keywords - separation management; self-separation; airspace capacity measurement; traffic stability; decentralized traffic management; Domino Effect Parameter (DEP)

I. Introduction

To cope with ever increasing air traffic volumes, many researchers have suggested novel airspace design concepts where traffic separation responsibility is transferred from the ground to the cockpit [1]. The resulting decentralized traffic separation paradigm is often referred to as 'self-separation'. While different forms of decentralization have been explored in the past [2], methods to identify the theoretical capacity limits of these concepts are not well defined, and most conventional capacity measurement methods relate to aspects that are not relevant for decentralized control, such as air traffic controller workload.

To develop appropriate capacity measurement methods for decentralized airspace concepts, it is first necessary to establish a commonly accepted definition of capacity within the context of self-separation. At a fundamental level, capacity can be considered equivalent to the density at which the airspace becomes saturated, i.e., the density beyond which no additional demand can be accommodated without causing significant deterioration to system-wide performance characteristics.

The current paper focuses on the contribution of traffic stability to airspace capacity. For decentralized concepts, previous research has found that at very high densities, Conflict Resolution (CR) maneuvers have been shown to trigger additional conflicts due the scarcity of airspace [3], [4]. This phenomenon can affect traffic stability by negatively influencing important intrinsic system-wide properties such as safety and efficiency of travel; conflict propagation clearly affects safety, but it also has a significant impact on efficiency due to the increased flight distances caused by resolution trajectories that deviate aircraft from their intended destinations. Thus traffic stability, particularly its variation with density, is an indication of system performance, and can therefore be used to determine the saturation density of decentralized concepts.

The concept of traffic stability is not new. Previous studies have presented the Domino Effect Parameter (DEP) as a measure of traffic stability, and which has been used to develop a model to determine the saturation density of a direct routing decentralized airspace concept [5]. This model assumes that the rate of conflicts for a uniformly distributed traffic scenario is independent of conflict resolution. Recent simulation findings, however, show that this assumption does not hold, at least for particular conflict resolution (CR) methods [6], [7].

In this paper, it is hypothesized that CR algorithms can display a self-structuring behavior. A model is therefore presented that can take this into account. The paper begins by describing the DEP and how it measures traffic stability. Then the previous DEP model and its incompatibilities with our simulation results are briefly discussed. This is followed by the derivation of an alternate DEP model that can relate airspace stability to capacity. Although the derivation of the alternate model is not complete as of writing this paper, its current form provides interesting insights on the relationship between stability and capacity, via safety and efficiency.

II. Previous Research on Traffic Stability

One of the primary concerns of decentralizing traffic separation tasks is the possible occurrence of conflict chain reactions at high traffic densities. These conflict chain reactions can destabilize the airspace by causing a large number of aircraft to exist simultaneously in a conflicted state, a phenomena that has been measured in literature using the Domino Effect Parameter (DEP). This section describes the DEP, as well as a model that relates stability to capacity via the DEP.

A. Measuring Traffic Stability: The Domino Effect Parameter

The DEP is best described with the aid of the Venn diagram pictured in Figure 1. Here, S1 is the set of all conflicts without resolutions, and S2 is the set of all conflicts with resolutions, for identical scenarios. Furthermore, three regions can be identified in Figure 1, with $R1 = S1 - S2$, $R2 = S1 \cup S2$ and $R3 = S2 - S1$. By comparing R3 with R1, the proportion of additional 'destabilizing' conflicts that are triggered by resolution maneuvers can be determined. Thus, the DEP is defined as [3], [4]:

$$DEP = \frac{R3 - R1}{S1} = \frac{S2}{S1} - 1 \tag{1}$$

A high value of DEP implies high airspace instability. For additional clarification, in this paper, conflicts are defined as

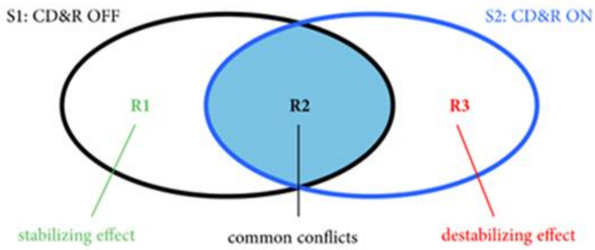

Figure 1: The Domino Effect Parameter (DEP) relates the additional conflicts caused by resolution maneuvers to traffic stability

predicted intrusions/loss of separations. The DEP can also be defined from the perspective of each single flight:

$$DEP = \frac{N_{WR}}{N_{NR}} \frac{E_{C_{WR}}}{E_{C_{NR}}} - 1 \tag{2}$$

Here, N_{WR} and N_{NR} are the total number of flights with and without conflict resolution, while $E_{C_{WR}}$ and $E_{C_{NR}}$ are the expected number of conflicts, with and without conflict resolution, for a single flight. Thus (1) and (2) are approximately equal to each other, particularly for uniformly distributed traffic conditions, where there are no preferred directions in the traffic flow.

