Nonlinear Dynamics Approach for Modeling of Air Traffic Performance Disruption and Recovery

Norbert Fürstenau, Monika Mittendorf
Institute of Flight Guidance
German Aerospace Center (DLR)
Braunschweig, Germany
Norbert.fuerstenau@dlr.de

Shumpei Kamo
Dept. of Aeronautics and Astronautics
The University of Tokyo
Tokyo, Japan
shunkam35@gmail.com

***Abstract*—The inclusion of a priori knowledge on random disturbance into stochastic and robust optimisation of arrival and departure scheduling for controller assistance is a topic of ongoing research. One direction is the development of advanced algorithms utilizing (microscopic) stochastic models of empirical traffic rate and delay time statistics derived from hundreds of flights over limited periods of time. The typically asymmetric delay distributions usually include some extreme disturbances ("Xevents", e.g. low pressure weather situations) which often are disregarded as outliers. In this contribution we focus on the modeling of disruption and recovery dynamics of performance during single Xevents. Based on previous work in the literature on formalizing performance disruption and systems resilience we analyse the potential of a simple logistic dynamics approach for deriving characteristic performance parameters (e.g. disruption / recovery time constants) from empirical data. We continue with initial results obtained with time dependent control parameters dependent e.g., on wind/gust speed variation, for dynamic performance simulation using nonlinear first and second order dynamics. This macroscopic low dimensional order and control parameter approach has the potential for anticipative disruptive events management within the Viability theory framework and complements microscopic (high dimensional) stochastic and robust optimization based scheduling.**

performance; airport; extreme events; disruption; recovery; resilience; logistic model; nonlinear dynamics; Viability theory

I. Introduction

One specific bottleneck of air traffic management (ATM) is the airport due to capacity limitations. Propagation of departure delays from origin to destination airports and adverse weather conditions en route and on the airports represent major causes for arrival delays. Recent approaches for supporting optimized management actions under normal traffic conditions utilize a-priori knowledge on empirical average delay statistics based on hundreds of flights as input into advanced stochastic and robust optimization algorithms [1][2]. Besides delays measured under normal traffic conditions there are periods of time with disruption effects and reduced performance. Performance disruption is defined here as decrease of airport capacity so that the typical average number of arrivals and departures decreases significantly within a given time interval due to an external extreme or rare event (termed Xevent in [3]), e.g. a heavy storm with large wind speed and gusts in a low pressure situation, deviating significantly from the normal situation. Disruption may be avoided in many cases if the system resilience and robustness of the system can be increased. E.g. when predicted strong sidewinds and gusts during a low pressure event leads to expectation of runway closing a suitable anticipating management action might consist in preparing landing and transport capacity at a neighbouring airport with different conditions (weather, runway orientation), possibly separately for the different aircraft weight classes. These anticipatory actions ideally would be accompanied by optimized re-scheduling (e.g. [1][2]).

Understanding and modeling the statistics, dynamics, and propagation of air-traffic arrival and departure delays is a prerequisite of any attempt to optimize the punctuality of schedules and airport capacity (e.g [4][5]. In the present report we present a simple nonlinear dynamics performance model that formalizes with macroscopic state variables previously published concepts [6][9][11]. It is expected that such a dynamical systems model could also pave the way for the application of the well founded Viability Theory framework [7] that was developed for quantifying the resilience dynamics of complex systems far from equilibrium by means of a small number of system state parameters. The purpose of the present approach is the derivation and initial validation of the proposed model for supporting anticipative management actions, with arrival scheduling as a specific example. It aims at keeping the system within an operational state under extreme (rare) disturbance and avoid or quickly recover from disruption of performance. It complements the (microscopic) optimized scheduling of individual flights that uses robust and stochastic algorithms with inclusion of averaged disturbance statistics for protecting planning against normal (everyday) uncertainties.

The resilience and robustness of air traffic systems has emerged as a research topic of increasing importance in recent years (e.g.[11][13]). From a formal point of view the sociotechnical ATM system has to be treated as a complex system: many participating agents (airlines, aircraft/ pilots, air traffic control / controllers, airport operators) on different hierarchical levels interact with delays under ressource and capacity limitations (nonlinearity). This gives rise to characteristic features such as bifurcation and phase transitions between qualitatively different system states, and onset of deterministic chaos [8]. A number of theoretical resilience models and metrics have been published recently, however mostly focused to different fields, such as infrastructure [9] and ecological viability [7]. Only few approaches like the viability theory [7] make explicit use of the results and methods available within complexity research. Relevant concepts for predictability, prediction and anticipation of traffic disruption through extreme events can be derived from the dynamical systems based interpretation of “Xevents” [3]. Kantz et al. [10] provide an example for a Markov chain based stochastic time series analysis of turbulent wind gust that indicates the potential for prediction of disruptive disturbance as precondition for anticipatory resilient systems. The dynamical systems approach of the present work integrates also stochastic components for Monte Carlo simulations within a Langevin formalism [8] that will be the topic of a followup paper.

In what follows, a simple logistic performance model will be derived and tested with two empirical examples of capacity disruption during heavy storm events at a large German airport. In section II the empirical wind/gust and traffic time series are presented. The dynamical model for Xevent modeling and simulation is derived in section III. In section IV the model is applied to the specific empirical traffic and disturbance data. Performance and Wind profile fitting yields the disturbance and recovery time constants, stationary states and extrema that are discussed in section V. Perspectives for dynamical simulations are illustrated in section V.B. A conclusion follows in section VI.

