Analysis of Wind-Shear Effects on Optimal Aircraft Cruise

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***Abstract*—An analysis of the effects of wind shear on optimal aircraft cruise trajectories is presented. The procedure considered is cruise at constant Mach number and constant altitude, which is commonly flown by airlines, following air-traffic-control rules. The optimal trajectories correspond to the case of minimum direct-operating-cost cruise with given range, and are obtained using parametric optimization theory. The main objective of the paper is to analyze the influence of the wind shear on the optimum altitude and speed. The results show that, for a given cost index, the effect of the wind shear on the optimum altitude is quite large. The effect of the cost index on the optimum results is also analyzed. Results are presented for a model of a Boeing 767-300ER.**

Keywords-aircraft trajectory optimization; optimal cruise; wind shear

I. INTRODUCTION

Trajectory optimization is an important subject in air traffic management, which aims at defining optimal flight procedures that lead to energy-efficient flights. In practice, the airlines consider a cost index (CI) and define the direct operating cost (DOC) as the combined cost of fuel consumed and flight time, weighted by the CI. Their goal is to minimize the DOC. This problem has been treated extensively in the literature. Among many others, for example, minimum-DOC trajectories have been studied by Barman and Erzberger [1], Erzberger and Lee [2] and Burrows [3] who analyze the minimum-DOC problem for global trajectories (climb-cruise-descent); they consider steady cruise, and take the aircraft mass as constant. Burrows [4] also analyzes the minimum-DOC problem for global trajectories, without the assumption of constant mass, but with the assumption that the cruise segment takes place in the stratosphere. Bilimoria et al. [5] and Chakravarty [6] analyze the minimum-DOC, steady cruise as the outer solution of a singular perturbation approach, where the aircraft mass is taken as constant. Lidén [7] proposes computing algorithms to be used in flight management systems which optimize the cruise profile. However, these works either do not consider wind-shear effects, as in [2-5], or only consider one particular wind profile, as in [1,6,7], not analyzing the effects of changing the wind profile.

In this paper we address the problem of analyzing the effects of wind shear on minimum-DOC aircraft cruise trajectories. The procedure considered is cruise at constant Mach number and constant altitude, which is commonly flown by airlines, following air-traffic-control (ATC) rules; the cruise range is fixed. The cruise is unsteady, with variable mass, subject to a horizontal, altitude-dependent wind profile. The main objective of this work, with respect to the published literature, is to provide an understanding of the influence of the wind shear on the optimum cruise altitude and cruise speed, taking into account the cost index value fixed by the operator. For completeness, the influence of the average wind speed (headwinds and tailwinds) is also analyzed.

To optimize the cruise procedure, a parametric optimization approach is presented. Parametric trajectory optimization has been also treated extensively. For example, Betts and Cramer [8] apply the direct transcription technique, which combines nonlinear optimization with a discretization of the trajectory dynamics, to the optimal design of trajectories (for several performance indices) subject to realistic constraints that represent the trajectory phases of a mission profile. Soler et al. [9] relax some of the constraints imposed in [8] to give more room for planning more efficient trajectories, formulating a single optimal control problem which is solved as a nonlinear optimization problem. Menon et al. [10] optimize flight strategies for conflict resolution parameterizing the trajectories in terms of four-dimensional waypoints, and approximating the trajectories by piecewise-linear paths. Wu and Zhao [11] optimize the trajectory from liftoff to touchdown and quantify the deviation from actual trajectories due to modeling errors and/or flight conditions, defining the trajectory by a series of flight segments specified by a set of flight objectives, such as speeds, altitudes or throttle settings. Torres et al. [12] formulate a multi-objective optimization problem that minimizes noise and pollutants emissions of the departure procedures, parameterizing the trajectory through two sets of variables that describe the evolution of the aircraft speed and thrust. Valenzuela et al. [13] optimize discrete cruise procedures constrained to have Mach numbers multiple of 0.01, and altitudes defined by flight levels.

