Probabilistic Analysis of Aircraft Fuel Consumption Using Ensemble Weather Forecasts

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Motivation

- Project TBO-Met (H2020 Ref. 699294)

Meteorological Uncertainty Management for Trajectory Based Operations.
SESAR 2020 Exploratory Research; Topic: Meteorology.

- **Specific challenge:** "to research enhanced meteorological capabilities and their integration into the ATM planning".

- **Expected impact:** "to enhance ATM efficiency by integrating meteorological information".

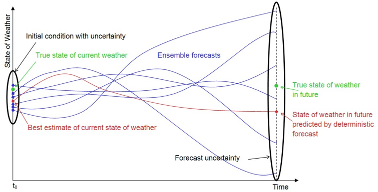
Objectives

- Development of a methodology to **manage weather uncertainty** suitable to be integrated into the **trajectory planning process** (general framework).
- Assessment of the impact of **wind uncertainty** on aircraft trajectory, and in particular on **cruise fuel consumption**. (Wind is one of the **main sources** of uncertainty that affect trajectory prediction.)
- Presentation of a **probabilistic trajectory predictor** that transforms the wind uncertainty into fuel load uncertainty.

Ensemble weather forecasting (1)

- Wind uncertainty is defined by **Ensemble Weather Forecasts** (EWF's).

- An EWF is obtained by
  - slightly **altering the initial conditions** and/or physical parameters, and/or
  - considering time-lagged or **multi-model** approaches.



- An EWF constitutes a **representative sample** of the possible realizations of the **potential weather outcome**.

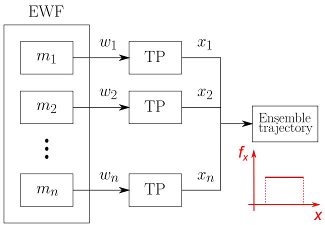
- The **uncertainty information is in the spread** of the various forecasts of the ensemble.

Ensemble weather forecasting (2)

- European Ensemble Prediction Systems:
  - **PEARP** (Météo France): 35 members.
  - **MOGREPS** (UK Met Office): 12 members.
  - **ECMWF** (European consortium): 51 members.
  - **SUPER** (Multi-model ensemble): 98 members.
- American Ensemble Prediction Systems:
  - **MEPS** (Air Force Weather Agency): 10 members.
  - **SREF** (National Centers for Environmental Prediction): 21 members.

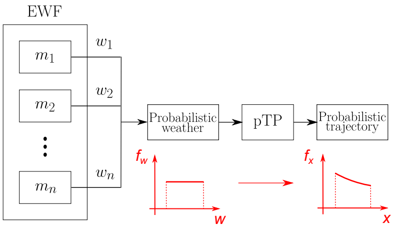
Trajectory prediction considering EWF uncertainty (1)

Ensemble TP (IMET):



1. Process the ensemble of trajectories.

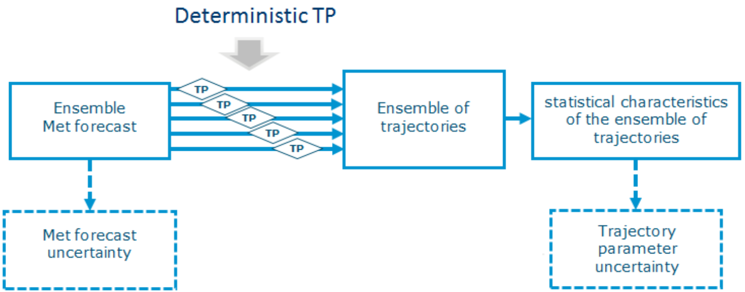
Probabilistic TP (present work):