II. Traffic disturbance under disruptive weather

Fig.1 depicts for a large German airport time series of normalized arrival rates for four consecutive days during a week in October 2013. Data are normalized with respect to nominal maximum capacity = 27 arrivals/h and corrected for different yearly averages between weekdays (Mo – Fr: -2 to -12 %, Sa / Su: +23% / +11 %). At this airport there is no night traffic between 23:00 and 06:00. On day 2 of the four days the low pressure storm event “Christian” (the Xevent) hit the North Sea coast around 12:00 h local time, with a strong increase of wind and gust speed at the airport (see Fig.4).

Typically, a strong increase of arrivals is observed for day 1 of each week (Sunday night) and a decrease around noon each day. In addition to this quasi-periodic traffic decrease day 2 exhibits a significantly decreased arrival rate as compared to the yearly average. The Xevent induced traffic decrease is superimposed to the quasi-periodic arrival rate minimum around noon. We want to demonstrate our dynamical performance model to allow for quantitative discrimination between the normal weather situation and the Xevent.

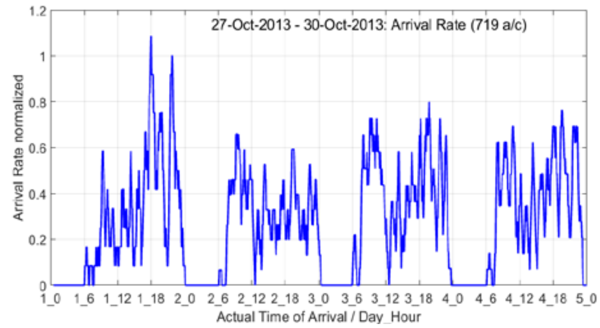

Figure 1. Time series of 719 arrivals over four consecutive days (30 min average rate, Sunday, Oct. 27 – Wednesday, Oct. 30/2013, no traffic from 11 pm to 6 am) with normalized (to maximum capacity 27 arr./h) rates corrected for different weekday averages (time scale: local, CET). “Christian” low pressure disturbance on Oct. 28: traffic disruption starting at day 2, around noon

A second example is depicted in Figure 2. It shows the normalized and corrected arrival rate time series of four days (Thursday to Sunday) including disruption and recovery during the “Xaver” low pressure event. The 4-day time series exhibits traffic disruption around 1 pm of day 1, with gradual recovery around noon of day 2.

Analysing the traffic disruption due to external disruptive events requires consideration of normal periodicities, trends and random noise in the time series. In the two examples of Fig.1 and Fig.2 we recognize the periodic interruption of traffic during nighttime (11:00 – 6:00 h), as well as daily traffic minima around noon for most days. Various approaches for modeling the periodicities and seasonal effects of traffic time series under normal conditions have been published recently (e.g. [4]). In what follows we focus on the potential of our logistic model approach for discriminating normal (mostly periodic) deviations from (more or less) stationary traffic time series as baseline, from Xevents induced performance disruption and on developing a corresponding simulation algorithm with the (medium term) goal of predicting Xevents.

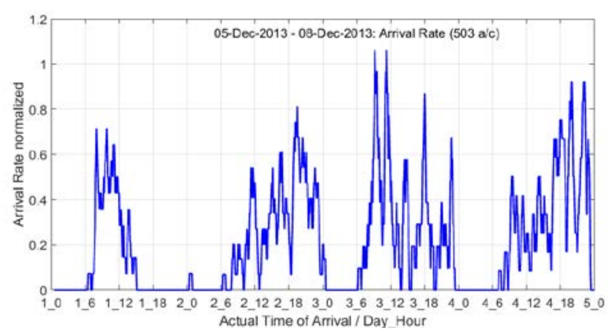

Figure 2. Time series of 503 arrivals for four consecutive days (30 min averages) (Thursday Dec. 5 (day 1) – Sunday Dec. 8), rate normalized to maximum capacity (27 arr./h) and corrected for different daily averages. Time scale: local time CET). “Xaver” low pressure event inducing traffic breakdown at day 1 around noon, with recovery starting day 2.

III. DYNAMIC PERFORMANCE MODEL FOR X-EVENTS

Recently generic frameworks and metrics were suggested for the analysis and quantitative approaches for system resilience as a function of time [6][9]. Starting with the schematic in [11] that depicted an idealized linear decrease ΔF_D of system functionality F(t) or performance at disruption time μ_D due to disruptive event, with subsequent recovery at μ_R after a delay $\Delta\mu = \mu_R - \mu_D$ we derive a simple formal model that allows for quantifying the time course of performance disruption and recovery as well as the disturbance dynamics by means of characteristic event times, functionality levels and changes, disruption and recovery time constants and delays.

A. *Logistic Performance Model with constant rate parameter*

The most simple dynamical continuous two-state model of relevance for the problem of system state disruption and recovery may be represented by the well known logistic differential equation (ODE, Verhulst equation) with quadratic nonlinearity. It is used, e.g. for describing the birth rate within a system of limited ressources or carrying capacity C. An example is a simplified model of Laser action that describes the transition between the two stationary states, i.e. from normal light (random photon) emission to coherent photon emission as external pump energy induced bifurcation at the laser threshold [8]. It is achieved by means of the homogeneous first order second degree equation for the time variation of photon number in the laser cavity $dn/dt = \dot{n} = k_1 n(k/k_1 - n)$ with control parameters k, k_1 where increasing pump power leads to a sign change of k at the instant of the phase transition to coherent laser light emission.