In this paper the cruise procedure is defined by a trajectory pattern formed by five segments commonly flown by airlines, which is in fact a flight intent that defines unambiguously how the aircraft is to fly. The segments are as follows: 1) starting at the initial altitude h_i, a transition segment at the initial Mach

M_i (descent/climb with idle/maximum cruise engine rating) ending at the cruise altitude h_c, 2) a transition segment at constant altitude h_c (deceleration/acceleration with idle/maximum cruise engine rating) ending at the cruise Mach M_c, 3) the main cruise segment at constant Mach M_c and constant altitude h_c, ending when a distance r_c is flown, 4) a transition segment at constant altitude h_c (deceleration/acceleration with idle/maximum cruise engine rating) ending at the final Mach M_f, and 5) a transition segment at constant Mach M_f (descent/climb with idle/maximum cruise engine rating) ending at the final altitude h_f. In this work, the initial and final conditions (h_i, M_i, h_f, M_f) are given, so that the cruise altitude h_c, the cruise Mach number M_c and the distance r_c are free variables, on which the optimization is performed.

Results are presented for a model of a Boeing 767-300ER, with compressible aerodynamics and general specific fuel consumption and thrust models, which is described in [14]. The results for linear wind profiles show that the effect of the wind shear on the optimum altitude is quite large, depending strongly on the cost index; on the contrary, the effect of the average wind speed is much smaller. In particular, it is found that depending on the value of the wind shear the optimal cruise takes place either in the troposphere or in the stratosphere, with opposite behaviors as a function of the cost index, namely, the optimum altitude decreases with the cost index in the troposphere, whereas it increases in the stratosphere.

II. PROBLEM FORMULATION

A. Equations of Motion

In this work, cruise flight in a vertical plane is considered. The model adopted to describe the aircraft motion is that of a point mass with three degrees of freedom, commonly used for trajectory prediction (see Slattery and Zhao [15]); the equations then describe the motion of the aircraft center of mass, considered as a mass-varying body. The case of altitude-dependent horizontal winds contained in the flight plane is considered. The equations of motion for symmetric flight with thrust parallel to the aircraft aerodynamic velocity are the following (see Jackson et al. [16]):

$$\begin{aligned}
&m\frac{\mathrm{d}V}{\mathrm{d}t} = T - D(V,h,L) - mg\gamma - mV\frac{\mathrm{d}w}{\mathrm{d}h}\gamma,\\
&L = mg,\\
&\frac{\mathrm{d}m}{\mathrm{d}t} = -c(V,h)T,\\
&\frac{\mathrm{d}r}{\mathrm{d}t} = V + w(h),\\
&\frac{\mathrm{d}h}{\mathrm{d}t} = V\gamma,
\end{aligned} \tag{1}$$

where the following simplifying assumptions have been made: $\gamma \ll 1$ and $V\dot{\gamma}/g \approx 0$. In the previous equations, V and γ are the aerodynamic velocity modulus and the aerodynamic path angle; m the aircraft mass; r and h the horizontal distance and the altitude; w the wind speed; g the gravity acceleration; t the time; T, L, and D the thrust, the lift, and the aerodynamic drag; and c the specific fuel consumption.

Each flight segment is defined by two flight constraints (for example, to fly at constant altitude and constant speed), which together with (1) form a system of differential algebraic equations (DAE). The resolution of the DAE systems for the different flight segments is based on the reduction of the system of equations to a system of ordinary differential equations (ODE) through the explicit utilization of the flight constraints. The ODE systems are then solved using MATLAB's *ode45* [17] (based on an explicit Runge Kutta formula).

The computation of each flight segment starts with the corresponding initial conditions and ends when the appropriate stopping condition is reached (for instance, reaching a given altitude or a given Mach number). Additionally, to compute the flight segments, some supplementary models are needed: Earth, aerodynamic and propulsion models. In this paper, the Earth has constant gravity, the atmospheric model is ISA, and realistic aerodynamic and propulsion models are considered, which are described in [14]. The aircraft model provides the following functions: compressible drag polar $C_D(M, C_L)$, specific fuel consumption $c(M,h)$, and available thrust $T_{MCRZ}(M,h)$ for maximum-cruise engine rating and $T_{IDLE}(M,h)$ for idle engine rating. The atmosphere model provides the density $\rho(h)$ and the wind speed profile $w(h)$. The lift and drag coefficients are defined by $L = \rho V^2 S C_L / 2$ and $D = \rho V^2 S C_D / 2$, where S is the reference wing surface.

