Modeling and Inferring Aircraft Takeoff Mass from Runway ADS-B Data

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***Abstract*—Aircraft mass is an important parameter in many ways, either to build aircraft performance models, to predict flight trajectories, or to simulate air traffic. Mass data is usually considered as sensitive information for airlines and is, therefore, not disclosed to researchers publicly. In this paper, we use two methods to infer the mass of an aircraft at its takeoff phase. The first is by studying the kinetic model at lift-off moment. The second is to look at the motion of aircraft on the runway at each sample moment to estimate the mass recursively.**

***Keywords*—aircraft mass, performance modeling, weight estimation, BlueSky.**

I. Introduction

Aircraft takeoff weight is an essential element of aircraft performance. Parameters such as lift, thrust, fuel consumption, and climb/descent rate are all related to the initial takeoff weight. Current implementations of performance models, such as BADA [1], use a reference mass for each aircraft type. This might, however, not be sufficient in all user cases. In more fundamental ATM studies the ability to model the takeoff weight is also important [2]. Due to the confidential nature of airline takeoff weight data, researchers are only able to infer aircraft weight using aircraft motion models. Previous studies have been conducted in order to estimate aircraft weight using track data during the climb phase [3], [4].

In the current study, the takeoff phase is used to estimate aircraft mass. Giving standard atmospheric and wind conditions for the same aircraft model under similar thrust settings, the aircraft acceleration profile is in close correlation with its weight. Even with slight deviations from standard conditions, extraction of averaged weight estimation models is still possible through observation of large amounts of data. This study will employ ADS-B data from aircraft takeoff, in combination with physical kinetic models, to establish two different estimation methods that are able to infer aircraft takeoff weight.

II. Aircraft models at takeoff

In order to study aircraft weight, simplified kinetic point-mass models are used, where there are six forces acting at the takeoff phase.

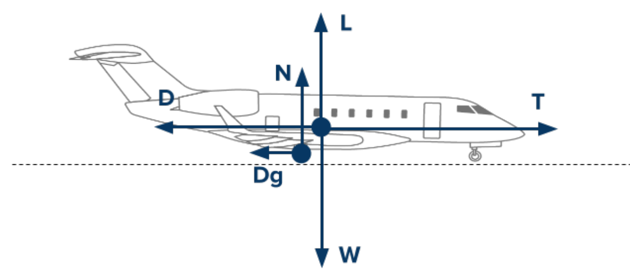

Fig. 1. Forces acting on aircraft at takeoff

In cruising flight, forces that act on the aircraft relate to thrust (T), drag (D), lift (L), and weight (W). During takeoff, two additional forces need to be taken into account; the normal force (N) and the ground drag (D_g). Here, D_g can be assumed proportional to the normal force N:

$$Dg = \mu N = \mu(W - L)$$

where μ is the friction coefficient. The weight of an aircraft can be considered as constant due to the low amount of fuel burnt in such a short time period. The normal force decreases as the lift increases together with the acceleration of the aircraft.

While the aircraft is in its ground phase, there is no vertical acceleration or movement. Assuming a flat runway, it can be modeled solely as horizontal motion described as follows:

$$T_i - D_i - \mu(W - L_i) = m \cdot a_i \tag{1}$$

Aircraft may sometime use a reduced power takeoff. As such a coefficient is introduced to represent the thrust used by the aircraft. The lift and drag of the aircraft can also be calculated with known lift and drag coefficients. The following equations represents these relations:

$$T_i = \eta T_{max}$$
$$L_i = \tfrac{1}{2} C_L \rho V_i^2 S$$
$$D_i = \tfrac{1}{2} C_D \rho V_i^2 S$$
$$C_D = C_{D0} + K C_L^2$$

Here, S is the surface area of the wings, ρ is the air density, and V represents the airspeed of the aircraft. C_L, C_D, C_{D0}, and K are coefficients for lift, drag, zero drag, and lift-induced drag, respectively.

In the remainder of this paper, we will try to estimate the mass with two approaches. The first is to look at the lift-off movement acceleration and velocity to infer the aircraft weight. The second is to recursively estimate the weight at the entire takeoff phase at each data point.

