Deconstructing Delay Dynamics

An air traffic network example

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Air transportation system

- A large-scale, interconnected network
- Delays or disruptions in one part of the system can propagate to many others
- Significant portion of this propagation occurs at airports
  - Incoming aircrafts continue on subsequent legs
  - Crew members connect to other flights
  - Passengers connect to other flights
Delay dynamics depend on aircraft connectivity

Flow of aircraft (and crew) through airport is important in delay dynamics

Aircraft connectivity is a key driver of flight delays
Objectives

1. Develop a model that relates the flow of traffic to the propagation of delay in air traffic networks

2. Use the model to analyze the dynamics of delay on the network (resilience of the air traffic network to delays)
Prior literature

Numerous studies have highlighted complexities and challenges in modelling the system

- Network connectivity

- Build-up of queues can lead to persistence of delays even after delayed aircraft depart or weather subsides
  [Klein et al. 2007, Ciruelos et al. 2015, Flerquin et al. 2015]

- Buffer or slack in flight schedules can help mitigate delay
  [Ahmed Beygi et al. 2008]

- Interaction between airports happen at different time scales
  [Beatty et al. 1999, Jetzki 2009, Xu et al. 2005]
Outline

- Model formulation and analysis
- Performance metrics for resilience of network
- Illustrative examples
  - Impulse delay
  - Sustained delay
- Conclusions and future work
Air traffic flow network

N vertices (airports)
Edge \((i, j)\) has weight \(\theta_{ij}\)

\(\theta_{ij}\) is the number of flights from airport \(i\) to airport \(j\)

Adjacency matrix \(\Theta = [\theta_{ij}]\)

Delay state of system \(\vec{x}(t) = [x_1(t) \ x_2(t) \ \cdots \ x_N(t)]\)

Average departure delay per flight at airport 1 at time \(t\)
Redistribution of delays

Delays tend to be redistributed among the traffic traversing through the airport.
Assuming delays are preserved,

\[ x_j(t) = 10 \text{ min/flight} \]

\[ x_k(t) = 25 \text{ min/flight} \]

\[ x_i(t + 1) = \frac{10 \times 25 + 5 \times 10}{10 + 5} = 20 \text{ min/flight} \]
Slack in schedules

Adapted from Skaltsas 2011
Modeling slack in the system

- Flight schedules contain some buffer, or slack to mitigate delay propagation
- Slack of $\beta$ min/flight for each link

Consider an example with $\beta = 3$ min/flight

$x_j(t) = 10$ min/flight

$x_k(t) = 25$ min/flight

$x_i(t+1) = \frac{10 \times (25 - 3) + 5 \times (10 - 3)}{10 + 5} = 17$ min/flight

Depends on traffic connectivity $\Theta = [\theta_{ij}]$
Persistence of delays

- Delay levels at an airport tend not to change abruptly
- Queues get built up and take some time to clear
- Delay at an airport depends on the delay at connected airports

\[
\text{delay}(t + 1) = \alpha \times \text{delay}(t) + (1 - \alpha) \times \text{delay from connected airports}(t)
\]

\(\alpha \in [0, 1]\) determines the amount of persistence or inertia to change in delay
Time scales

Flight durations determine the time scales of interaction between airports.
Multiple time scales

\( \theta_{ij} \) is the number of flights from airport \( i \) to airport \( j \)

\[ V = \{ n_1, n_2, n_3 \} \]

\[ \Theta = \begin{bmatrix} 0 & \theta_{12} & \theta_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Augmented network

\[ V = \{ n_1, n_2, n_3, p_1, p_2, p_3 \} \]

\[ A = \begin{bmatrix} 0 & 0 & 0 & \theta_{12} & 0 & \theta_{13} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{12} & 0 \\ 0 & \theta_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{13} & 0 & 0 & 0 \end{bmatrix} \]
Delay dynamics

For all airport nodes

\[ x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t) \]

- **Persistence**
- **Network effect**
- **Exogenous input**

Traffic weighted incoming delay

\[(x_j(t) - \beta)^+ = \max\{x_j(t) - \beta, 0\}\] ensures that the delays are non-negative

Airport delay at \(t+1\) depends on delay at previous time step (persistence) and average delay of incoming flights (network effect)
Delay dynamics

For all pseudo nodes, the delay is simply propagated

\[ x_i(t + 1) = \sum_j \mathbb{I}_{a_{ij} > 0} x_j(t) \]

\( \mathbb{I}_{a_{ij} > 0} \) an indicator variable which is 1 when there is traffic from node \( i \) to \( j \)
Exogenous inputs

We consider two types of exogenous input functions

1. **Impulse input**
   - $u_i(0) > 0$ and $u_i(t) = 0 \ \forall t > 0$
   - Models transient delays or brief disruptions (1 hour)