1. Process the ensemble of forecasts.
2. Evolve the uncertainty.

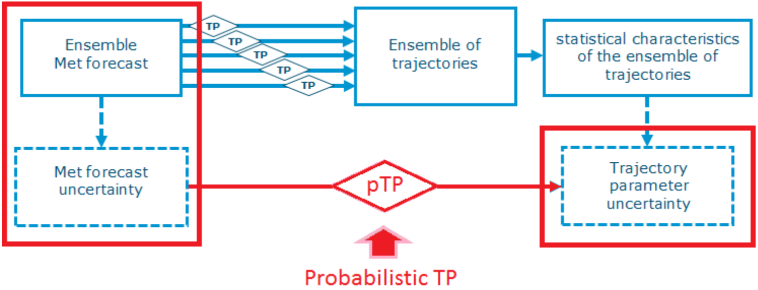
Trajectory prediction considering EWF uncertainty (2)

- **Ensemble TP** (IMET):

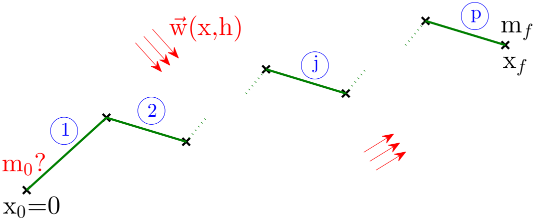


Trajectory prediction considering EWF uncertainty (2)

- **Probabilistic TP** (present work):



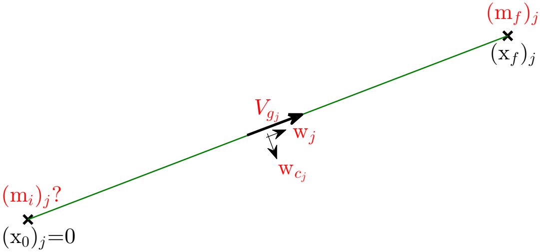
Fuel consumption in cruise flight (1)



- p segments.
- m_f, x_f given; m_0 unknown.
- Fuel consumption: $m_F = m_0 - m_f$.
- Wind uncertain $\Longrightarrow$ m_0 and m_F uncertain.

Fuel consumption in cruise flight (2)

- Segment j:



- w_j, w_{c_j} average values (different for each segment).
- Random variables:

$$w_j, \quad w_{c_j}, \quad V_{g_j} = \sqrt{V^2 - w_{c_j}^2} + w_j, \quad (m_f)_j = (m_i)_{j+1}.$$

- V, $(x_f)_j$ given.

Trajectory analysis (1)

- First step: **one cruise segment** defined by
  - **constant course**,
  - **constant speed**,
  - **constant altitude** (following ATC rules),
  - **constant average along-track wind** ($w_c = 0$).

- **Aircraft mass evolution:**

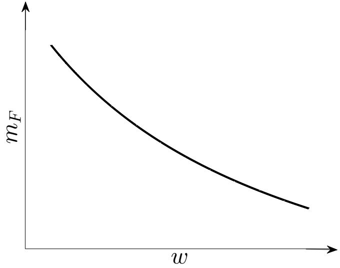
$$\frac{\mathrm{d}m}{\mathrm{d}x} = -\frac{A + Bm^2}{V + w}, \quad A = \frac{c}{2}\rho V^2 S C_{D_0}, \quad B = \frac{c C_{D_2} g^2}{\frac{1}{2}\rho V^2 S}, \quad A, B > 0$$

$m(x_f) = m_f$ (**Fixed final mass**; backwards resolution.)

Solution: $m(x; w)$ $\longrightarrow$ Random process.

Model: adequate, and yet very simple.

- **Fuel consumption:**

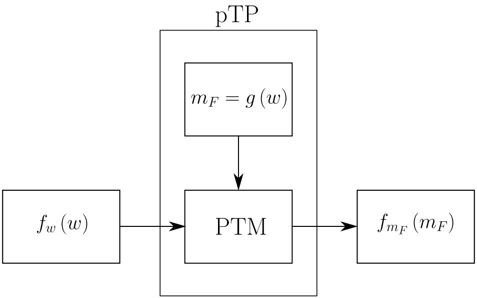


$$m_F(w) = m(0,w) - m_f$$

$$m_F = \frac{(m_f^2 + \frac{A}{B})\tan\left(\frac{\sqrt{AB}x_f}{V+w}\right)}{\sqrt{\frac{A}{B}} - m_f\tan\left(\frac{\sqrt{AB}x_f}{V+w}\right)} \equiv g(w)$$

Because w is uncertain, the **fuel consumption** defined by the transformation $m_F = g(w)$ **is also uncertain**.