A. Inferring aircraft mass at the lift-off moment

At the moment of lift-off, the airspeed has increased sufficiently for the lift L to equal the aircraft weight. At this point, the normal force and ground drag become zero, but there is still only horizontal speed and acceleration. Here, the model simplifies to:

$$L = W \Rightarrow \tfrac{1}{2} C_L \rho V_{lof}^2 S = mg$$
$$T - D = m \cdot a_{lof}$$

When substituting the thrust and drag equations, the above equation becomes:

$$\eta T_{max} - \tfrac{1}{2} C_{D0} \rho V_{lof}^2 S - \frac{2m^2 g^2 K}{\rho V_{lof}^2 S} = m \cdot a_{lof}$$
$$\Downarrow$$
$$\frac{2g^2 K}{\rho S} m^2 + a_{lof} V_{lof}^2 m + \tfrac{1}{2} C_{D0} \rho S V_{lof}^4 - \eta T_{max} = 0$$

This can be rewritten in the following simplified quadratic form of mass m:

$$\theta_1 m^2 + a_{lof} V_{lof}^2 m + \theta_2 V_{lof}^4 - \eta T_{max} V_{lof}^2 = 0 \qquad (2)$$

Where:

$$\theta_1 = \frac{2g^2 K}{\rho S} \text{ and } \theta_2 = \tfrac{1}{2} C_{D0} \rho S$$

It can be assumed that θ_1 and θ_2 are constant for the same aircraft model under standard atmospheric conditions. The aircraft mass can be found by solving the above quadratic equation and taking the positive root of m.

Zero drag coefficient and lift induced drag coefficients can be found by using performance libraries such as BADA or literature studies [5]. ADS-B data can be used to interpolate the ground velocity and acceleration at the lift off moment. In order to get the airspeed of the aircraft, aggregated weather data at the observed airports can be used to calculate the TAS (true airspeed) of aircraft at takeoff.

After identifying the coefficients, the calculation depends solely on the acceleration (a) and velocity (V) of the aircraft at lift-off. Figure 2 shows a theoretical grid of the relationship between velocity/acceleration and aircraft mass. For illustration, the estimations of around 10K flights of B738 and A320 aircraft each are displayed on top of the theoretical grid.

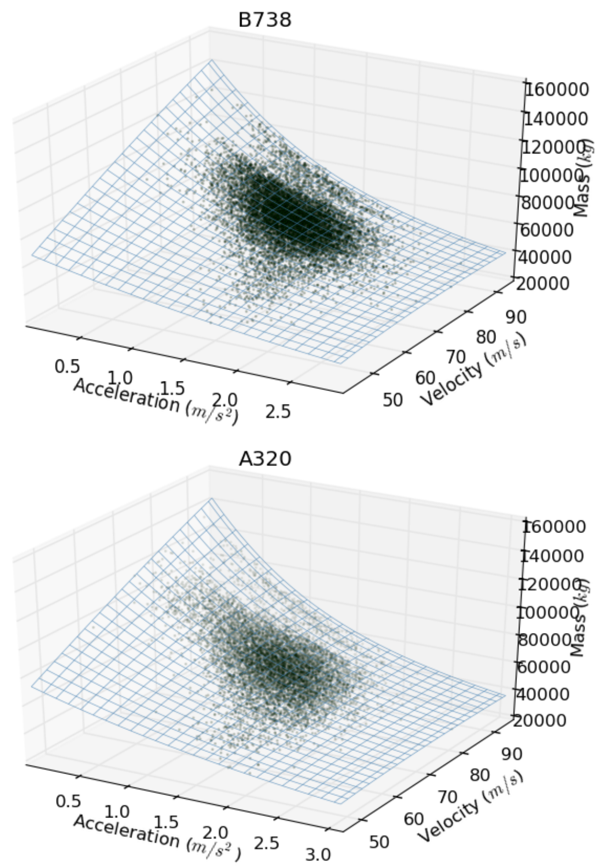

Fig. 2. Aircraft estimated mass distribution on V/A grid

B. Recursive takeoff mass estimation

The second method looks at the entire takeoff phase on the runway. Constant weight is still assumed in this short period of time. First, the previous Equation (1) can be rewritten in following form:

$$T - \tfrac{1}{2}\rho V_i^2 S(C_D - \mu C_L) - (\mu g + a)m = 0 \quad (3)$$

During the takeoff, thrust can be modeled as a function of speed using the following empirical equation [6]:

$$T = T_0(1 - \epsilon V^2) = \eta T_{max}(1 - \epsilon V^2)$$

where T_0 is the static thrust at zero speed. This coefficient ϵ can be positive, negative, or zero. Equation 3 then becomes:

$$\eta T_{max}(1 - \epsilon V_i^2) - \tfrac{1}{2}\rho V_i^2 S(C_D - \mu C_L) - (\mu g + a_i)m = 0$$

$$\Downarrow$$

$$\begin{aligned}\eta T_{max} - \left[\eta\epsilon T_{max} + \tfrac{1}{2}\rho S(C_{D0} + KC_L^2 - \mu C_L)\right] V_i^2 \\ -(\mu g + a_i)m = 0\end{aligned} \quad (4)$$

By measuring the velocity at each sample point, the mass of the aircraft can be recursively calculated. The challenge lies in estimating all unknown coefficients. These coefficients are listed as follows:

- ϵ: thrust-related coefficient
- μ: ground friction coefficient
- C_{D0} and K: zero drag and induced drag coefficients
- C_L: lift coefficient

The thrust-related coefficient ϵ is found empirically [6]. To minimize the takeoff distance, the lift coefficient C_L usually is optimized in relation with the ground friction coefficient [6], as follows:

$$C_L = \mu / 2K$$

Thus, the equation (4) becomes:

$$\eta T_{max} - \left[\eta\epsilon T_{max} + \frac{1}{2}\rho S(C_{D0} - \frac{\mu^2}{4K})\right] V_i^2 - (\mu g + a_i)m = 0 \quad (5)$$

Both the zero-lift drag and induced drag coefficients can be found using the BADA performance library for most aircraft types.

The ground drag plays a big role during the takeoff. However, very little accurate data can be found. To model the ground friction coefficient, a few previous studies have been conducted to build estimations for different runway conditions versus aircraft types [7], [8]. A small fixed value is used [6].

As such, the aircraft mass becomes a simple function of velocity and acceleration. The acceleration at each measurement point a_i is noisy due to the fact that it is estimated using the derivative of V_i. Instead, the average acceleration during takeoff can be used as an approximation for each sample point.

$$m_i = f(V_i^2, \bar{a}) \quad (6)$$

Then the final mass of the aircraft can be approximated by minimizing the following:

$$m = \underset{m}{\operatorname{argmin}} \sum_{i=1}^{n} \left[m - f(V_i^2, \bar{a})\right]^2 \quad (7)$$

III. Experiments

A. Data processing

Previous related research has presented methods to extract individual flights from large amounts of data using machine learning methods [9]. This method can be used to process scattered flight data from ADS-B sources. With global ADS-B networks such as FlightRadar24, a total of around 40 GB air traffic data on a wide range of aircraft models can be recorded within the period of a week. These data can be processed to extract around 10,000 takeoffs of A320 and B738 aircraft for this experiment. These data cover the majority of operational aircraft of those two models from airlines around the world.

ADS-B position and velocity measurements originate from aircraft GPS/GNSS receivers. These data can often be noisy due to measurement uncertainties [10]. To extract takeoff trajectories with proper data, a second degree polynomial function is fitted on the velocity data of each takeoff. By studying the root squared error of the prediction, a margin is applied to exclude unrealistic data. Figure 3 illustrates 10% of the speed profiles of B738 in the experiment data sets, which indicates filtered data at the runway during takeoff.

