Uncertainty Assessment for ETA prediction towards 4D Trajectory Operation

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***Abstract*— One concept in future air traffic management is 4D trajectory based operation (TBO) which aims at improving airspace capacity and decrease the environmental impact. In order to maintain separation and exploit the benefits of these 4D-trajectories, aircraft must stay within very small volumes around their negotiated reference track. As a preliminary step initial 4D trajectories are planned that define one explicit time constraint for a target waypoint, for example on the final approach segment. Real benefits for other stakeholder come into play if accurate predictions for the Estimated Time of Arrival (ETA) at that waypoint can be provided together with a statistical measure for meeting the negotiated Required Time of Arrival (RTA) window. Given this uncertainty measure, other users can weight the ETA prediction for their purposes and for example include additional buffers only if necessary.**

This paper analyzes the sensitivity of ETA predictions to uncertainties in the wind forecast for an exemplarily 4D approach in the Terminal Area (TA). It further outlines the necessary steps in order to understand the mitigating effect of closed-loop speed control on the ETA uncertainty prediction. The final goal is to extend the current definition of a typical aircraft state vector with a reliability measure for the ETA, expressed for example in form of a standard deviation. The latter shall not only consider current aircraft performance metrics but also take into account the mitigation options by closed-loop control in the background of ATC regulations.

Keywords: 4D-Trajectory, ETA, Required Time of Arrival, Flight Management.

I. INTRODUCTION

For future TBO it is envisaged that aircraft share their trajectory prediction including the ETA which is calculated in the on-board Flight Management System (FMS) with other ATM stakeholder. The latter additionally need information about how trustworthy the provided data is. For this purpose, we use the approach of a system transfer function to represent how uncertainties that occur during the flight affect the ETA uncertainty. In order to study the propagation of uncertainties in 4D trajectory prediction, a simulation framework was developed to induce wind forecast errors together with initial altitude errors that result from the navigation system. It then analyzes how these errors transfer to the errors in the ETA prediction at the target waypoint. The paper will approach the problem in two separate steps. First, the effect of statistical wind forecast distributions will be investigated with the focus on the 4D errors at the target waypoint. The analysis is conducted under the assumption that no active 4D speed control for the RTA is applied. This step is a pure forward propagation and aims at investigating the uncertainties if no countermeasures are engaged to meet the ETA. In a next step, parameters are evaluated that correlate with the robustness of the ETA prediction like speed control margins for compensating wind forecast errors. In the future, this framework will allow investigating the robustness of the ETA prediction if mitigating 4D control strategies are applied. This paper is divided into two parts. First, the simulation framework including the aircraft model is presented. The second part discusses parameters that have the potential to represent the uncertainty propagation to the ETA uncertainty.

II. METHODOLOGY

Different studies were conducted before in the field of uncertainty propagation along 4D trajectories, see **[3]**, **[1]**. In **[3]** EUROCONTROL'S BADA model was applied via the web interface in order to study the interaction between vertical and along-track errors in 4D trajectories for the special case of continuous descent approaches. One drawback of this approach is that the intrinsic BADA model only considers pure forward state propagation. Therefore, mitigating effects like for example closed-loop 4D speed control cannot be accounted for.

For filling the gap, this paper applies a three degree of freedom point mass model as introduced in **[5]** and implements aircraft performance parameters according to BADA **[1]** in a combined framework. Furthermore, it inherits the BADA speed constraints in order to simulate realistic aircraft speed envelops that tie to current airspace procedures in terms of current practice and restrictions. The advantage is that different 4D speed control strategies can be applied later which are currently not supported by the BADA version 3.9. Moreover, it enables a dedicated investigation of single influences by separately turning on/off different error simulation models. Fig. 1 shows four different simulated curved approaches on Atlantic City International Airport. The approach 1) on runway RW04 was selected for the simulation in this paper. The total length of the flight path is 10 nautical miles. The target waypoint is set to the over-the-threshold runway crossing point.

Figure 1. Different approach trajectories with RTAs for the selected approach 1) on RW04.

The following TABLE I. lists the wind forecast uncertainty, assuming mean moderate wind conditions.

TABLE I. SIMULATED WIND PARAMETER

Parameter	Mean	Standard Deviation
Direction	359 °	10 °
Speed	11 m/s	5 m/s

III. TRAJECTORY SIMULATION

This section describes the aircraft model that was used for the simulation. The basic model is represented by a nonlinear discrete time state space aircraft model with three degrees of freedom. The model is extended with BADA aircraft performance and procedure parameters for an A320 aircraft type. The state vector is defined as

$$\underline{x} = \left(p_n, p_e, p_h, V_{GS}, V_{TAS}, \gamma, \chi, m\right)^T \qquad (1)$$

and holds the position in north, east and up direction p_n, p_e, p_h, the aircraft ground speed V_{GS}, the true airspeed V_{TAS} the glide path angle γ, the course angle χ and the current aircraft mass m. Speed control was implemented for the true airspeed V_{TAS} and converted to ground speed by adding the simulated wind. The states are propagated via the following equation for each discrete time step k.