B. Trajectory Pattern

To model the cruise flight in a vertical plane, the trajectory pattern shown in Fig. 1 is considered; the arrows in the figure indicate the stopping criterion for each flight segment, and ER stands for fixed engine rating. The pattern starts from the initial altitude h_i and initial Mach number M_i, and ends at the final conditions h_f and M_f. The pattern is formed by five flight segments. The first one is a transition segment, a descent/climb at constant Mach M_i and with idle/maximum cruise engine rating ending at the cruise altitude h_c. The second one is also a transition segment, a deceleration/acceleration at constant altitude h_c and with idle/maximum cruise engine rating, ending at the cruise Mach number M_c. The third one is a segment at constant Mach M_c and constant altitude h_c ending when a distance r_c is flown. Finally, because the cruise flight has to end at the final conditions h_f and M_f, two more transition segments, as those just described, complete the pattern; that is, a transition segment at constant altitude h_c (deceleration/acceleration with idle/maximum cruise engine rating) ending at the final Mach M_f, and a transition segment at constant Mach M_f (descent/climb with idle/maximum cruise engine rating) ending at the final altitude h_f .

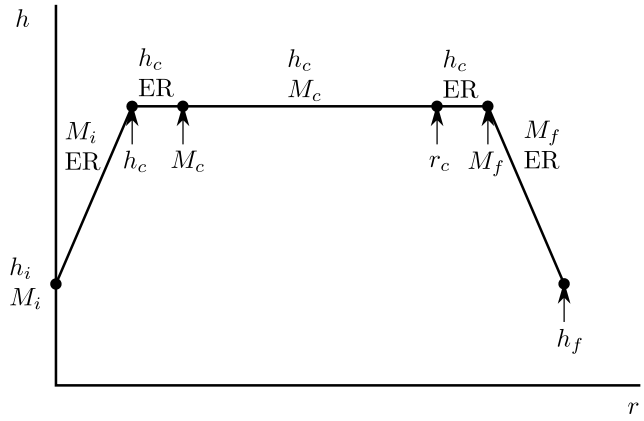

Figure 1. Trajectory pattern.

Hence, one can see that three different types of flight segments are considered, which comply with usual ATC rules, namely, segments with constant Mach and constant altitude, transition segments with fixed engine rating and constant Mach (for descent/climb segments), and transition segments with fixed engine rating and constant altitude (for decelerating/accelerating segments).

The procedure is defined by seven parameters: altitude h_c, Mach number M_c, distance r_c, initial and final altitudes, h_i and h_f, and initial and final Mach numbers, M_i and M_f. Depending on the application, some of these parameters can be fixed, whereas the rest are free and used as variables in the optimization problem. In this paper, the initial and final values of altitude and Mach number are fixed.

C. Parametric Optimization

Once the free parameters of the trajectory are defined, they are collected in a vector $\mathbf{x}$. The optimization problem is formulated as a nonlinear programming (NLP) problem:

$$\begin{aligned} &\text{minimize } J(\mathbf{x}) \\ &\text{subject to } \mathbf{f}(\mathbf{x}) = \mathbf{0}, \mathbf{g}(\mathbf{x}) \le \mathbf{0}, \mathbf{x} \in X, \end{aligned} \tag{2}$$

where X is the feasible region of the variables. In this formulation, the optimality criterion defining the cost function can be the minimization of any property or combination of properties of the trajectory that can be derived from the computation of the trajectory. The equality and inequality constraints and the feasible region depend on the application.

Different techniques can be used to solve NLP problems [18]. In this work, MATLAB's *fmincon* is used, a sequential quadratic programming (SQP) method, which is proposed by Schittkowsky [19] as the most efficient to solve nonlinear programming problems. It must be noted that SQP methods, as gradient-based methods, are only able to find one local minimum within the feasible region; in case that several local minima exist, the global minimum can be obtained by subdividing the feasible region into appropriate subregions, solving the optimization problem on each subregion, and finally taking the best local minimum found.

