Modelling Flexible Thrust Performance for Trajectory Prediction Applications in ATM

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ICRAT 2016
Overview

1. Introduction
2. Background
3. Assumed Temperature Model
4. Thrust Models
5. Model Validation
6. Results
Section 1

Introduction
Introduction

Trajectory Prediction in ATM

Trajectory predictors

Estimate future trajectory of an aircraft based on mathematical models.

Applications:

- Simulation in ATM research
- Environmental assessment tools (EUROCONTROL’S ESCAPE)
- Decision Support Tools
- Trajectory Based Operations
- Many others...
Aircraft Performance Models (APM)

Mathematical framework to compute aircraft performance and dynamics.

- ATM simulation: take-off and departure procedures are of special interest.

- **Thrust models** provide information for environmental and noise assessment: noise, emissions, consumption...

- Accurate thrust models are available for conventional procedures (standard thrust ratings: MTO, MCT, MCL, MCR...).
**Introduction**

Reduced thrust operations

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**Reduced thrust operations**

Take-off operations with a thrust level **lower than the maximum takeoff thrust**. They aim to reduce noise and engine deterioration.

- Reduced thrust can be achieved by different techniques:
  - Thrust derate
  - **Assumed temperature (AT)**

- Current APMs do not provide a validated methodology to **compute reduced thrust**.

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**Proposed model**

In this paper we propose a validated methodology to **compute reduced thrust with the assumed temperature method**.
Section 2

Background
Background

Engine ratings

**Thrust rating**

Maximum certified thrust that an engine can provide under certain conditions

![Diagram showing thrust rating vs Outside Air Temperature (OAT)]

**Engine operational limits:**
- Combustion and inlet pressure
- Turbine inlet **temperature**
- Fan rotation speed
Assumed temperature method (Flexible thrust or FLEX)

Takes advantage of the **thrust rating limitation at high temperatures** to limit the thrust to the **minimum necessary** for a safe take-off.

For a fixed set of take-off conditions (take-off weight, airfield elevation, wind, etc.), the take-off distance is determined by the take-off thrust:

\[ \text{TOD} = \text{TOD}(F_T) \]

In conventional operations, \( F_T = MTO \).

If \( \text{TOD}(MTO) < \text{TODA} \) then:

\[ \exists F_T = F_{T,\text{min}} < MTO : \quad \text{TOD}(F_{T,\text{min}}) = \text{TODA} \]
The assumed temperature method

At the **maximum assumed temperature (MAT)**, the maximum certified thrust equals $F_{T,\text{min}}$.

The **regulatory limit** for the MAT is the one that results in **25% thrust reduction** with respect to the maximum rating.

Since the TOD depends on many factors, the MAT depends on the **take-off conditions**.
**Background**

Model structure

- **Assumed temperature model**: Polynomial model identified with manufacturer take-off performance data.
- **Thrust model**: ATM thrust models validated with AT input.
Section 3

Assumed Temperature Model
Assumed Temperature Model

Model dependencies

The MAT is determined by the **take-off conditions**:  
- Airframe configuration  
- Take-off weight  
- Airfield elevation  
- Outside air temperature  
- Wind  
- Runway length  
- Runway slope  
- Runway contamination
The MAT is determined by the **take-off conditions:**

- Airframe configuration
- Take-off weight
- Airfield elevation
- **Outside air temperature**
- Wind
- Runway length
- Runway slope
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Assumed Temperature Model

Polynomial model

\textbf{AT Polynomial Model}

For a given airframe configuration $i$:

\[ \text{MAT}_{\text{temp}}^{(i)} = f(h, w, l, m, \theta) \]

\[ \text{MAT}_{\text{max}}^{(i)} = \text{cnt} \]

MAT is the minimum:

\[ \text{MAT}^{(i)} = \min\{ \text{MAT}_{\text{max}}^{(i)}, \text{MAT}_{\text{temp}}^{(i)} \} \]

$i$: Airframe configuration
$h$: Airfield elevation
$w$: Wind speed
$l$: Runway length
$m$: Take-off weight
$\theta$: Polynomial coefficients
$f(h, w, l, m)$: polynomial function of order $n$
Assumed Temperature Model
Parameter identification

AT Polynomial

\[ \text{MAT}_{\text{temp}}^{(i)} = f(h, w, l, m, \theta) \]

- The polynomial \( f(h, w, l, m, \theta) \) is **linear in the coefficients \( \theta \).**