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$$\begin{pmatrix} p_n \\ p_e \\ p_u \\ V_{GS} \\ V_{TAS} \\ \gamma \\ \chi \\ m \end{pmatrix}_{k+1} = \underline{x}_k + \begin{pmatrix} V_{GSk} \cdot \cos(\gamma_k) \cdot \cos(\chi_k) \\ V_{GSk} \cdot \cos(\gamma_k) \cdot \sin(\chi_k) \\ V_{GSk} \cdot \sin(\gamma_k) \\ 0 \\ 0 \\ 0 \\ 0 \\ -c_{fuel} \end{pmatrix} \cdot T_s + \begin{pmatrix} 0 \\ 0 \\ 0 \\ a_{input} \\ a_{input} \\ \dot{\gamma}_{input} \\ \dot{\chi}_{input} \\ 0 \end{pmatrix}_k \cdot T_s + \begin{pmatrix} 0 \\ 0 \\ 0 \\ V_w - V_{GS} + V_{TAS} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_k$$

(2)

Here a_{input} represents a commanded acceleration of V_{TAS} which is calculated by BADA, $\dot{\gamma}_{input}$ a commanded change in the glide path angle and $\dot{\chi}_{input}$ a possible commanded change in the course angle which is used by the course controller to follow the trajectories defined in Fig. 1.

T_s refers to the sampling time of the system and is selected such that the discretization errors are negligible for this application. The position error due to discretization is over-bounded by

$$\left|p_{error}\right| < \left|V_{\max}\right| \cdot T_s . \qquad (3)$$

As a consequence, the sampling time was set to $T_s = 0.1s$ for the simulation, which leads to maximum position errors of 12m for the speed profile in the presented approach scenario. Additionally, the simulation calculates the cross-track, vertical track and the along-track error for every time step.

IV. MONTE CARLO SIMULATION

Fast-time simulations were performed based on Monte Carlo simulations which included 500 repetitive runs for all four approach trajectories shown in Fig. 1. The statistical wind distribution from TABLE I. was applied as well as a statistical initial altitude error. Additional settings of the simulation are listed in TABLE II.

TABLE II. ADDITIONAL SIMULATION PARAMETER

Number of simulation runs	500
RTA Window	+-20s
Initial altitude error mean	0m
Initial altitude error standard deviation	20m
Approach glide path angle	-3°

The abort criterion for the simulation was set to the arrival at the RTA waypoint, more precisely the along-track error to that waypoint being within a perimeter of 300m. First evaluations of trajectory 1) showed that the initial altitude error had a minor effect on the actual arrival time ($\sigma \approx 4s$). When additionally the wind distribution was enabled, the arrival times widened up to $\sigma \approx 19s$. No active 4D speed control was applied in that scenario. However, aircraft speed was reduced to fulfill ATC speed regulations and simulate a typical approach speed profile. The skewed distribution of the arrival time at the target waypoint is depicted in Fig. 2, where the RTA is marked in green and the targeted time window indicated in red. The skewed nature is due to the different headings of the trajectory's legs. Still 71% of the flights met the RTA. So far, the simulation environment provides analysis of the RTA success rate in the presence of different wind disturbances. As a full Monte Carlo simulation requires high computational capacity and therefore cannot be performed in real time, the goal is to derive a direct formula in the form of an approximated transfer function which is based on the data that were gained by the simulation.

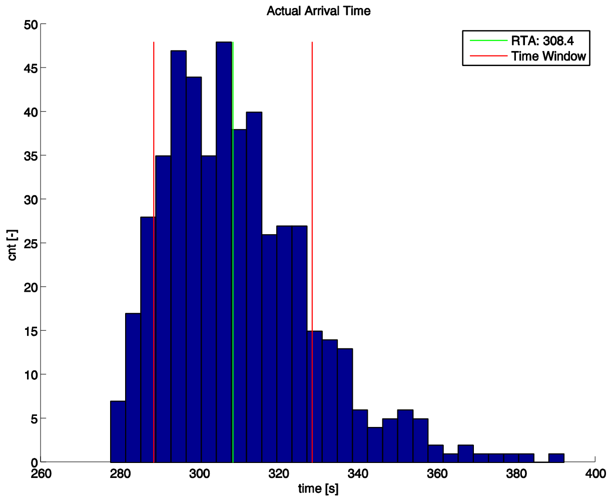

Figure 2. Distribution of the actual arrival time at the target waypoint for the approch 1) on RW04.

