A Study on Trajectory Optimization for the Terminal Area

Keywords: BADA, conflict resolution, terminal area

2014/05/30 ICRAT2014 Doctoral session
Yokohama National University
Daichi Toratani
Seiya Ueno
1, Introduction
   - Background and target of this study

2, Problem formulation (without conflict)
   - Model of the trajectory

3, Simulation results (without conflict)

4, Problem formulation (Conflict resolution)
   - Introducing conflict resolution

5, Simulation results (Conflict resolution)

6, Conclusion and future plan
Continuous descent operation (CDO)

- Descending constant rate
- Constant thrust

CDO is able to improve;
- Fuel consumption
- Noise pollution
  etc.

Tokyo international airport (Haneda airport), HND

Proposed for the CDO
Crossing
Air traffic management in the terminal area

- Conventional descent operation (Step-by-step)

- Continuous descent operation

The purpose of this study is ... the optimal conflict-free trajectory for the CCO.
Optimal trajectory theory and trajectory optimization

- **State equation (Dubins car)**
  \[
  \frac{d}{dt} \begin{pmatrix} \theta \\ x \\ y \end{pmatrix} = \begin{pmatrix} u \\ \cos \theta \\ \sin \theta \end{pmatrix}
  \]
  Input: \( u \)

- **Criterion (Minimum input)**
  Minimizing
  \[
  J = \int_{t_0}^{t_f} \frac{1}{2} u^2 \, dt
  \]

- **Boundary conditions**
  \[
  X_0 = (90 \, [\text{deg}] \quad 0 \quad 0)
  \]
  \[
  X_f = (0 \, [\text{deg}] \quad 1 \quad 1)
  \]
  where
  \[
  X^T = (\theta \quad x \quad y)
  \]
  0: Initial
  \( f \): Terminal

- Example: Cruising aircraft
  \( V = 1 \) (const.)

- Optimal trajectory
Related previous studies (Optimal control approach)

- A. Andreeva-Mori et al., “Scheduling of Arrival Aircraft Based on Minimum Fuel Burn Descents”
  - Fuel burn model
  - Optimal trajectory $\rightarrow$ CDO

- J. Hu et al., “Optimal Coordinated Maneuvers for Three-Dimensional Aircraft Conflict Resolution”
  - Constraint for conflict resolution
  - Multiple aircraft conflict resolution
  - Constant velocity
Problem of the trajectory optimization

In the practice of the air traffic control, …

Vectoring (Spatial control)  Changing velocity (Temporal control)

Spatial conflict resolution  Temporal conflict resolution  Spatial and Temporal conflict resolution

! It is difficult to treat spatial and temporal conflict resolutions simultaneously.
1, Introduction 6/6

Space-time coordinate system (STCS)

In the STCS,

- The vertical axis means time.
- It is able to treat the time along with the position.
- It is also able to calculate the altitude.

4D trajectory

The target of this study is ...

To develop the optimization method in the STCS.
- Conflict resolution, minimum fuel, minimum time, etc.

Which conflict resolutions (spatial or temporal) are optimal to resolve conflict?
2, Problem formulation (w/o conflict) 1/5

Base of aircraft data (BADA)

- Total-energy model (TEM)
  
  \[(Thr - D)V_{TAS} = mg_0 \frac{dh}{dt} + mV_{TAS} \frac{dV_{TAS}}{dt}\]

\[\leftrightarrow \frac{dV_{TAS}}{dt} = \frac{1}{m} (Thr - D - mg \sin \gamma)\]

- Azimuth angle
  
  \[\frac{d\psi}{dt} = \frac{g_0}{V_{TAS}} \tan \phi\]

- Fuel flow
  
  \[FF = C_{f1} \left(1 + \frac{V_{TAS}}{C_{f2}}\right) Thr\]

- Maximum climb thrust
  
  \[Thr = C_{Tc,1} \left(1 - \frac{h_p}{C_{Tc,2}} + C_{Tc,3} h_p^2\right)\]
Base of aircraft data (BADA)