- The coefficients can be identified with **linear regressor estimators** from manufacturer take-off performance data providing the **MAT for different take-off conditions.**
Section 4

Thrust Models
Thrust Models

Base of Aircraft Data (BADA) thrust model: 3DoF, total energy model.
- BADA 3: almost 100% coverage
- BADA 4: 70% coverage

Aircraft Noise and Performance (ANP) database thrust model: intended to support environmental and noise assessments (ECAC Doc.29 and ICAO Doc 9911).
BADA thrust model

Net thrust contribution from all engines:

\[ F_T = \delta \ W_{mref} \ C_T, \quad (1) \]

Thrust coefficient:

\[ C_T = \sum_{i=0}^{5} \delta_T^i \left( \sum_{j=1}^{6} a_{6i+j} \ M^{j-1} \right) \quad (2) \]

Throttle parameter is captures the **thrust rating**:

\[ \delta_T = \begin{cases} 
\delta_{T,\text{flat}} & \text{if } \Delta T_{\text{ISA}} \leq \Delta T_{\text{ISA},k} \\
\delta_{T,\text{temp}} & \text{if } \Delta T_{\text{ISA}} > \Delta T_{\text{ISA},k}. 
\end{cases} \quad (3) \]
Thrust Models
BADA thrust model

- Flat rated area (temperature-independent):
  \[
  \delta_{T,\text{flat}} = \sum_{i=0}^{5} \delta^i \left( \sum_{j=1}^{6} b_{6i+j} M_{i-1} \right) \tag{4}
  \]

- Temp rated area (temperature-dependent):
  \[
  \delta_{T,\text{temp}} = \sum_{i=1}^{5} c_i M_{i-1} + \sum_{j=1}^{4} \theta_T \left( \sum_{i=0}^{4} c_{5(j-1)+(i+1)+5} M_{i} \right) + \sum_{j=1}^{4} \delta^j \left( \sum_{i=0}^{4} c_{5(j-1)+(i+1)+25} M_{i} \right) \tag{5}
  \]

  \[
  \delta = \frac{p}{p_0}; \quad \theta_T = \frac{T_T}{T_0}; \quad T_T = \left( 1 + \frac{\gamma - 1}{2} M^2 \right) T \tag{6}
  \]

Kink point \( T_K \) is explicitly given.
ANP-based thrust model

Net thrust for a given rating:

\[ F_T = n \delta (E + F \ V_{CAS} + G_A \ h + G_B \ h^2 + H \ T), \] (7)

\( (E, F, G_A, G_B, H) \) vary in the two areas of the thrust rating.

- Flat rated area (temperature-independent):

  \( (E_L, F_L, G_{A,L}, G_{B,L}, H_L = 0) \)

- Temp rated area (temperature-dependent):

  \( (E_H, F_H, G_{A,H}, G_{B,H}, H_H < 0) \)

Kink point is the intersection between \( F_{T,H} \) and \( F_{T,L} \):

\[ T_K = \frac{1}{H_H} [(E_L - E_H) + V_{CAS}(F_L - F_H) + h(F_{A,L} - F_{A,H}) \]
\[ + h^2 (G_{B,L} - G_{B,H})] \] (8)
Section 5

Model Validation
MAT model identification

- MAT polynomials identified for **3 aircraft models**.
- **Reference assumed temperature data** was generated with Boeing’s Standard Take-off Analysis Software (STAS).
- STAS provides **take-off performance tables** from which the MAT can be extracted for **different take-off conditions**.
Parameter identification $\theta$ with Minimum Mean Square Error (MMSE) estimator.

For each aircraft, 3 polynomial structures were evaluated:
- 1st order $MAT^{(i)} = \theta_1 h + \theta_2 l + \theta_3 w + \theta_4 m + \theta_5$
- 2nd order $MAT^{(i)} = \theta_1 h^2 + \theta_2 hl + \theta_3 hw + \theta_4 hm + \theta_5 h + \ldots \theta_{15}$
- 3rd order $MAT^{(i)} = \theta_1 h^3 + \theta_2 h^2 l + \theta_3 h^2 w + \theta_4 h^2 m + \theta_5 h^2 + \ldots \theta_{35}$

Fitting quality assessment based upon RMS between reference data and MAT polynomials.