As the DEP compares scenarios with and without CR, it can only be measured using simulation experiments. In this sense, the DEP represents a theoretical notion of stability as real operations always make use of CR. Nonetheless, the DEP makes it possible to assess the direct impact of tactical CR maneuvers on traffic stability. As mentioned in the introduction, this is a key consideration for capacity measurement of decentralized airspace designs.

B. *Previous Research Relating Stability and Capacity*

Previous research modeled the relationship between stability and capacity through the DEP [5]. In that model, the total number of conflicts *without* conflict resolution, $N_{NR}E_{C_{NR}}$, for an unstructured airspace concept was modelled using a binomial random variable model. From the total number of conflicts without resolution, the total number of conflicts *with* resolution, $N_{WR}E_{C_{WR}}$, was computed using the following two assumptions:

1. Conflict resolution maneuvers increase the amount of airspace and time spent searching for conflicts
2. The rate of conflicts per unit distance/time is the same with and without conflict resolution

The result is a closed form analytical expression which relates the DEP to density via airspace design parameters, and can be used to determine the stability related capacity limit for decentralized direct-routing airspace concepts.

Although the aforementioned model relates stability to capacity via airspace parameters, research done within our group suggests that the rate of conflicts with and without resolution is *not necessarily equal*, invalidating the second assumption listed above [6], see Figure 2. Here, simulation results for an unstructured direct-routing airspace concept shows that the *conflict rate with resolution can be different to the conflict rate without resolution*, and depends on the traffic density considered.

The difference in conflict rates is hypothesized to be caused by a change in the traffic flow pattern when conflict resolution is enabled, caused by the logic of the specific tactical CR algorithm used. In fact, our results show that this difference between conflict rates caused tactical CR to reduce the total number of conflicts relative to the no resolution case, for some traffic densities. In other words, tactical CR caused a net stabilizing effect on the airspace, resulting in negative DEPs for some traffic densities.

A negative DEP implies that the majority of conflicts are resolved without pushing adjacent traffic into existing conflicts. Our results indicate that the DEP decreases with traffic demand for a range of low densities [7], see Figure 3, while it increases from negative to positive values for extreme traffic densities [6], see Figure 4. Since the conflict rate was considered to be independent of resolution maneuvers in [5], negative DEP values are *not* predicted using the model in [5].

III. Alternate Model Relating Stability and Capacity

To overcome the limitations mentioned above, an alternate model of the DEP is derived here for a decentralized direct routing airspace concept. This derivation consists of three main parts where the expected number of conflicts without conflict resolution, during one conflict resolution maneuver, and with conflict resolution, are considered separately for a single flight. In contrast to the previous model from [5], conflict rates with and without resolution are not required to be equal. Similar to [5], the number of conflicts detected is assumed to be proportional to the time spent searching for conflicts, for all three steps. The derivation below considers the 2D case, but can be easily extended to 3D.

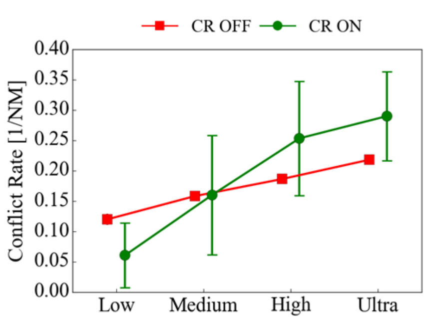

Figure 2: The conflict rate with and without Conflict Resolution (CR) is found to be unequal. Data taken from [6].