- State equations

$$\begin{pmatrix}
\frac{d}{dt} \psi \\
\frac{d}{dt} \gamma \\
\frac{d}{dt} x \\
\frac{d}{dt} y \\
\frac{d}{dt} h_p
\end{pmatrix} =
\begin{pmatrix}
\frac{g}{V_{TAS}} \tan \phi \\
\frac{1}{m} (Thr - D - mg \sin \gamma) \\
V_{TAS} \cos \gamma \cos \psi \\
V_{TAS} \cos \gamma \sin \psi \\
V_{TAS} \sin \gamma
\end{pmatrix}$$

- Fuel flow

$$FF = C_{f1} \left( 1 + \frac{V_{TAS}}{C_{f2}} \right) Thr$$

$\rho$: Rate of flight path angle
Space-time coordinate system (STCS)

- **State equation**
  \[
  \frac{d}{ds} \begin{pmatrix} \psi_s \\ \psi_t \\ x \\ y \\ t \end{pmatrix} = \begin{pmatrix} \kappa_s \\ \kappa_t \\ \cos \psi_s \cos \psi_t \\ \sin \psi_s \cos \psi_t \\ \sin \psi_t \end{pmatrix}
  \]
  \(\kappa\): Curvature

- **Velocity and acceleration**
  \[
  V = \frac{dl}{dt} = \tan(90^\circ - \psi_v) = \frac{1}{\tan \psi_t}
  \]
  \[
  a = -\frac{\kappa_t}{\sin^3 \psi_t} \quad \leftarrow \kappa_t = -as \sin^3 \psi_t
  \]
**2, Problem formulation (w/o conflict)**

- **BADA (Independent variable: time)**

\[
\begin{align*}
\frac{d}{dt} \begin{pmatrix}
\psi_s \\
V_{TAS} \\
\gamma \\
x \\
y \\
h_p
\end{pmatrix}
= & \begin{pmatrix}
g \tan \phi \\
\frac{g}{V_{TAS}} \\
\frac{1}{m} \left( Thr - D - mg \sin \gamma \right) \\
p \\
\frac{V_{TAS} \cos \gamma \cos \psi_s}{V_{TAS} \cos \gamma \sin \psi_s} \\
\frac{V_{TAS} \cos \gamma \sin \psi_s}{V_{TAS} \sin \gamma}
\end{pmatrix} + \\
& \begin{pmatrix}
\frac{d\psi_t}{ds} = -\sin^3 \psi_t \\
\frac{dt}{ds} = \sin \psi_t \\
V_{TAS} = \frac{1}{\tan \psi_t}
\end{pmatrix}
\end{align*}
\]

- **BADA (STCS)**

\[
\begin{align*}
\frac{d}{ds} \begin{pmatrix}
\psi_s \\
\psi_t \\
\gamma \\
x \\
y \\
h_p \\
t
\end{pmatrix}
= & \begin{pmatrix}
g \sin \psi_t \tan \psi_t \tan \phi \\
-\sin^3 \psi_t \frac{1}{m} \left( Thr - D - mg \sin \gamma \right) \\
\sin \psi_t \ p \\
\cos \psi_s \cos \psi_t \cos \gamma \\
\sin \psi_s \cos \psi_t \cos \gamma \\
\cos \psi_t \sin \gamma \\
\sin \psi_t
\end{pmatrix}
\end{align*}
\]
Optimal control problem and calculation method

• **Optimal control problem**

  **Constraint equation**  **Criterion**  **Boundary conditions**

  \[
  \frac{dX}{ds} = F \\
  J = \int_0^{s_F} L \, ds \\
  X(s_0) = X_0 \\
  X(s_f) = X_f
  \]

  State equation: \( \frac{dX}{ds} = F \)  
  Fuel flow: \( J = \int_0^{s_F} L \, ds \)  
  Initial and terminal state

  Optimal control theory

• **Two-point boundary value problem (TPBVP)**

  Simultaneous non-linear differential equations

  Linear approximation

• **Simultaneous non-linear equations**

  Simultaneous non-linear equations solver
3, Simulation results (w/o conflict) 1/3

Simulation conditions

Terminal condition

\[(\psi_{sf} \ V_{TASf} \ \gamma_f \ x_f \ y_f \ h_{pf} \ t_f) = (0 \ 250.0 \ F \ F \ 0 \ 10000 \ F)\]