$$\varepsilon = \frac{1}{n} \sum_{i=1}^{N} (MAT_{\text{ref}} - MAT_{\text{model}})^2$$ (9)
Thrust models validation

- **Reference thrust data** obtained from simulated take-off trajectories.
- 56,700 trajectories simulated with Boeing Climbout Program (BCOP).
Using AT in the BADA thrust model

- The temperature of the intake airmass (OAT) is captured by $M$.
- The temperature used by the FADEC to derive EPR or NE is **captured by the total temperature ratio $\theta_T$**.

$$C_T = \sum_{i=0}^{5} \delta_T^i \left( \sum_{j=1}^{6} a_{6i+j} M^{j-1} \right)$$

$$\delta_{T,\text{temp}} = \sum_{i=1}^{5} c_i M^{i-1} + \sum_{j=1}^{4} \theta_T^j \left( \sum_{i=0}^{4} c_5(j-1)+(i+1)+5 M^i \right)$$

$$+ \sum_{j=1}^{4} \delta^j \left( \sum_{i=0}^{4} c_5(j-1)+(i+1)+25 M^i \right)$$

Outside Air Temperature  Assumed Temperature
Using AT in the ANP-based model

- Only one temperature input, also captures **physical effect** of OAT on flight performance.
- Actual OAT no longer used for thrust computation.

\[
F_T = n \delta \left( E + F V_{\text{CAS}} + G_A h + G_B h^2 + H T \right),
\]

Assumed Temperature
Section 6

Results
AT Model Results

- **Single flat region** corresponding to $MAT_{\text{max}}$.
- **Resolution** of reference AT data from STAS was 5°.

**Figure**: 1st order polynomial (5 coeff.), no wind, flaps 10 and TOW 65 t, 900ft airfield elevation.
Results

AT Model

Three flat regions: 3 possible choices for $MAT_{\text{max}}$:

- **Highest**: model higher than actual MAT in some regions.
- **Maximal range**: best RMS, too conservative in some regions.
- **Lowest**: never above the actual MAT, but too conservative.

Choice of $MAT_{\text{max}}$ influences overall accuracy.

Option with **best overall accuracy** was chosen in each particular case.

**Figure**: 1st order polynomial (5 coeff.), no wind, flaps 10 and TOW 65 t, 1100ft runway.
Overall accuracy (RMSE)

- 1st order polynomials (5 coefficients): 6 – 11 °C.
- 2nd order polynomials (15 coefficients): 5 – 9 °C.
- 3rd order polynomials (35 coefficients): 3 – 6 °C.

Resolution of reference data was 5°C.

Trade-off (Complexity ↔ Accuracy): 2nd order

Table: RMSE of the AT model for different aircraft and airframe configurations

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>RMSE (°C) – Second-order polynomial, 15 coef.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flaps 1</td>
</tr>
<tr>
<td>B737</td>
<td>7.90</td>
</tr>
<tr>
<td>B757</td>
<td></td>
</tr>
<tr>
<td>B777</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Flaps 5</th>
<th>Flaps 15</th>
<th>Flaps 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>B757</td>
<td>7.58</td>
<td>7.54</td>
<td>6.77</td>
</tr>
<tr>
<td>B777</td>
<td>8.52</td>
<td>7.84</td>
<td>7.68</td>
</tr>
</tbody>
</table>
Thrust Models Results

Overall accuracy (RMSE) in terms of:
- Net thrust: ANP / BADA thrust models.
- Thrust reduction.

Thrust Reduction

\[ TR^{(n)} = \frac{T_{BCOP}^{(n)}(OAT) - T^{(n)}(AT)}{T_{BCOP}^{(n)}(OAT)} \cdot 100 \]
Results

Thrust Models

ANP

Thrust reduction vs. AT - Mach 0.235
Flaps=10, Hp=1500ft, Runway=9000ft, TOW=65T, OAT=20ºC

BADA

Thrust reduction vs. AT - Mach 0.235
Flaps=10, Hp=1500ft, Runway=9000ft, TOW=65T, OAT=20ºC
Table: Thrust RMSE for BADA and ANP

<table>
<thead>
<tr>
<th>APM</th>
<th>Absolute error [lbf]</th>
<th>Relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Model</td>
</tr>
<tr>
<td>BADA</td>
<td>873</td>
<td>833</td>
</tr>
<tr>
<td>ANP</td>
<td>685</td>
<td>495</td>
</tr>
</tbody>
</table>

Error increment: **0.78%** BADA and **1.14%** ANP.