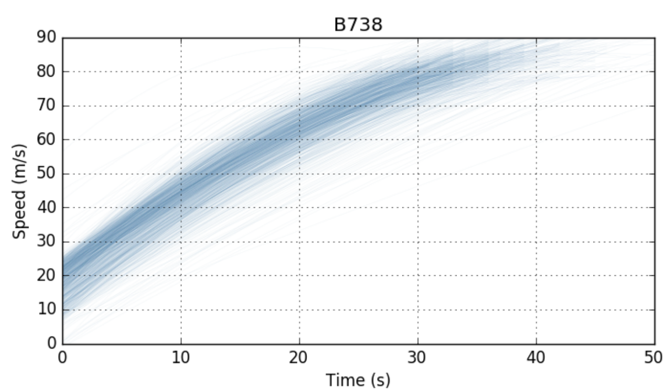

Fig. 3. Velocity profile at runway takeoff

As for the coefficients related to the calculation, from empirical data, η, ϵ, and μ are chosen as 1, 2.7e-5, and 0.02 respectively. T_{max}, C_{D0}, and K are from existing BADA performance model.

B. Mass Estimation Results

Both methods were applied to the same datasets of the two aircraft models. The results are shown in Figure 4. Blue plots correspond to the mass estimated from the lift-off moments, while the red plots are generated from the recursive method. The green areas represent the valid masses, of which the left bound is operational empty weight (OEW) and right bound is the maximum takeoff weight (MTOW).

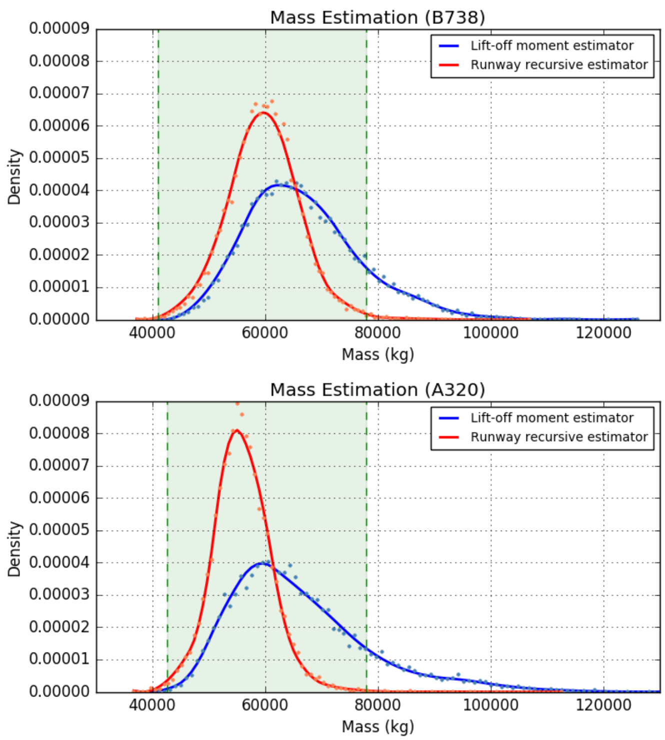

Fig. 4. Velocity profile at runway takeoff (approx. 10k flights)

In the recursive method, most of the estimations fall into the valid area, which can be considered as a better estimation. However, the BADA thrust model presented in the simulation uses maximum climbing thrust rating, where the actual rated thrust is likely to be higher during the takeoff phase. Both A320 and B738 are equipped with turbofan engines, hence the thrusts are slightly different than the model presented in reference [6]. Both facts can influence the estimation of takeoff weight.

Without proper reference data on the mass of aircraft, it is hard to validate the accuracy of the results at each takeoff sample. However, the two independent methods show fairly similar results. This can be considered as a statistical cross validation. Looking at other studies on weight estimation, the methods in this paper also yield similar results, despite the fact that different data sources were used [3].

IV. Conclusions

This paper proposed two mass estimation methods that incorporate flight takeoff data. The results of this paper show that both methods lead to reasonable estimations of aircraft mass.

Open method alternatives to BADA that consist of values obtained empirically or from other open sources can be implemented [11]. The BADA thrust model lacks precision for ground segments. The implementation of a more accurate thrust model can also improve the results [12].

Due to the use of a large set of global traffic data, wind is a factor that is not compensated for in the data. Instead, the aircraft ground speed was used to approximate the airspeed at takeoff. Statistically, the positive and negative influences of wind should cancel out. However, further studies can be conducted from fewer selected airports to factor in the wind effect.

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