In what follows the dynamical variables of interest are the performance decrease or disruption of functionality F(t) from an initial normal (stationary) state F_0, to an (asymptotic) disruptive state. Disruption is followed, after a delay $\Delta\mu$, by performance increase or recovery from the disruptive state F_D back to normal functionality. In the present model the asymptotic $F_D \approx$ minimum performance F_{min} is reached only if $\Delta\mu >>$ time constants τ_D, τ_R of disruption and recovery. Under strong disturbance we assume the performance decrease dF/dt to start proportional to the momentary value F(t) from an initial stationary value $F_0 \approx C$ (exponential decay). For the lower asymptotic limit F_D of F(t) we have $0 < F_D < F(t) < C$. The (exponential) reduction of the F(t)-decrease rate when approaching F_D is realized by a factor $(F(t) - \Delta F)$, $\Delta F = C - F_D$. It turns out that for our practical problems it is sufficient to assume $F_D = 0$, i.e. $\Delta F = C$, and like the laser phase transition both disruption and recovery can be represented by the first order second degree (nonlinear) Verhulst equation

$$df/dt = \dot{f}(t) = kf(t)\big(1 - f(t)\big) \tag{1}$$

with normalization $f = F/\Delta F$ and constant rate parameter $k = 1/\tau =$ inverse time constant) that generally has different values for disruption (k_D) and recovery (k_R). The sign of the rate parameter k, i.e. $k > 0$ for increasing rate (recovery from disrupted state), $k < 0$ for decreasing rate (disruption of functionality) determines the sign of change of initial state. The well known general solution of (1) is a sigmoid function

$$f(t) = \frac{1}{1 + exp\{-k(t - \mu)\}} \tag{2}$$

where $\tau = 1 / |k|$ represents the time constant, the sign of the exponent represents decreasing (+) or increasing (-) transition (sigmoid), and μ = half transition time: $f(t = \mu) = ½$ (characterizing time of state change). It is easily verified that e.g. for the negative exponent the two stationary states are achieved as $\lim f(t \rightarrow +\infty) = 1$ (increasing) and $\lim f(t \rightarrow -\infty) = 0$, corresponding to $\lim F(t \rightarrow -\infty) = 0$ and $\lim F(t \rightarrow +\infty) = C$, respectively.

At $t = 0$ the normalized functionality for the increasing sigmoid has the value $f_0 = 1/ (1 + \exp(\mu/\tau)$, yielding for the time-shift parameter μ

$$\mu = \tau ln\left(\frac{1}{f_0} - 1\right) \tag{3}$$

so that in this simplified model for a single sigmoid $\mu >> \tau$ is required in order to use $f_0 \approx \lim f(t \rightarrow -\infty) = 0$ as the quasi-stationary initial state.

The time course of the disruptive event with increasing and decreasing external disturbance and correspondingly varying performance decrease and increase requires a time dependent rate parameter, like the varying pump power with the laser phase transition. For the purpose of fitting the shape of the generic average performance variation an analytical (heuristic) approximation can be constructed by considering discrete instances of time only (μ_D, μ_1, $\mu_2 = \mu_R$), with performance decrease from initial stationary level C at disruption time μ_D, possibly an initial quick recovery to an intermediate level F_1 at μ_1 after a short delay $\Delta\mu_1 = \mu_1 - \mu_D$, followed by the sustainable recovery towards the stationary final state $F_2 = F_R = C$ at $\mu_2 = \mu_R$ after appropriate management actions (also $F_R \neq C$ is possible). Our basic model for time series fitting consists of a succession of three sigmoids (2), starting with a decreasing (disruption: $k < 0$) and followed by one or two increasing functions (recovery: $k > 0$):

$$F(t) = \frac{\Delta F_D}{1 + exp\left\{+\frac{t - \mu_D}{\tau_D}\right\}} + \sum_{i=1}^{2}\left[\frac{\Delta F_i}{1 + exp\left\{-\frac{t - \mu_i}{\tau_i}\right\}}\right] \tag{4}$$

F = non-normalized performance scale (e.g. arrivals/h), $\Delta F = F_0 - F_{min}$. For the examples provided below it turned out that the sum of two sigmoids only, with $\Delta F_D = C - F_D$, $\Delta F_1 = 0$, $\Delta F_2 = \Delta F_R = F_R - F_D$, and $F_D = 0$ is sufficient for fitting the disruption – recovery time series, so that ΔF_D and ΔF_R represent the stationary levels before and after the disturbance. By inspecting the asymptotic values $F(t \rightarrow \pm\infty)$ one finds that for $F(\lim t \rightarrow -\infty) = C$ and $F(\lim t \rightarrow +\infty) = F_R$.

Equation (4) formalizes concepts presented in [9][11] and allows to attribute characteristic stationary performance levels, performance minima, time constants, and delays to the Xevent. The performance value in the center between decrease and increase at $(\mu_R + \mu_D)/2 = \Delta\mu/2$ is obtained as

$$F_{min} \approx \frac{C}{1 + exp\left\{+\frac{\mu_R - \mu_D}{2\tau_D}\right\}} + \frac{F_R}{1 + exp\left\{+\frac{\mu_R - \mu_D}{2\tau_R}\right\}} \quad (5)$$

which equals the minimum performance F_{min} in case of symmetric time constants $\tau_D = \tau_R$. For $\Delta\mu >> \tau$ it converges to F = 0. In section IV a 2-state version of (4) will be demonstrated to be sufficient for fitting the empirical arrival rate disruption measured during extreme storm events.