2. **Constant input**
   - The control input $u_i(t) > 0$ is varied such that the delay $x_i(t)$ is kept at a constant value of $\overset{*}{x}_i$
   - Models sustained delay or long duration Traffic Management Initiatives
Impulse input

- An impulse input at an inducing airport $k$ is if the form

\[ u_k(0) > 0 \text{ and } u_k(t) = 0 \quad \forall t > 0 \]

- The exogenous input is zero at all other airports

- Recall: $\alpha$ is the persistence of delays and $\beta$ is the slack in the schedule

\[
x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t)
\]

- Different dynamics based on $\alpha$ and $\beta$ values
**Impulse input: Possible dynamics**

\[ x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t) \]

> **Case 1:** \( \alpha = 1 \) (no network effect)

Inertia driven, delay will be isolated and persist indefinitely

Impulse input of \( u_k(0) > 0 \) and \( u_k(t) = 0 \ \forall t > 0 \)

\[ x_i(t + 1) = \begin{cases} 
  x_i(0) + u_i(0) & \forall t = 0 \\
  x_i(t) & \forall t \geq 1 
\end{cases} \]
Impulse input: Possible dynamics

\[ x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t) \]

Case 2: \( \alpha \in (0, 1) \) and \( \beta = 0 \) (no slack)
Delays will disperse and converge to a non-zero value

\[ x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} x_j(t)}{\sum_j a_{ji}} \quad \forall t \geq 1 \]

In fact, at steady state \( x^{SS} = \frac{x_k(0) \text{deg}(k)}{\sum_{i \in N} \text{deg}(i)} \)
Impulse input: Possible dynamics

\[ x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} \ (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t) \]

Case 3: \( \alpha = 0 \) and \( \beta = 0 \)

Delays could oscillate and need not converge:

\[ x_i(t + 1) = \frac{\sum_j a_{ji} x_j(t)}{\sum_j a_{ji}} \quad \forall t \geq 1 \]

\( x_1(t) = 15 \)
\( x_2(t) = 0 \)

\( x_1(t + 1) = 0 \)
\( x_2(t + 1) = 15 \)
Impulse input: Possible dynamics

\[ x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t) \]

- Case 4: \( \beta > 0 \) (slack is positive)
  Delays will decay to zero in finite time because slack will absorb delays

Interesting dynamics when \( \alpha \in (0, 1), \beta > 0 \)
Sustained input

An exogenous input is engineered to maintain a delay of $x^*$ min/flight for a particular airport $k$

- When $\beta = 0$ (i.e. there is no slack) at steady state, all the airports will have the same delay of $x^*$ min/flight
- When $\beta > 0$ (i.e. delay is decreased because of slack), it reduces the exposure of other airports to the sustained delay of $x^*$ min/flight
- At steady state, the delay is independent of $\alpha$ (the persistence)

$$x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t)$$
Key features of our delay model

1. Weighted and directed air traffic flow network
   - Traffic intensities is different on different OD pairs
   - They need not be symmetric

2. Characteristics of air traffic delays
   - Multiple time scales
   - Persistence of delays
   - Slack in schedules
   - Redistribution of delays

3. Incorporates exogenous control inputs
   - Impulse input
   - Sustained inputs
Performance metrics

- **Total delay**
The sum of the delay seen at all airports at any time step $t$, i.e., $\sum_{j \in N} x_j(t)$

- **Average induced delay**
Average delay level seen across all airports when an exogenous delay is introduced at an *inducing airport*

$$\bar{ID}(t) = \frac{\text{total delay}(t)}{|N|}$$

- **Largest impacted cluster**
Largest set of connected airports that have a non-zero delay
Data

- Bureau of Transportation Statistics 2011
- 158 airports and 1102 edges
- Only consider US domestic operations in this work

Fig: Average daily traffic

Table: Top 10 airports by traffic

<table>
<thead>
<tr>
<th>Airport</th>
<th>Avg. no. of daily departures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta (ATL)</td>
<td>912</td>
</tr>
<tr>
<td>Chicago (ORD)</td>
<td>662</td>
</tr>
<tr>
<td>Los Angeles (LAX)</td>
<td>489</td>
</tr>
<tr>
<td>Dallas (DFW)</td>
<td>486</td>
</tr>
<tr>
<td>Denver (DEN)</td>
<td>468</td>
</tr>
<tr>
<td>Phoenix (PHX)</td>
<td>417</td>
</tr>
<tr>
<td>San Francisco (SFO)</td>
<td>300</td>
</tr>
<tr>
<td>Las Vegas (LAS)</td>
<td>285</td>
</tr>
<tr>
<td>Houston (IAH)</td>
<td>279</td>
</tr>
<tr>
<td>Charlotte (CLT)</td>
<td>239</td>
</tr>
</tbody>
</table>
Impulse input