D. Minimum-DOC Cruise

The objective is to minimize the direct operating cost in cruise flight with fixed range. The DOC is a combination of fuel and time costs, $DOC = m_F + CI\, t_f$ (measured in kg), where m_F is the fuel consumption, t_f the flight time, and CI the cost index which measures the relative importance of both costs (the case $CI = 0$ corresponds to minimum fuel). Note that although airlines define the CI in units of \$/hour divided by cents/lb, in this paper international units of measure are used, hence, the CI is measured in kg/s. Representative values of the CI are in the range 0 to 3 kg/s, which is considered in the numerical simulations. The initial and final altitudes and speeds are fixed, $h_i = h_f = 30000$ ft and $M_i = M_f = 0.79$. The same initial and final conditions are chosen, to be able to compare all cases considered in the analysis; M_i and M_f correspond to typical climb and descent Mach values and $h_i = h_f$ is a typical cruise altitude. The range to be flown is $r_A = 2000$ km.

The cost function can be written as

$$J(\mathbf{x}) = m_F(\mathbf{x}) + CI\ t_f(\mathbf{x}), \tag{3}$$

where the fuel consumption and the flight time depend on the free parameters $\mathbf{x}$. Because the total flown distance r_f is a function of the free parameters, the given range r_A is imposed by the equality constraint

$$r_f(\mathbf{x}) - r_A = 0. \tag{4}$$

The inequality constraints reduce to requiring that the speeds and altitudes be within the aircraft operational envelope.

Because the initial and final conditions are given, the total number of free parameters is three: the Mach number M_c, the altitude h_c, and the distance flown during the third pattern segment r_c. The feasible region is given by $M_c \in [0.60, 0.86]$, $h_c \in [20000, 43000]$ ft, and $r_c \in [0, 2000]$ km.

Considering the different behavior of the atmosphere in the troposphere and in the stratosphere, one could expect the objective function to be non convex in the feasible region, possibly having a local minimum in each layer. For this reason, it has been decided to subdivide the feasible region into two subregions: $h_c \in [20000, 36089]$ ft in the troposphere and $h_c \in [36089, 43000]$ ft in the stratosphere. For the cases analyzed in Section III, it has been observed that the objective function is convex in each subregion. The optimization problem is then solved in both subregions and the best local minimum found is taken as the global minimum.

E. Wind Profile

For the wind model, linear profiles are considered, with the absolute value of the wind speed increasing with altitude (see [6,20,21]). The profiles, between two given altitudes h_1 and $h_2 > h_1$, are defined as follows

$$w(h) = \overline{w} + \Delta w \frac{h-\overline{h}}{h_f - \overline{h}}, \quad (5)$$

where $\overline{w}$ is the average wind, Δw the wind-shear parameter and $\overline{h} = (h_1 + h_2)/2$ the average altitude. For given values of h_1 and h_2, Δw defines the wind shear $\mathrm{d}w/\mathrm{d}h$, and, in particular, $\Delta w = 0$ defines a uniform wind profile. Note that the average wind speed $\overline{w}$ is given by

$$\overline{w} = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} w(h)\,\mathrm{d}h, \quad (6)$$

and, also, since the wind profiles are linear, $\overline{w}$ is the wind speed at the average altitude, that is, $\overline{w} = w(\overline{h})$. In the following, both tailwinds (TW) and headwinds (HW) are considered, with the linear profiles defined as follows: for TW one has $\overline{w} > 0$ and $\Delta w \geq 0$, and for HW $\overline{w} < 0$ and $\Delta w \leq 0$. To define the wind profile, the following altitudes are considered: $h_1 = 10000$ ft, $h_2 = 33000$ ft; the average altitude is $\overline{h} = 21500$ ft. The average wind ranges from -40 kt to 40 kt, and the absolute value of the wind-shear parameter ranges from 0 to 40 kt.

III. RESULTS

In this section optimization results are presented for $W_f = 1200$ kN. Note that instead of fixing the initial aircraft weight, which would depend on the value of the CI, the final weight is fixed, the same for all the simulations. Hence, the integration of the equations of motion is performed backwards.

The effects of the average wind and of the wind shear on the optimal procedures are analyzed in Sections III.A and III.B, respectively.

A. Effect of the Average Wind

The evolution of the optimum altitude h_c^* with the average wind is shown in Fig. 2 for different values of the CI and for a wind-shear parameter $\Delta w = 0$. It can be seen that the altitudes decrease as the CI increases; this same behavior was found, for example, by Barman and Erzberger [1] for short-haul aircraft with constant mass and no wind. For a fixed value of the CI, the altitude slightly increases as the average wind increases; the smaller the CI, the weaker the altitude increase. For instance, for $CI = 0$, one has that the difference in the altitude (for the range of average wind considered) is 129 ft, and for $CI = 3$ kg/s is 727 ft.