B. *Time dependent parameters and external control*

While the logistic model with constant rate parameters k_D, k_R at discrete transition times μ_D, μ_R turns out to be sufficient for empirical performance time series fitting, any realistic dynamic simulation for real-time prediction requires time dependent parameters for the external disturbance including noise. In the present approach a dynamic disturbance parameter is obtained by fitting empirical wind / gust speed time series v(t) of the Xevent by means of the same logistic model (4) as the traffic performance data. Because, to a first approximation, speed increase dv/dt > 0 is assumed to correlate with performance decrease dF/dt < 0, the succession of sigmoids is exchanged and parameters in (4) correspondingly renamed. Wind/gust speed increase Δv_{inc} at μ_{inc} is followed by a decrease by Δv_{dec} to normal level at $\mu_{dec} = \mu_{inc} + \Delta\mu$, where μ_{inc} replaces $\mu_2 = \mu_R$ in (4), μ_{dec} replaces μ_D in the first summand and $\mu_{dec} > \mu_{inc}$. We now have a speed maximum v_{max} near $(\mu_{inc} + \mu_{dec})/2$:

$$v_{max} \approx \frac{\Delta v_{inc}}{1 + exp\left\{-\frac{\Delta\mu}{2\tau_{inc}}\right\}} + \frac{\Delta v_{dec}}{1 + exp\left\{-\frac{\Delta\mu}{2\tau_{dec}}\right\}} \quad (6)$$

For large intervals $\Delta\mu$ between disturbance increase and decrease it approaches $v_{max} = \Delta v_{inc} + \Delta v_{dec} = 2v^*$ if stationary levels before and after the storm are equal $\Delta v_{inc} \approx \Delta v_{dec} = v^*$. This model provides the disturbance parameters, i.e.wind / gust speed profile and via fit residuals also noise as input into dynamical simulations.

In order to allow for a time dependent rate parameter k(t) with the potential of generalization to two independent rate parameters and external control q(t) an extension of the basic Verhulst model leads to the (inhomogenous) Ricatti equation

$$\dot{f} + p(t)f + r(t)f^2 = q(t) \quad (7)$$

with different rate parameters for the linear and quadratic term and external disturbance or control q(t). If we keep the single parameter approach (1) we have time dependent parameter functions p(t) = - k(t), r(t) = +k(t). This problem is solved with the Ansatz f = u + y via transformation into a 2[nd] degree Bernoulli equation for y(t) if a particular solution u(t) for the Ricatti equation can be determined (e.g. [12]):

$$\dot{u} - ku + ku^2 = q(t) \quad (8)$$

$$\dot{y} - k(1 - 2u)y + ky^2 = 0 \quad (9)$$

As a simple approach we use q(t) ~ k(t) yielding $u = f_p$ = const. as particular solution:

$$f_p = \frac{1}{2}\left(1 \pm \sqrt{1 + 4q/k}\right) \quad (10)$$

The general solution of (9) is obtained by standard procedures [12] *as*:

$$y = \frac{exp\left\{c_p \int_{t_0}^{t} k(\vartheta)d\vartheta\right\}}{\left[c + \int_{t_0}^{t} k(\vartheta)\, exp\left\{c_p \int_{\vartheta_0}^{\vartheta} k(\zeta)d\zeta\right\}d\vartheta\right]} \quad (11)$$

with $c_p(q) = 1 - 2f_p$. For k = const. the solution (2) of the Verhulst equation is confirmed, although now including a modification of k due to the external control c_p:

$$y(t) = \frac{c_p}{1 + c\, exp\left\{-c_p kt\right\}} \quad (12)$$

with c_p = 0 or 1 for q = 0, and constant c determined by the time-shift parameter μ of (2). The general solution $f = y + f_p$ of the Ricatti equation in principle provides an analytical model for the performance disruption if we include an integrable explicit function for the rate parameter k(t). The latter is obtained, e.g. from a fit to the wind/gust speed data. This becomes a problem, however, when including noise into k, q, that is required for realistic modeling.

That is why based on the logistic approach we derive the structure of a simple dynamical performance model that allows for simulating experimental time series with time dependent rate parameter k(t) and recovery (management) actions modeled as adaptive feedback gain G(f). From the general solution $f = y + f_p$ we obtain

$$\dot{f} = k(t)c_p(q)f(1 - f) + k\, h(f_p) \quad (13)$$

Here we will neglect $h(f_p)$ for simplicity. Following [8] for a dynamic two-state system, $G(f) \sim c_p(t)$ due to its dependence on the external control q is introduced as a new gain control parameter realizing adaptive feedback. We arrive at the coupled state space equations for the normalized performance (order parameter f(G)) and feedback gain control parameter G(f):

$$\dot{f} = k_0 w(v, \dot{v}) G\, f(1 - f)$$
$$\dot{G} = \frac{f_b - f}{\tau_f} + \frac{G_b - G}{\tau_G} = c_G - f/\tau_f - G/\tau_G \qquad (14)$$

with $k(t) = k_0 w$, $k_0 = 1/\tau_0$, $c_G = (f_b + \eta G_b)/\tau_f$, $\eta = \tau_f/\tau_G$, $\Delta G = G_b - G$. $w(v, dv/dt)$ is defined via the external disturbance, i.e. in our case the profile of wind speed and speed change:

$$w(v, \dot{v}) = (1 - v/v_{th})\dot{v}/\dot{v}_0 \qquad (15)$$

Speed profile is obtained from logistic function (4) fitting to empirical data or alternatively through dynamical model (8) fitting with suitably selected rate function k(t) and q(t) = 0. With constant control G, this disturbance model changes its sign when speed v > threshold v_{th} under increasing speed dv/dt > 0, so that an initially recovery type performance function becomes disruptive as a kind of phase transition into the alternate system state. After v starts decreasing, performance f starts to recover due to transition back into the original state. Wind speed/gust profiles and derivatives are obtained by logistic models for fitting empirical disturbance time series as demonstrated in the following section. This process is modulated and stabilized by management actions via adaptive feedback control G(f), through adjustment of bias values f_b, G_b and time constants τ_f, τ_G as illustrated in Fig. 6 of section V.B.