- An impulse input at an inducing airport $k$ is if the form
  
  $$u_k(0) > 0 \text{ and } u_k(t) = 0 \quad \forall t > 0$$

- The input is zero at all other airports

- Models a short 1 hour disruption at an airport $k$

Recall: $\alpha$ is the persistence of delays and $\beta$ is the slack in the schedule

$$x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} (x_j(t) - \beta)^+}{\sum_j a_{ji}} + u_i(t)$$

Interesting dynamics when $\alpha \in (0, 1), \beta > 0$
Role of persistence ($\alpha$)

Impulse input of 120 min/flight at Chicago O’Hare airport, $\beta = 10$ min/flight

The persistence ($\alpha$) determines whether system will have low delay for long time or high delay for a short time.
Role of slack ($\beta$)

Impulse input of 120 min/flight at Chicago O’Hare airport, $\alpha = 0.2$

When the slack ($\beta$) increases, delay levels scale down
Time for delays to decay

- When the slack ($\alpha$) is small, less sensitive to the persistence ($\beta$)
- When the persistence is high, less sensitive to slack

Higher $\beta$ and lower $\alpha$ decrease the time for delays to decay
Sustained input

An exogenous input is engineered to maintain a constant delay of $x^*$ for a particular airport $k$

$$x_i(t + 1) = \alpha x_i(t) + (1 - \alpha) \frac{\sum_j a_{ji} \left( x_j(t) - \beta \right)^+}{\sum_j a_{ji}} + u_i(t)$$

- The delays will spread to other airports depending on the slack $\beta$

- The next few plots show
  - Effect of slack on the performance metrics
  - Geographical spread of delays
  - Time for delays to stabilize
Role of slack ($\beta$)

Set point of $x^* = 120 \text{ min/flight}$ for Chicago ORD

- Two phases of decreasing delay
  - First phase: Decrease in number of impacted airports
  - Second phase: Delay decrease in airports directly connected to ORD
Effect of slack on spread of delays

Variation in $\beta$ also affects the geographical spread of the delay.

Sustained delay of $x^* = 120 \text{ min/flight}$ for Chicago ORD.

Color and size both represent the magnitude of delay at the airport.

$\beta = 10 \text{ min/flight}$

$\beta = 5 \text{ min/flight}$
Geographical spread of sustained delay originating at different airports

Sustained delay of $\beta = 5$ min/flight at different airports
Impact of sustained delay originating at different airports

<table>
<thead>
<tr>
<th>Inducing Airport</th>
<th>Average Induced Delay (min/flight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta (ATL)</td>
<td>31.75</td>
</tr>
<tr>
<td>Chicago (ORD)</td>
<td>17.82</td>
</tr>
<tr>
<td>Denver (DEN)</td>
<td>8.30</td>
</tr>
<tr>
<td>Dallas (DFW)</td>
<td>7.97</td>
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<tr>
<td>Los Angeles (LAX)</td>
<td>7.28</td>
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<tr>
<td>Phoenix (PHX)</td>
<td>5.42</td>
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<tr>
<td>San Francisco (SFO)</td>
<td>4.73</td>
</tr>
<tr>
<td>Baltimore (BWI)</td>
<td>4.37</td>
</tr>
<tr>
<td>Houston (IAH)</td>
<td>4.25</td>
</tr>
<tr>
<td>Honolulu (HNL)</td>
<td>3.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inducing Airport</th>
<th>Number of impacted airports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta (ATL)</td>
<td>125</td>
</tr>
<tr>
<td>Chicago (ORD)</td>
<td>74</td>
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<tr>
<td>Los Angeles (LAX)</td>
<td>68</td>
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<tr>
<td>Dallas (DFW)</td>
<td>40</td>
</tr>
<tr>
<td>Denver (DEN)</td>
<td>39</td>
</tr>
<tr>
<td>San Francisco (SFO)</td>
<td>52</td>
</tr>
<tr>
<td>Phoenix (PHX)</td>
<td>45</td>
</tr>
<tr>
<td>Houston (IAH)</td>
<td>40</td>
</tr>
<tr>
<td>Boston (BOS)</td>
<td>37</td>
</tr>
<tr>
<td>Orlando (MCO)</td>
<td>32</td>
</tr>
</tbody>
</table>

Impact of the New York airports?
Time for steady state

Higher slack and lower persistence decreases the time for steady state to be attained

- When slack is higher than the sustained delay, there is no delay propagation
- Similar plots for other airports

Sustained delay of 120 min/flight at Chicago
Summary

1. Proposed a model of delay propagation in air traffic networks, accounting for several key features of such systems

2. Developed and quantified some candidate metrics for resilience of air traffic networks to delays

Extensions:

- Different values of slack, inertia, etc. for different airports
- Time-varying model parameters
- Estimating model parameters from data and subsequent validation
- Wider range of exogenous inputs, correlated delay between airports