Probabilistic trajectory predictor (pTP)



- PTM - **Probability Transformation Method** $f_{m_F} = \dfrac{f_w\left(g^{-1}(m_F)\right)}{\left|g'\left(g^{-1}(m_F)\right)\right|}$

- The method **evolves** the pdf of the wind.

- **Fuel pdf:**

$$f_{m_F}(m_F) = \begin{cases} \dfrac{f_w\left(g^{-1}(m_F)\right)}{\left|g'\left(g^{-1}(m_F)\right)\right|} & \text{for } m_F \in [m_{F,1}, m_{F,2}] \\ 0 & \text{for } m_F \notin [m_{F,1}, m_{F,2}] \end{cases}$$

g^{-1} and g' easily follow from $m_F = g(w)$; $m_{F,1} = g(w_m)$, $m_{F,2} = g(w_M)$.

Mean: $\mathrm{E}[m_F] = \int_{m_{F,1}}^{m_{F,2}} m_F f_{m_F}(m_F)\mathrm{d}m_F$

Standard Deviation: $\sigma[m_F] = \left[\int_{m_{F,1}}^{m_{F,2}} m_F^2 f_{m_F}(m_F)\mathrm{d}m_F - (\mathrm{E}[m_F])^2\right]^{\frac{1}{2}}$

Probabilistic wind model (1)

- $\{w_1, ..., w_n\}$ are the **average winds** along the cruise segment defined by the n members of the ensemble.
- We must assume a **particular wind distribution** (uniform, beta, gaussian, ...).
- How to **get the wind pdf** from the ensemble information is an **open challenge**.

Probabilistic wind model (2)

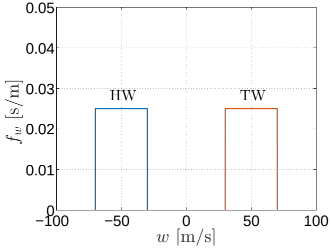
- **Uniform wind distribution:**

pdf:
$$f_w(w) = \begin{cases} \dfrac{1}{(w_M - w_m)} & \text{for } w \in [w_m, w_M] \\ 0 & \text{for } w \notin [w_m, w_M] \end{cases}$$

$$\begin{cases} w_m = \min\{w_1, ..., w_n\} \\ w_M = \max\{w_1, ..., w_n\} \end{cases}$$

Mean: $$E[w] = \frac{w_m + w_M}{2}$$

Standard Deviation: $$\sigma[w] = \frac{w_M - w_m}{2\sqrt{3}}$$



Probabilistic wind model (3)

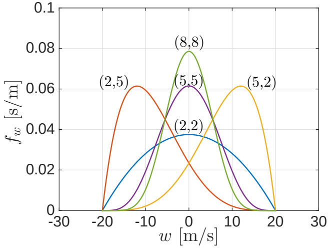
- **Beta wind distribution:**

pdf:
$$f_w(w) = \begin{cases} \dfrac{(w-w_m)^{\alpha-1}(w_M-w)^{\beta-1}}{(w_M-w_m)^{\beta+\alpha-1}\mathbb{B}(\alpha,\beta)} & \text{for } w \in [w_m, w_M] \\ 0 & \text{for } w \notin [w_m, w_M] \end{cases}$$

$$\mathbb{B}(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \qquad \begin{cases} w_m \le \min\{w_1,\ldots,w_n\} \\ w_M \ge \max\{w_1,\ldots,w_n\} \end{cases}$$