IV. Logistic Function Fitting of X-Event Time Series

For the nonlinear fits equation (4) is used as disruption – recovery model with bias v_D=0, F_D = 0 for wind and traffic respectively, and two logistic functions, disruption and recovery, with parameters sets for wind speed increase (v_{inc}, μ_{inc}, τ_{inc}), traffic rate decrease (ΔF_D, μ_D, τ_D), and decreasing wind (v_{dec}, μ_{dec}, τ_{dec}), increasing traffic rate (ΔF_R, μ_R, τ_R) respectively.

A. Baseline Time Series

Although the traffic data are corrected for systematic variations between daily averages the numerical arrival rate parameter values from model based fits during to the X-event time series usually can not be taken as the disruption – recovery characteristics because they are not corrected for periodic intra-day variations. A baseline for comparison with the disruption at X-event days may be derived from the normal weather arrival rate time series, with corresponding analysis depicted in the following Fig. 3 two weeks after Christian and one week before Xaver respectively, for the same day of the respective week. The numerical results shown in the figure are collected also in TABLE II. of section V.

The traffic disruption induced by "Christian" occours around noon and afternoon of day 2 where also under standard conditions a characteristic arrival rate reduction is observed each day (between 11:00 am and 3:00 pm), as can be seen in the top graphics of Fig.3. On the other hand, "Xaver" extends over a two-day interval including standard traffic interruption between 23:00 pm and 6:00 am.

It can be seen that the logistic model provides a reasonable fit even under rapid changes of arrival rate levels at the beginning and end of traffic interruption during nighttime.

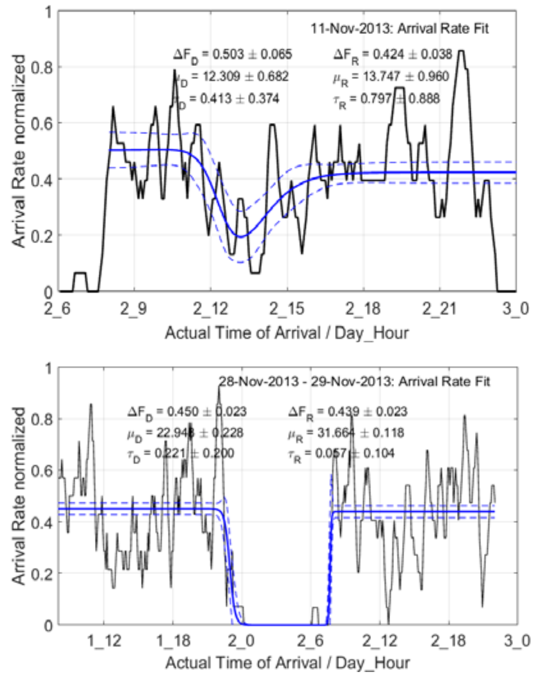

Figure 3. Time series of arrival rate data for two normal days of traffic (corrected for systematic deviation from average), two weeks after "Christian" (top graphics) and one week before "Xaver" respectively (bottom graphics), together with logistic function fit (±95% confidence).

B. X-Event time series

Fig. 4 depicts logistic function fits of wind speed (METAR data with 10 / 2 min averages for wind / gust provided in 30 min intervals) and of corresponding traffic rate data for day 2 in Fig. 1 of low pressure storm event "Christian" on Oct. 28 2013. Maximum wind /gust speed around 30 / 50 kt are observed around t = $(\mu_{inc} + \mu_{dec})/2 \approx$ 12:30 pm.

Fig. 5 depicts logistic function fits of wind/gust and corresponding traffic rate data for day 1 - 2 in Fig.2. It shows X-event "Xaver" with maximum wind and gust speed approaching 45 kt between noon Dec 5 and noon Dec 6 2013. Although the figures clearly exhibit a correlation between the wind / gust speed data, and the traffic rate decrease and recovery towards the end of the storm this is of course no proof for a causal relationship. Other weather factors that could be the cause for traffic disruption are cross and tail wind velocity (i.e. velocity vector with combination of speed and direction), and accompanying rain and snowfall as mentioned above. TABLE I. lists the quantitative model parameters derived from the wind-speed and traffic rate parameters.

With regard to model based predictability it appears worthwhile to highlight the fact that the maximum wind speed of ca 23 – 24 kt (Xaver) and 31 – 32 kt (Christian) as estimated from the fit in the graphics corresponds precisely to the theoretical model based estimates at the time near speed maximum $t(v_{max}) \approx (\mu_{dec} + \mu_{inc})/2$. The latter values are derived with (6) from the wind parameter estimates of TABLE I. based only on the time constants, delays, and stationary states before and after the X-event: v_{max}(Xaver) = 23.3 kt, v_{max}(Christian) = 32.5 kt (the latter reduced by 1 kt through $\exp\{\Delta\mu/(2\ \tau)\}$ in the denominator of (6)), indicating the logistic approach a reasonable hypothesis for a deterministic wind profile disturbance model.

V. Discussion

A. Static fitting of empirical data

If wind speed increase over a critical threshold is in some way related to the traffic disruption this should be reflected in different values of arrival rate fit parameters under Xevent and normal environmental conditions.