V. Towards a closed-Loop Mitigation Performance

As mentioned before, not only disturbances that occur along the trajectory are important when assessing the arrival time prediction uncertainty. Another significant budget results from the control strategy if appropriate closed loop control for the RTA is applied. Basically, the uncertainty of the ETA prediction decreases if the control effort is augmented, assuming that the control is able to compensate for the amount of steady state error. The quantification of that control effort is not straight forward and affected by many parameters such as look-ahead time, the slope of speed variations, the update period for control actions etc. However, a certain control strategy can be evaluated with respect to a modified cost index (CI), see [equ. 4] which describes the ratio between control efforts (represented by additional fuel costs) and costs for being off-schedule.

$$CI = \frac{C_{off-schedule}}{C_{control}}. \quad (4)$$

Whilst today, only flight delay costs are considered in the budget for cost of time, future shifts to 4D business trajectory operation and an "on-schedule, first serve" prioritization of aircraft make it necessary to consider missing the RTA constraint in both ways, being delayed or ahead of time. Both cases could lead to a renegotiation of the trajectory and incur costs for the airline. This is clearly a difference to the cost index as it is known today, for which only delays are assumed to cause airline costs. Different research projects developed tools to determine an optimal cost index dynamically, taking into account not only fuel costs, but also passenger connectivity, flight plans and slot allocation (for example see [**[6]**]). The missing part towards initial 4D capabilities in those frameworks is the correlation between the necessity for a renegotiation and its consequential costs.

In contrast to the control effort, the other necessary input is the degree of freedom to allow the speed controller to mitigate or even compensate time errors during the flight by applying closed-loop control. In this case it can be represented by the speed envelope. The following Fig. 3 shows the speed profile for one simulated approach, indicating the true airspeed (magenta), the reference speed calculated by the FMS (dark blue), the speed profile according to BADA (red) and the minimum speed in landing configuration (cyan). Additionally, the speed margins to minimum and maximum operating speed $V_{ref} \pm 7\%$ are outlined by the green-shaded area.

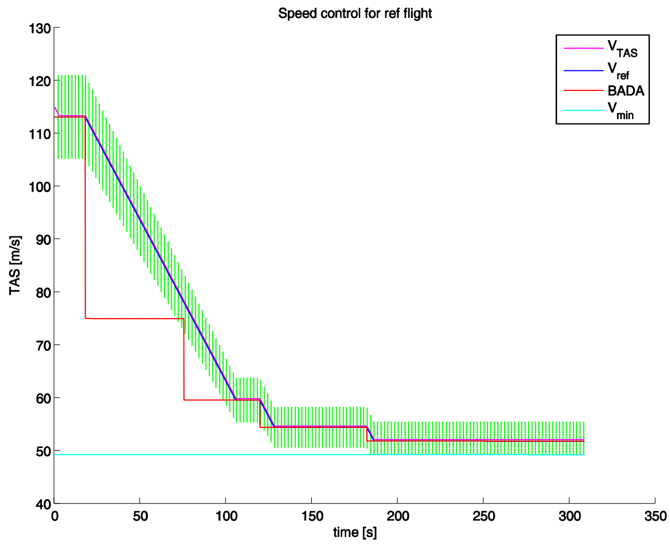

Figure 3. Speed envelope for reference flight.

The area (A) directly correlates with the mitigating effect of closed-loop control. It can be divided into two parts, the one above and the one below the reference speed, which are A_{up} and A_{lo} respectively.

$$A_{up}(t) = \int_{t}^{ETA} \left(V_{\max}(\tau) - V_{ref}(\tau)\right) d\tau \quad (5)$$

Those parameters can be interpreted as a measure for the current robustness against time delays. At the same time, they immediately reduce the ETA prediction uncertainty, because deviations of the input parameters, e.g. wind, can be compensated and do not propagate to the ETA error. The parameters consider remaining time to the target waypoint and margins to increase and decrease the reference true airspeed.

Thus, A_{up} can be interpreted as a factor to compensate for time delays. Analogously, the compensation factor for being ahead of time reads

$$A_{lo}(t) = \int_{t}^{ETA} \left(V_{ref}(\tau) - V_{\min}(\tau) \right) d\tau \ . \qquad (6)$$

At the same time, the formula accounts for the effect that with a decreasing remaining time to the target waypoint, the mitigation effect of controls decreases to zero.

In a next step, both transfer parameters shall be combined to describe the error propagation from input parameters (wind direction, wind speed) to the ETA uncertainty by evaluating the actual time of arrival.

VI. Conclusion and Outlook

The paper introduced a light framework to study the effects of wind forecast errors and initial position uncertainties on the ETA prediction uncertainty. The basic idea is to model the uncertainty propagation in the form of transfer functions that can be determined via Monte Carlo Simulation. Three basic influences are taken into account, which are the trajectory path in combination with the wind uncertainty on the one hand, as well as control strategy together with control margins on the other. Therefore, two different budgets are considered. The first describes the error propagation of input uncertainties along the trajectory. The second evaluates the mitigation effect of closed-loop control, which is part of current studies. Different strategies will be investigated to optimize ETA prediction uncertainty versus control effort which is always at the cost of an increased fuel burn and higher environmental impacts. The aim is to find a direct formula that provides a reliable estimate of the ETA uncertainty as well as a success probability for meeting the RTA. The latter will be of high benefit for all depending stakeholders.

VII. Acknowledgements

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