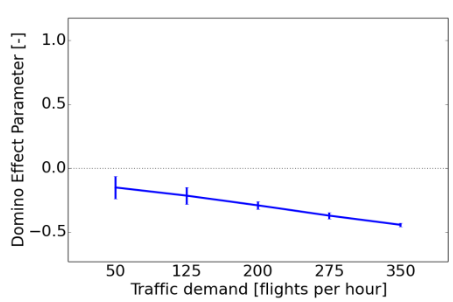

Figure 3: The Domino Effect Parameter (DEP) was found to be negative and decreasing for relatively low densities. Data taken from [7].

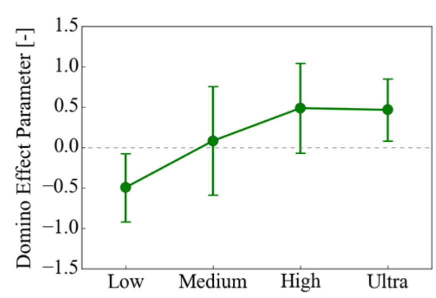

Figure 4: The Domino Effect Parameter (DEP) was found to increase from negative to positive values for relatively high traffic densities. Data taken from [6].

A. Modelling the Number of Conflicts Without Conflict Resolution

The expected number of conflicts without conflict resolution for a single flight, $E_{C_{NR}}$, is dependent on the rate of conflicts without conflict resolution, $r_{C_{NR}}$, and the total time searched for conflicts. For the no resolution case, this time is equivalent to the total fight time, $t_{f_{NR}}$:

$$E_{C_{NR}} = r_{C_{NR}} t_{f_{NR}} \tag{3}$$

Here, $r_{C_{NR}}$ is defined as the product of three terms: 1) the ratio between the area searched for conflicts per unit time and the total area A, 2) the ratio between the conflict detection look-ahead time, t_l, and the total observation time, T and 3), the probability of conflict between two aircraft *without* resolution, $P_{2_{NR}}$:

$$r_{C_{NR}} = \frac{d_{sep}\bar{V}}{A}\frac{t_l}{T}P_{2_{NR}} \tag{4}$$

In (4), d_{sep} is the separation minima and $\bar{V}$ is the average speed of the aircraft under consideration. Therefore, the term $d_{sep}\bar{V}t_l$ in (4) is the area searched for conflicts during the CD look-ahead time. Substituting (4) in (3) results in the following expression for $E_{C_{NR}}$:

$$E_{C_{NR}} = \frac{d_{sep}\bar{V}t_l P_{2_{NR}} t_{f_{NR}}}{AT} \tag{5}$$

B. Modelling the Number of Conflicts During One Conflict Resolution Maneuver

The expected number of conflicts during one resolution maneuver for a single flight, $E_{C_{RM}}$, is computed in a similar manner. However, during one conflict resolution maneuver, only the area corresponding to the look-ahead time, t_l, is searched for conflicts. Hence, $E_{C_{RM}}$ is equal to:

$$E_{C_{RM}} = r_{C_{WR}} t_l \tag{6}$$

The conflict rate with conflict resolution, $r_{C_{WR}}$, is defined similarly to $r_{C_{NR}}$. However, in this case, the probability of conflict between two aircraft *with* conflict resolution enabled, $P_{2_{WR}}$, is considered:

$$r_{C_{WR}} = \frac{d_{sep}\bar{V}}{A}\frac{t_l}{T}P_{2_{WR}} \tag{7}$$

Substituting (7) in (6) results in the following relation for $E_{C_{RM}}$:

$$E_{C_{RM}} = \frac{d_{sep}\bar{V}t_l^2 P_{2_{WR}}}{AT} \tag{8}$$

C. Modelling the Number of Conflicts With Conflict Resolution

The final step involves the computation of the expected number of conflicts with conflict resolution for a single flight, $E_{C_{WR}}$. When conflict resolution is enabled, aircraft use resolution maneuvers to avoid potential intrusions. Therefore, for each conflict detected, an area of airspace, corresponding to a look-ahead time of t_l, is searched for conflicts, but not flown through. This 'extra' airspace scanned for conflicts needs to be taken into account when computing $E_{C_{WR}}$, see Figure 5.