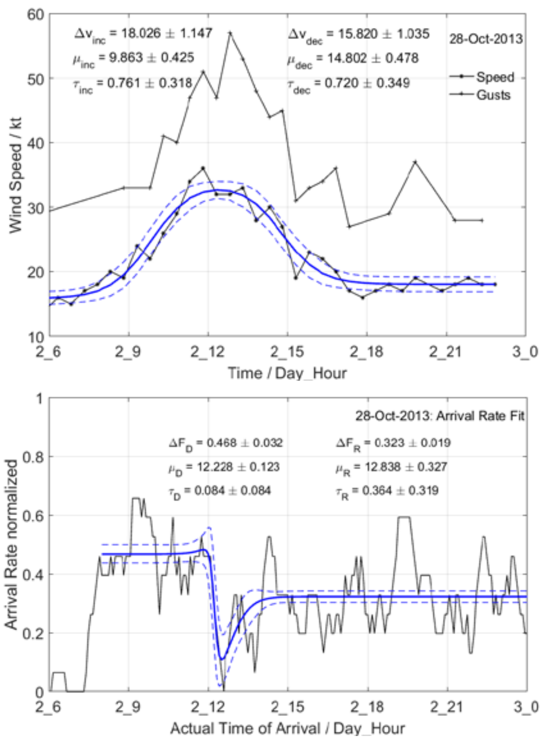

Figure 4. Wind/gust speed data (top, 10 / 2 min averages) and normalized (for maximum capacity = 27 arrivals/h) arrival rate data (bottom, 30 min averages) for low pressure storm event "Christian" on Oct. 28 2013. Time scale: Local time CET. Logistic function fits according to (4) (smooth blue lines) with 95% confidence interval (dotted lines). For numerical results see also text and TABLE I.

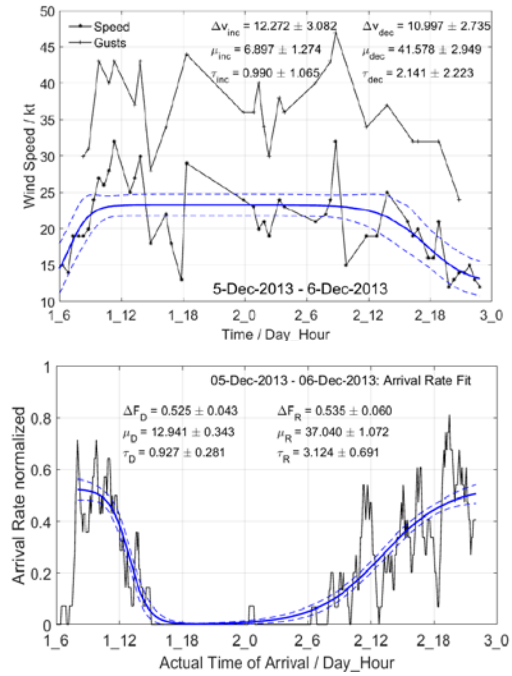

Figure 5. Wind/gust speed (top) and normalized arrival rate data (bottom) for low pressure storm event "Xaver" on Dec.5-6 2013. Logistic function fits according to (4) (smooth blue lines) with 95% confidence interval (dotted lines). For details see text.

TABLE I. PARAMETER ESTIMATES FOR WIND SPEED AND ARRIVAL RATE (95% CONFIDENCE INTERVALS IN BRACKETS).

Xevent	Logistic Model Parameters: Wind Speed					
	Δv_{inc} /kt	μ_{inc}/ h	τ_{inc} / h	Δv_{dec}/kt	μ_{dec} / h	τ_{dec} / h
Christian	18.0 (1.1)	9.9 (0.4)	0.8 (0.3)	15.8 (1.0)	14.8 (0.5)	0.7 (0.3)
Xaver	12.3 (3.1)	6.9 (1.3)	1.0 (1.1)	11.0 (2.7)	41.6 (2.9)	2.1 (2.2)
	Logistic Model Parameters: Arrival Rate					
	ΔF_D	μ_D / h	τ_D / h	ΔF_R	μ_R / h	τ_R / h
Christian	0.47 (0.03)	12.2 (0.1)	0.08 (0.08)	0.32 (0.02)	12.8 (0.3)	0.4 (0.3)
Xaver	0.53 (0.04)	12.9 (0.3)	0.9 (0.3)	0.54 (0.06)	37.0 (1.1)	3.1 (0.7)

Using wind speed as hypothetic cause for traffic disruption in our context serves mainly for testing the power of the logistic model. For a more realistic quantification of the possible different environmental causes for disruption (precipitation and icing are other possibilities) an evident improvement would be the use of wind velocity vector (including direction, relative to runway) and (the significantly higher) gust values. The latter represent the real possible danger for landing and starting aircraft due to their turbulent nature. Gust profiles (over time) and speed fluctuations could provide predictability information that may be used for anticipation of the disruptive potential [10].

The following table collects the values of the model parameter estimates, where for the baseline time series of two normal days (only one shown in Fig.3, in most cases they agree within 1 σ) the two-day means are listed (bold indicates relevant and significant differences between baseline and Xevent, F_{min} from parameter estimates using (5)).

TABLE II. Comparison of Baseline and Xevent

	Arrival Rate Fit Parameters: Baseline and XEvent							
Xevent		ΔF_D	μ_D / h	τ_D / h	ΔF_R	μ_R / h	τ_R / h	**F_{min}**
Christian	mean Baseline	0.47	12.1	-0.5	**0.46**	12.5	0.4	**0.2**
	XEvent	0.47	12.2	0.08	**0.32**	12.8	0.4	**0.1**
Xaver	mean Baseline	0.45	**23.0**	**0.2**	0.44	**31.7**	**0.06**	$< 10^{-5}$
	XEvent	0.53	**12.9**	**0.9**	0.54	**37.0**	**3.1**	$< 10^{-5}$

For “Christian” the duration ($\mu_R - \mu_D$) of the arrival rate disturbance around noon (12 – 1 pm) appears not significantly different as compared to the baseline traffic reduction at noon. However, F_{min} exhibits a significant decrease from baseline = 0.2 to disturbed rate = 0.1, when comparing Fig.4 with the baseline in Fig.3. This is confirmed formally through quantitative parameter estimates in TABLE I. Moreover a significant stationary performance reduction ΔF_R is quantified for the rest of the day, corresponding to increased wind speed.