Mean:
$$E[w] = \frac{\beta w_m + \alpha w_M}{\alpha+\beta}$$

Standard Deviation:
$$\sigma[w] = \frac{w_M - w_m}{\alpha+\beta}\sqrt{\frac{\alpha\beta}{1+\alpha+\beta}}$$



$\alpha = \beta$ symmetric

$\alpha \neq \beta$ non-symmetric

Results (1)

- **Parameters:** (B767-400; BADA 3.13.)

$$V = 240 \text{ m/s} \quad h = 10000 \text{ m} \quad C_{D_0} = 0.01744 \quad C_{D_2} = 0.04823$$
$$c = 1.49 \cdot 10^{-5} \text{ s/m} \quad S = 283.5 \text{ m}^2 \quad m_f = 150000 \text{ kg} \quad x_f = 3000 \text{ km}$$

- To assess the impact of wind uncertainty, a **parametric study** is performed with arbitrary wind distributions (not real winds).

Uniform wind distribution.

Parameters:

$$\bar{w} = \frac{w_M + w_m}{2} = -50, 50 \text{ m/s (HW, TW)}$$

$$\delta_w = \frac{w_M - w_m}{2} = 5, 10, 15, 20, 25 \text{ m/s}$$

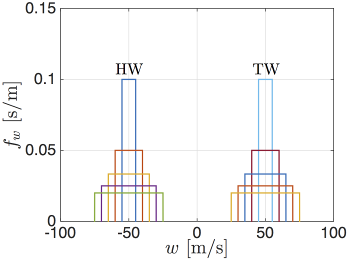
Comments:

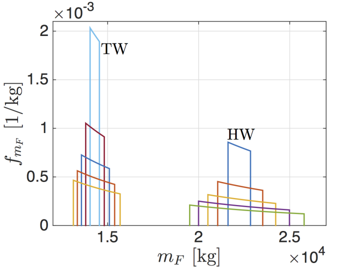
as δ_w increases:

- $E[m_F]$ almost constant
- $\sigma[m_F]$ increases

as $\bar{w}$ increases:

- $E[m_F]$ decreases (as expected)
- $\sigma[m_F]$ decreases (larger for HW than for TW)





Results (3)

Uniform wind distribution.

Parameters:

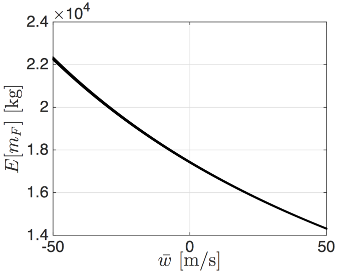
$\bar{w}$ ranges from -50 m/s to 50 m/s

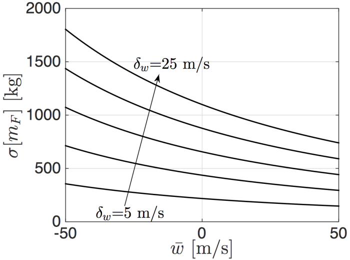
$\delta_w = 5, 10, 15, 20, 25$ m/s

Comments:

$E[m_F]$ is almost independent of δ_w.

It can be shown that the increase of $\sigma[m_F]$ with δ_w is almost linear.





Results (4)

Beta wind distribution (symmetric).

Parameters:

$$\alpha = \beta = 2$$

$$\bar{w} = \frac{\beta w_m + \alpha w_M}{\alpha + \beta} = -50, 50 \text{ m/s}$$

$$\delta_w = \frac{w_M - w_m}{2} = 10, 20, 30 \text{ m/s}$$

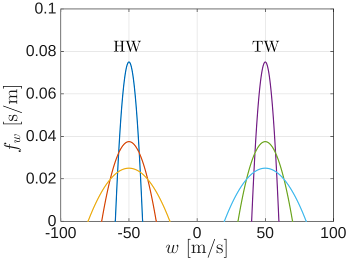
Comments:

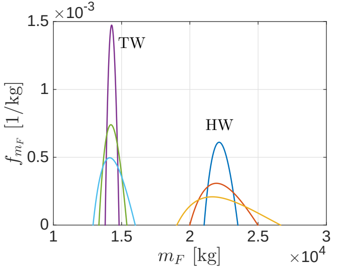
as δ_w increases:

- $E[m_F]$ almost constant
- $\sigma[m_F]$ increases

as $\bar{w}$ increases:

- $E[m_F]$ decreases (as expected)
- $\sigma[m_F]$ decreases (larger for HW than for TW)





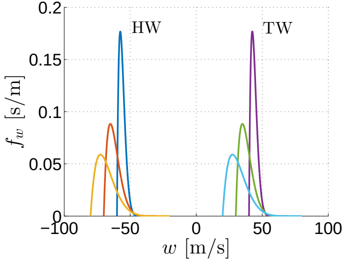
Results (5)

Beta wind distribution (non-symmetric).

Parameters:

$\alpha = 2, \beta = 8$

HW		TW	
w_m	w_M	w_m	w_M
-60 m/s	-40 m/s	40 m/s	60 m/s
-70 m/s	-30 m/s	30 m/s	70 m/s
-80 m/s	-20 m/s	20 m/s	80 m/s



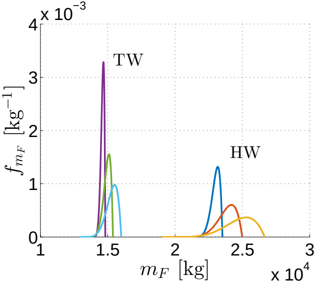
Comments:

As δ_w increases, $\bar{w}$ decreases (for $\beta > \alpha$).

As $\bar{w}$ decreases, $E[m_F]$ increases.

As δ_w increases, $\sigma[m_F]$ increases.

The skewness of the fuel mass distribution is opposite to the wind's.



Conclusions (1)

- This work has provided an **assessment of the impact of wind uncertainty** on cruise fuel load.
  - Main result: For given wind uncertainty, the uncertainty in the fuel consumption is much larger in the case of HWs than in the case of TWs. (**HWs increase uncertainty; TWs decrease uncertainty.**)
- It is expected that by considering the wind uncertainty one could adjust the **contingency fuel** depending on the uncertainty obtained for the fuel consumption.
  - Translating wind uncertainty into fuel uncertainty may lead to a **more effective decision making** process.

Conclusions (2)

- The pdf $f_{m_F}(m_F)$ has been obtained explicitly; for more complex cases a **numerical approach** is available

  (Vazquez and Rivas, JGCD 36 (2), 2013, pp 415-429).

- The pTP presented can take **any type of wind distribution** as input (uniform, beta, gaussian, ...).

- The determination of the wind pdf from the uncertainty information contained in the EWFs is an **open challenge**: it is a **multidisciplinary task** (meteorologists, statisticians, ATM experts).

Future Work

- Application to trajectories composed of several cruise segments.
  - This problem involves **two random variables**: the wind and the final fuel mass of each segment (except the last one).
- Analysis of scenarios including both wind uncertainty and the presesence of **convective regions**.

Announcement

- **Workshop** on "Meteorological Uncertainty and ATM":

Madrid, 24-25 November, 2016.

Schedule:
- Thursday 24, Afternoon, 14:00 - 18:00
- Friday 25, Morning, 9:00 - 13:00

Equations of motion

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = V + w, & T = D \\ \dfrac{\mathrm{d}m}{\mathrm{d}t} = -cT, & L = mg \end{cases}$$

$$D = \tfrac{1}{2}\rho V^2 S C_D, \quad C_D = C_{D_0} + C_{D_2} C_L^2, \quad C_L = \frac{mg}{\frac{1}{2}\rho V^2 S}$$

$$\left.\begin{array}{lllll} c & = & c(V,h) & = & const. \\ C_{D_0} & = & C_{D_0}(M) & = & const. \\ C_{D_2} & = & C_{D_2}(M) & = & const. \end{array}\right\} \longrightarrow$$

Corresponding to the values of h, V (or M) set for the flight.