If $E_{C_{WR}}$ conflicts are detected, then the total area searched for conflicts, but not flown through, corresponds to a time equalling $t_l E_{C_{WR}}$, and the grand total time searched for conflicts, including the time when no conflicts are detected, equals $t_{f_{WR}} + t_l E_{C_{WR}}$, where $t_{f_{WR}}$ is the flight time with resolution. $E_{C_{WR}}$ can then be computed as:

$$E_{C_{WR}} = r_{C_{WR}}\left(t_{f_{WR}} + t_l E_{C_{WR}}\right) \tag{9}$$

Substituting the definition of $r_{C_{WR}}$ from (7) into (9) gives:

$$E_{C_{WR}} = \frac{d_{sep}\bar{V}t_l P_{2_{WR}} t_{f_{WR}}}{AT} + \frac{d_{sep}\bar{V}t_l^2 P_{2_{WR}}}{AT} E_{C_{WR}} \tag{10}$$

Using the relation for $E_{C_{RM}}$ from (8), and solving (10) for $E_{C_{WR}}$:

$$E_{C_{WR}} = \frac{d_{sep}\bar{V}t_l P_{2_{WR}} t_{f_{WR}}}{AT\left(1 - E_{C_{RM}}\right)} \tag{11}$$

The above relations for $E_{C_{NR}}$ and $E_{C_{WR}}$ can be used to derive an expression for the DEP by substituting (5) and (11) into (2):

$$DEP = \frac{N_{WR}}{N_{NR}}\frac{d_{sep}\bar{V}t_l P_{2_{WR}} t_{f_{WR}}}{AT\left(1 - E_{C_{RM}}\right)}\frac{AT}{d_{sep}\bar{V}t_l P_{2_{NR}} t_{f_{NR}}} - 1 \tag{12}$$

Simplifying (12) yields:

$$DEP = \underbrace{\frac{P_{2_{WR}}}{P_{2_{NR}}}}_{\text{Safety}} \cdot \underbrace{\frac{N_{WR}}{N_{NR}}\frac{t_{f_{WR}}}{t_{f_{NR}}}}_{\text{Efficiency}} \cdot \underbrace{\frac{1}{\left(1 - E_{C_{RM}}\right)}}_{\text{Domino}} - 1 \tag{13}$$

Note that the $E_{C_{RM}}$ term in (13) relates the DEP to airspace parameters through (8). Additionally, (13) shows that the DEP is inversely proportional to $1 - E_{C_{RM}}$. Hence, if $E_{C_{RM}} = 1$, then the DEP, and its gradient, will tend to infinity, i.e., if every conflict resolution is expected to trigger one new conflict, for each and every aircraft, then the airspace becomes completely unstable.

D. The Three Components of the Domino Effect Parameter

The DEP model of (13) can be split into three main components. The first component is a ratio of the conflict probabilities, with and without resolution, and is therefore related to safety. The second component is a ratio of the total flight times for all aircraft, with and without resolution, and is therefore related to (in)efficiency. Finally, the third component relates the DEP to the conflicts that occur during resolution maneuvers, $E_{C_{RM}}$, hence this component is called the 'Domino'.

To link DEP to capacity, it is necessary to relate the three components to density. At the time of writing this paper, the development of such relations is not yet complete. However, using logical reasoning, the qualitative relationships between the three components and density can be hypothesized, see Figure 6.

The predicted relationship of the (in)efficiency and the Domino components with density is fairly straight forward. At very low densities, $E_{C_{RM}}$ is expected to equal zero, and the total flight times with and without resolutions are expected to be the same, due to the sparsity of the airspace. Thus both components are expected to be equal to 1 at low densities. At higher densities, previous studies have shown that the total number of conflicts, and corresponding number of conflict resolution maneuvers, increase non-linearly [6], [7]. The resulting increase in flight times with conflict resolution will increase airspace inefficiency. Similarly at higher densities, $E_{C_{RM}}$, and thus also the Domino term, will increase. As discussed earlier, when $E_{C_{RM}} = 1$, the Domino term tends to infinity.

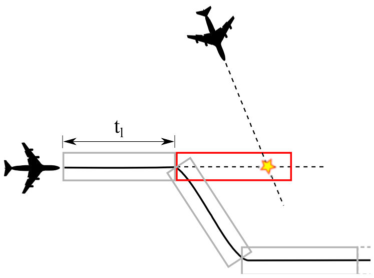

Figure 5: When conflict resolution is enabled, each conflict triggers an area of airspace, corresponding to a look-ahead time of t_l, to be searched for conflicts, but not flown through.