For the time constants τ_D, τ_R any potential difference is hidden within their uncertainties because their estimates exhibit the same order of magnitude as their confidence intervals. This is certainly due to the fact that any disruption effect is superimposed by the normal intra-day traffic decrease. A difference (slightly outside the single ≈ ±10% confidence intervals) is observed, however with the stationary states after recovery: $\Delta F_R(\text{Xevent}) \approx 0.32 < \Delta F_R(\text{normal}) \approx 0.46$. This indicates an ongoing traffic disturbance following the storm maximum during the rest of the day.

The “Christian” Xevent on the one hand shows that our logistic modeling approach is well suited for fitting wind and traffic data. On the other hand, as mentioned in the beginning, the different periodicities and seasonal effects of the traffic time series have to be carefully taken into account for interpretation of parameter estimates.

The disruption effect is more pronounced with the second Xevent, “Xaver” that extends over two days, although also in this case the disturbance is superimposed by a periodic traffic interruption: the break during nighttime 11 pm – 6 am. The long duration of the disturbance results in significant differences between baseline and Xevent beginning and end time μ_D, μ_R as well as time constants τ_D, τ_R. The latter are abrupt for the baseline with the more or less complete interruption during nighttime (Fig. 3) whereas the storm induced disturbance builds up and recovers with a time scale of hours, and forces an extension of the periodic traffic pause during nighttime. The flexibility of the logistic function fit is demonstrated by the fact that estimates of time constants down to the range of minutes can be obtained, although only with limited confidence due to the low number of (or even missing) measurements within these short time intervals. The logistic function model (4) with two or three stationary levels, based on two or three instants of time may be seen as a heuristic approximation to the full dynamical model with continuous time varying control parameters k(t), w(v, dv/dt), G(t).

B. Dynamic simulation

In order to predict the general behavior of performance dynamics f(t) as obtained with computer experiments using (14), the two state space equations for d[f;G]/dt may be written as a second order generalized force equation $d^2f/dt^2 = F(f;G,w)$:

$$\ddot{f} = k_0 w\, f[c_1(G) + c_2(G)f + c_3(G)f^2] \qquad (16)$$

where we neglect the w(t)-derivative for simplicity. If we also neglect the G^2-dependencies of the c_i-coefficients (originating from dissipative pseudo force df/dt) we obtain with $c_1/c_3 = f_b + \eta\Delta G$, $c_2/c_3 = -(1+f_b) - \eta\Delta G$, as stationary solutions f* ($d^2f/dt^2 = 0$), besides the trivial one ($f_1^* = 0$): $f_2^* = f_b + \eta\Delta G, f_3^* = 1$. The potential function U(f;G,w) is derived from (16) (for conservative forces) via U = -dF/df:

$$U(f;w,G) = \frac{w}{\tau_0\tau_f}\left[-\frac{c_1}{2c_3}f^2 - \frac{c_2}{3c_3}f^3 - \frac{1}{4}f^4\right] \qquad (17)$$

It visualizes the fact that the performance model f(w, G) exhibits two stable states at the potential minima $f_1^* = 0$ (minimum performance), $f_3^* = 1$ (maximum performance),: $U(f_1^*) = 0$, $U(f_3^*) = w\,(1/2 - f_b - \eta\Delta G)/6\tau_0\tau_f$, with a metastable state defined by the potential barrier between the stable states near the bias f_b at f_2^* with $U(f_2^*) = w\,(f_b + \eta\Delta G)^3\,[(f_b + \eta\Delta G)/2 - 1]/6\tau_0\tau_f$,. Details are determined by the control parameters (w, G), the bias values, and time constants of the c_i-parameters.

Figure 6 depicts example results of simulation runs using the “Christian” wind speed profile (roughly ~ gust speed, see Fig.4, 5) as disturbance function (15), with v_{th} = 18 kn as threshold for sign change of rate parameter k(t). This is not a realistic wind disturbance threshold, in contrast to the gust profile, and is used here for demonstration of principle only due to better data availability. The selected parameter pairs (G_b, η) illustrate how, in principle, the recovery actions $G(f;G_b,\eta)$ as adaptive feedback gain can be shaped through time constants and offset to optimize the system resilience, i.e minimize deviation from nominal state. For real life application concrete management actions like arrival re-scheduling and runway re-assignment have to be mapped onto G-parameters, in order to provide a link to standard scheduling and optimization procedures.

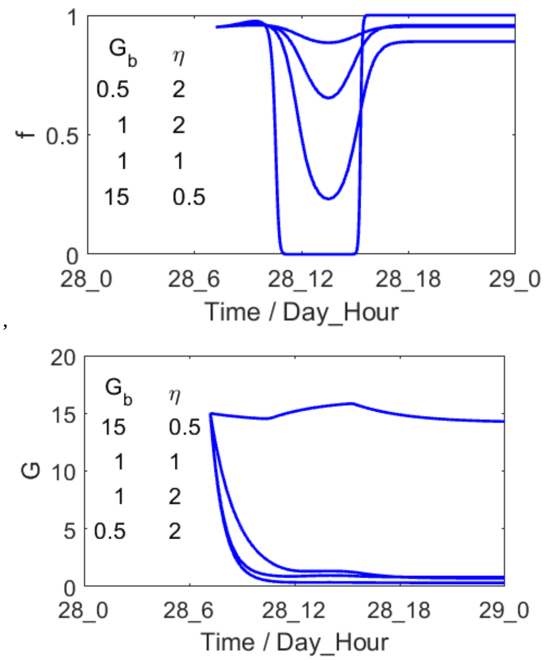

Figure 6. Simulation of performance disruption f(t) (top) and recovery action G(t) (bottom) dependent on offset G_b and time constant ratio η. Disturbance w(t) from "Christian". Model parameters v_{th} = 18, $[dv/dt]_0$ =35, $\tau_0 = \tau_f$ = 1.5 h, f_0 = 0.95, f_b = 0.6, η = 0.5, G_0 = 15. Succession of curves corresponds to that of [G_b, η]-parameter values