Data of aircraft

Boeing 777-200

Initial condition

\[(\psi_{s0} \ V_{TAS0} \ \gamma_0 \ x_0 \ y_0 \ h_{p0}^{-0.5} \ t_0) = (0 \ 150.0 \ 5 \ 0 \ 0 \ 3000 \ 0)\]

Units: (\(\psi_s\) \ \(V_{TAS}\) \ \(\gamma\) \ \(x\) \ \(y\) \ \(h_p\) \ \(t\))

\[= ([\text{deg}] \ \ [\text{m/s}] \ \ [\text{deg}] \ \ [\text{m}] \ \ [\text{m}] \ \ [\text{m}] \ \ [\text{m}] \ \ [\text{s}])\]

\(F\) : Terminal free
3, Simulation results (w/o conflict) 2/3

Trajectories in the 3D space and the STCS

3D space

Space-time coordinate system
• The optimal trajectory in the STCS is derived.
Spatial conflict resolution

Vectoring (Spatial control)

Temporal conflict resolution

Changing velocity (Temporal control)
4, Problem formulation (Conflict resolution) 2/2

Interior-point constraint

If you want to add a new constraint, …

![Diagram showing a 3D plot with a Waypoint and coordinates labeled x [m], y [m], and h [m].]
Interior-point constraint

Position1 = Position2 (Specified)
Angle1 = Angle2 (Free)

\[ \text{Position1} = \text{Position2} \quad (\text{Specified}) \]
\[ \text{Angle1} = \text{Angle2} \quad (\text{Free}) \]
Interior-point constraint

- The original trajectory optimization is transformed to the trajectory optimization problem with a new constraint.
- To see clearly which conflict resolutions are optimal, conflict resolution with specified point is shown.
Simulation conditions (Interior point constraint)

Conflict point
\[
(\psi_s \ V_{TAS} \ \gamma \ x \ y \ h_p \ t) = (0 \ 225.9 \ 2.410 \ 73842 \ 0 \ 7553 \ 384.4)
\]

32808 [ft]
10000

1

5000

6000

54.00 [nm]

0

0.5

1

107.99 [nm]

54.00 [nm]

-54.00 [nm]

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)

\(\times 10^5\)
Simulation conditions (Spatial conflict resolution)

Interior point conditions

\[(\psi_{sint} \quad V_{TASint} \quad \gamma_{int} \quad x_{int} \quad y_{int} \quad h_{pint} \quad t_{int}) = (0 \quad F \quad F \quad 73842 \quad 9260 \quad F \quad F)\]

\[39.87 \text{ [nm]} \quad 5 \text{ [nm]}\]
Spatial conflict resolution

- It is confirmed that the spatial conflict resolution is able to resolve the conflict with the spatial control.

3D space

x-y plane

Simulation results (Conflict resolution) 3/6
Simulation conditions (Temporal conflict resolution)

Conflict point
\[ t_{int} = 384.4 \text{ [s]} \]
\[ V_{TAS int} = 225.9 \text{ [m/s]} \]

Interior point conditions
\[ (\psi_{sint}, V_{TAS int}, \gamma_{int}, x_{int}, \gamma_{int}, h_{pint}, t_{int}) = (0, F, F, 73842, 0, F, 421.4) \]

\[ 5 \text{ [nm]} / 225.9 \text{ [m/s]} = 41.0 \text{ [s]} \]
\[ 384.4 + 41.0 = 421.4 \text{ [s]} \]
Temporal conflict resolution

Space-time coordinate system

- It is confirmed that the temporal conflict resolution is able to resolve the conflict decreasing its velocity.
Fuel flow and fuel consumption

- The fuel consumption with the spatial conflict resolution is lower than the fuel consumption with the temporal conflict resolution.
Conclusion

✓ The trajectory optimization method in the space-time coordinate system is developed, and the optimal trajectory is derived.

✓ By introducing the interior-point constraint, the optimal conflict-free trajectories with spatial and temporal conflict resolutions are obtained.

✓ The fuel consumption with the spatial conflict resolution is lower than the fuel consumption with the temporal conflict resolution.
Future plan

- The optimal trajectories will be obtained in various boundary conditions.
  - e.g.) Conflict resolution with accelerating velocity.

- The model of the trajectory will be improved.
  - Introducing mass change, $V_{CAS}$, clime rate, etc.

- Multiple aircraft will be introduced to the trajectory optimization.