Table: Thrust reduction RMSE for BADA and ANP

<table>
<thead>
<tr>
<th>APM</th>
<th>Thrust reduction RMSE [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>BADA</td>
<td>4.01</td>
</tr>
<tr>
<td>ANP</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Error increment: **-0.09%** BADA and **0.66%** ANP.
Results

Thrust Models

**Thrust**
Flaps: 25, Starting elevation: 1500ft, Runway: 15000ft, TOW: 65000 kg
Wind= 0kt, OAT= 25°C, AT= 35°C

```
Trajectory error [%]

<table>
<thead>
<tr>
<th>Error</th>
<th>Thrust</th>
<th>Fuel Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.10%</td>
<td>0.09%</td>
</tr>
<tr>
<td>σ</td>
<td>1.64%</td>
<td>0.82%</td>
</tr>
<tr>
<td>RMSE</td>
<td>3.03%</td>
<td>0.82%</td>
</tr>
</tbody>
</table>
```
Conclusions

- The Maximum Assumed Temperature can be modelled as a polynomial function of the take-off conditions.

- Polynomials of second order are a fair trade-off between complexity and accuracy.

- BADA and ANP can be used to compute reduced thrust with the AT method without significant degradation of their overall accuracy.

Future work

- Validate the model with more aircraft types.
Thank you for your attention!
Questions?
Section 7

Backup Slides
Model Validation

**MAT model validation**

<table>
<thead>
<tr>
<th>TEMP (°C)</th>
<th>-10</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>713*/43-46-52</td>
<td>738*/47-48-54</td>
<td>742*/47-49-54</td>
<td>746*/48-49-55</td>
</tr>
<tr>
<td>36</td>
<td>724*/44-47-53</td>
<td>750*/47-49-55</td>
<td>754*/48-50-56</td>
<td>758*/49-50-56</td>
</tr>
<tr>
<td>34</td>
<td>735*/44-48-54</td>
<td>762*/48-50-56</td>
<td>766*/49-51-57</td>
<td>770*/49-51-57</td>
</tr>
<tr>
<td>32</td>
<td>746*/45-49-55</td>
<td>774*/49-52-58</td>
<td>779*/50-52-58</td>
<td>783*/50-52-58</td>
</tr>
<tr>
<td>20</td>
<td>764*/47-50-57</td>
<td>792*/50-53-59</td>
<td>797*/51-54-60</td>
<td>801*/52-54-60</td>
</tr>
</tbody>
</table>
## Model Validation

MAT model validation

<table>
<thead>
<tr>
<th>TEMP (°C)</th>
<th>-10</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>702*/42-45-51</td>
<td>717*/46-47-53</td>
<td>720*/47-48-54</td>
<td>724*/47-48-54</td>
</tr>
<tr>
<td>38</td>
<td>713*/43-46-52</td>
<td>718*/47-48-54</td>
<td>724*/48-49-55</td>
<td>728*/48-49-55</td>
</tr>
<tr>
<td>34</td>
<td>735*/44-48-54</td>
<td>742*/48-50-56</td>
<td>750*/49-52-58</td>
<td>756*/49-52-58</td>
</tr>
<tr>
<td>30</td>
<td>757*/46-50-56</td>
<td>764*/50-53-59</td>
<td>770*/51-54-60</td>
<td>776*/51-54-60</td>
</tr>
<tr>
<td>28</td>
<td>761*/46-50-57</td>
<td>779*/50-52-58</td>
<td>785*/51-54-60</td>
<td>801*/52-54-60</td>
</tr>
</tbody>
</table>

**OAT 30°C, No Wind**

**TOW = 70,000 kg**
**Table:** Validity ranges of the MAT polynomial

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5</td>
<td>40</td>
<td>8000</td>
<td>15000</td>
<td>50000</td>
<td>80000</td>
</tr>
<tr>
<td>1000</td>
<td>-5</td>
<td>40</td>
<td>8500</td>
<td>15000</td>
<td>50000</td>
<td>80000</td>
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<tr>
<td>2000</td>
<td>-5</td>
<td>40</td>
<td>8500</td>
<td>15000</td>
<td>50000</td>
<td>77500</td>
</tr>
<tr>
<td>3000</td>
<td>-15</td>
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<td>8500</td>
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<td>15000</td>
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<td>67500</td>
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