It should be emphasized that the dynamical model acts on a macroscopic scale with a few so called order and control parameters [8] that dramatically reduce the dimensionality of the system and focus on the global pattern (e.g. system performance). The effect of anticipative control actions to be mediated e.g. through optimized re-scheduling on the micro level [1][2], depend on the predictive power (i.e. confidence) of the dynamic two state (macro) model parameters, to be implemented into the Viability framework [7].

VI. CONCLUSION

A nonlinear dynamics (Ricatti) performance model is presented that is validated with empirical air traffic disturbance time series measured during extreme storm events (Xevents). Logistic functions are used as heuristic, analytic solution for data fitting. It provides quantitative estimates of arrival rate disruption and the simultanous wind-profile disturbance parameters. Based on the Ricatti equation a dynamical state space model is derived that besides the (heuristic) logistic performance model includes adaptive feedback gain for modelling management actions to counteract disturbances under Xevents. Simplified stationary state analysis and initial simulations confirm the expected behaviour of the dynamical model. Future work will include noise into the model equations for Monte Carlo simulations, with the goal of disruption prevention through anticipatory management actions, in combination with stochastic optimization. For this purpose we also have to analyse in detail the nonstationary fluctuations during the Xevent timecourse that typically accompany phase transitions in nonlinear systems. The present model is expected to provide an appropriate basis for integration into the Viability theory framework for Xevents management in ATM, through formal optimization of system resilience [7] (R = 1/C) with C a cost function, defined e.g. through the system time constants and bias levels.

ACKNOWLEDGMENT

Part of this work was co-financed by EUROCONTROL acting on behalf of the SESAR Joint Undertaking and the EUROPEAN UNION as part of work package E in the SESAR programme. We are indebted to M. Kapolke, F. Liers (Erlangen University) and D. Schäfer (Eurocontrol) for fruitful discussions within the project RobustATM (E.02.19).

REFERENCES

[1] N. Fürstenau, A. Heidt, M. Kapolke, F. Liers, M. Mittendorf, C. Weiß, "Pre-tactical Planning of Runway Utilization Under uncertaintiy: Optimization and Validation. In D. Schäfer (Ed.), Proc. 5th SESAR Innovation Days. Bologna 2015. Eurocontrol-SESAR-WPE.: http://www.sesarinnovationdays.eu/files/2015/Papers/SIDs_2015_paper_9.pdf

[2] M. Kapolke, N. Fürstenau, A. Heidt, F. Liers, M. Mittendorf, C. Weiß, , "Pre-tactical optimization of runway utilization under uncertainty". J. of Air Transport Management, 2016, in print

[3] S. Albeverio, V. Jentsch, H. Kantz, Eds., „Extreme Events in Nature And Society" Berlin, Heidelberg, New York: Springer, 2006.

[4] Y. Tu, M. Ball, W. Jank, "Estimating Flight departure delay distributions - a statistical approach with long-term trend and short-term pattern", J. American Statistical Assoc., vol.103, pp.112-125, 2008.

[5] J.-T. Wong, S.-C. Tsai, A survival model for flight delay propagation . Journal of Air Transport Management, vol. 23, pp.5-11, 2012.

[6] H. Devanandham, J. Ramirez-Marquez, "Generic metrics and quantitative approaches for system resilience as a function of time", Reliability Engineering and System Safety, vol. 99, pp. 114-122, 2012.

[7] G. Deffuant, G. Nigel, Viability and Resilience of Complex Systems. Berlin, Heidelberg, New York, Springer, 2011.

[8] H. Haken, Synergetics, 2nd. edn., Berlin, Heidelberg, New York,: Springer-Verlag, 1978.

[9] F. Royce, B. Bekera, "A metric and frameworks for resilience analysis of engineered and infrastructure systems", Reliability Engineering and System Safety, vol.121, pp. 90-103, 2014.

[10] H. Kantz, E. G. Altmann, S. Hallerberg, D. Holstein, A. Riegert, „Dynamical Interpretation of Extreme Events: Predictability and Predictions", in S. Albeverio, V. Jentsch, & H. Kantz, Eds., *Extreme Events in Nature and Society*. Berlin, Heidelberg, New York: Springer 2006, pp. 69-93.

[11] K. Barker, E. Ramirez-Marquez, C. M. Rocco, "Resilience-based network component importance measures", Reliability Analysis and System Safety, vol. 117, pp.89-97, 2012

[12] L. Collatz, "Differentialgleichungen", Stuttgart: B.G. Teubner, 1990

[13] O. Gluchshenko, P. Foerster, "Performance based approach to investigate resilience and robustness of an ATM system". Proc. 10th USA/Europe ATM Research and Development Seminar (ATM2013) Chicago.http://www.atmseminar.org/seminarContent/seminar10/papers/277-Gluchshenko_0127130117-Final-Paper-4-8-13.pdf