The optimum Mach number M_c^* is shown in Fig. 3. As expected, it increases as the CI increases. It can be seen that the Mach number slightly decreases as the average wind increases; the larger the CI, the weaker the Mach decrease. For instance, for $CI = 3$ kg/s, one has that the difference in the Mach number is just 0.005, and for $CI = 0$ is 0.011.

In summary, it can be said that the effect of the average wind on the optimal procedures is small.

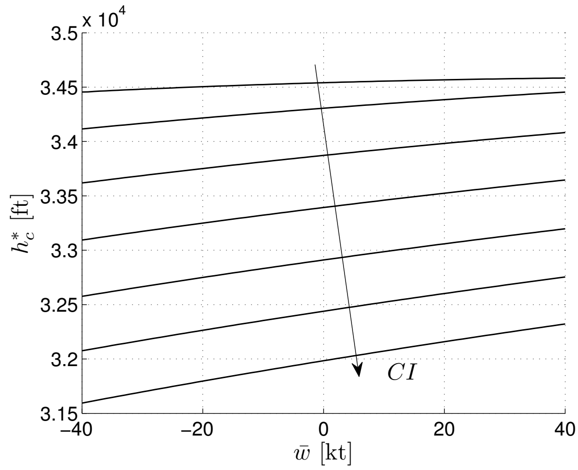

Figure 2. Optimum altitude vs average wind for CI = 0,0.5,1,1.5,2,2.5, 3 kg/s, and $\Delta w = 0$.

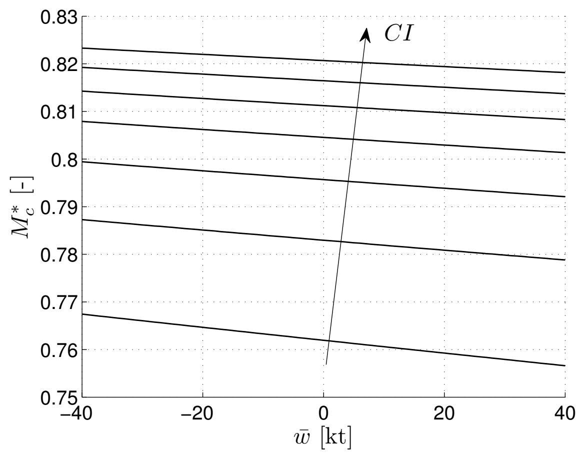

Figure 3. Optimum Mach number vs average wind for CI = 0,0.5,1,1.5,2, 2.5,3 kg/s, and $\Delta w = 0$.

The global properties m_F, t_f, and DOC are shown in Figs. 4, 5, and 6, respectively. These three global properties significantly decrease as $\overline{w}$ increases: they are, as expected, larger for headwinds than for tailwinds. As an example, for $CI = 1.5$ kg/s one has the following differences (for the range of average wind considered): 1756 kg in m_F, 1347 s in t_f, and 3777 kg in DOC.

As the CI increases, m_F increases and t_f decreases as expected, and DOC increases, because both m_F and the product $CI\, t_f$ increases.

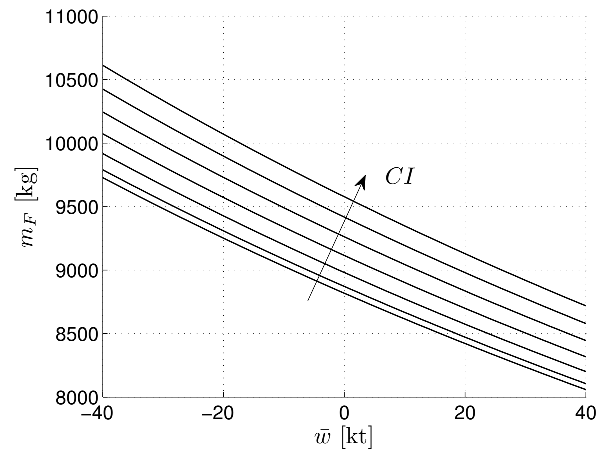

Figure 4. Fuel consumption vs average wind for CI = 0,0.5,1,1.5,2,2.5, 3 kg/s, and $\Delta w = 0$.