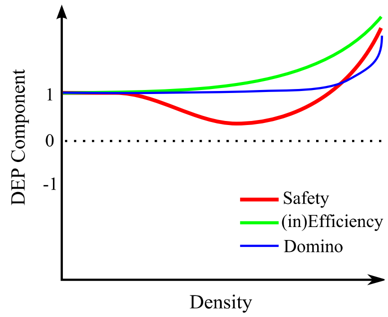

Figure 6: Hypothesized relationship between the three components of the Domino Effect Parameter (DEP) and density

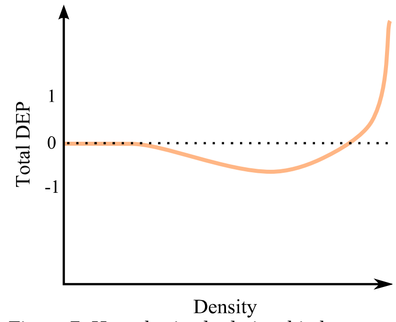

Figure 7: Hypothesized relationship between the Domino Effect Parameter (DEP) and density. Negative values are expected for a certain range of densities.

The hypothesized relationship between the safety component and density is somewhat more complicated. At low densities, the sparsity of the airspace is once again expected to cause no significant differences between the conflict probabilities with and without resolution, yielding a value close to 1 for the safety term. At higher densities, the type of CR method used is expected to heavily influence this component. The safety trend shown in Figure 6 applies for the Modified Voltage Potential (MVP) method, a type of voltage-potential based CR algorithm used in our previous studies [1],[7]. The charged particle behaviour of MVP has been shown to disperse traffic over the available airspace, reducing local traffic densities and the corresponding conflict probability, relative to the no resolution case, for a range of as yet unknown densities. However, at high densities, this self-structuring effect is likely to be outweighed by traffic congestion, causing the safety component to also increase.

Subtracting '1' from the product of the three hypothesized components of the DEP, as required by (13), results in the DEP trend pictured in Figure 7. Here, as implied by our previous experimental studies, the DEP is predicted to be negative for a range of densities, and increases to infinity when $E_{C_{RM}} = 1$.

E. Capacity Limits of Decentralized Airspace Concepts

Equation (13) has shown that safety and efficiency affects the DEP, a measure of traffic stability. Thus, safety, efficiency and stability need to be considered in *unison* when measuring the capacity of decentralized airspace designs.

Given the interrelations between safety, efficiency and stability, to empirically measure the capacity of a decentralized airspace concept using simulation experiments, it is necessary to determine the rate of change of safety, efficiency and stability metrics for a large range of traffic densities. The *theoretical* capacity limit for a particular airspace design can then be defined *as the lowest density at which the rate of change of safety/efficiency/stability metrics with density tends to infinity*. In other words, capacity, depending on the airspace design under consideration, is *limited by one of safety or efficiency or stability* metrics:

$$\rho_{capcity} = \min\left(\rho_{Sa}, \rho_{Ef}, \rho_{St} \left| \begin{array}{l} \lim\limits_{\rho \to \rho_{Sa}} \dfrac{\partial Sa}{\partial \rho} = \infty, \\ \lim\limits_{\rho \to \rho_{Ef}} \dfrac{\partial Ef}{\partial \rho} = \infty, \\ \lim\limits_{\rho \to \rho_{St}} \dfrac{\partial St}{\partial \rho} = \infty \end{array} \right.\right) \quad (14)$$

Using (14), the density for which $E_{C_{RM}} = 1$, corresponds to the density at which the stability related capacity limit is reached. While the above relation describes the *theoretical* capacity limit of a decentralized concept, in *practice*, society will not accept an asymptotic limit of safety as the capacity of the airspace. Moreover, airline economics, which is primarily focused on improving efficiency, will also be an influencing factor. However, determining the theoretical capacity limit is useful for comparing different decentralized concepts.

IV. Conclusions

This paper provided initial insights of a model for relating traffic stability to capacity, for decentralized airspace concepts using self-separation. In contrast to previous research, the model assumes that the conflict rate with and without conflict resolution can be different. The model showed that the stability related capacity limit occurs for the density at which all aircraft exist perpetually in a conflicted state. Furthermore, the model indicated that safety and efficiency affects traffic stability, implying that the capacity measurement of decentralized concepts should consider these three intrinsic system properties in unison to measure capacity. A number of hypotheses were made on the relationship between the model components and traffic density. Future work will further develop this model, and verify these hypotheses using targeted simulation experiments over a wide range of densities.

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