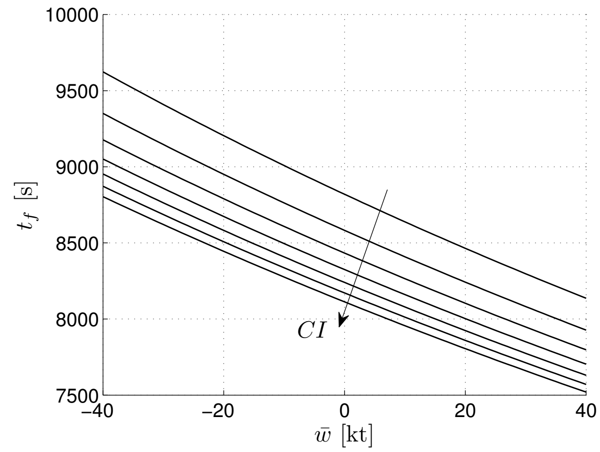

Figure 5. Flight time vs average wind for CI = 0,0.5,1,1.5,2,2.5,3 kg/s, and $\Delta w = 0$.

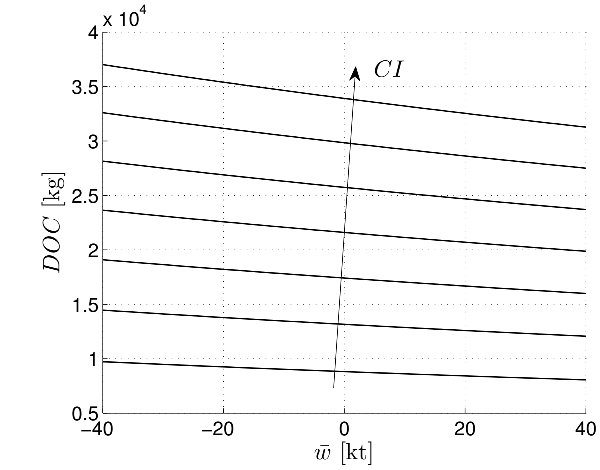

Figure 6. DOC vs average wind for CI = 0,0.5,1,1.5,2,2.5,3 kg/s, and $\Delta w = 0$.

B. Effect of the Wind Shear

Now, the effect of the wind shear is analyzed. In this analysis one has $\overline{w} = -30$ kt and $\Delta w < 0$ for HW, and $\overline{w} = 30$ kt and $\Delta w > 0$ for TW.

The evolution of the optimum altitude with the wind-shear parameter is shown in Fig. 7. It can be seen that the altitude increases as the wind shear increases. This variation is quite large and strongly depends on the CI. For instance, for $CI = 0$, one has that the difference in the altitude (for the range of wind shear considered) is 5791 ft, and for $CI = 3$ kg/s is as large as 15977 ft. The jumps at $\Delta w = 0$ correspond to the different average winds considered.

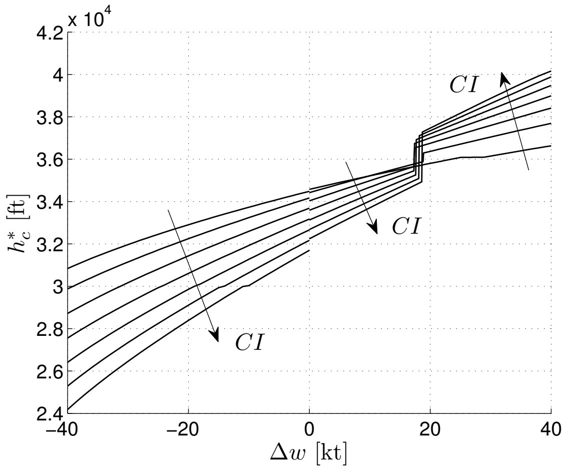

Figure 7. Optimum altitude vs wind shear for CI = 0,0.5,1,1.5,2,2.5,3 kg/s; $\overline{w} = -30$ kt for HW, $\overline{w} = 30$ kt for TW.

For the range of CI considered, the optimum altitude is found to be in the troposphere for negative and small positive values of Δw and in the stratosphere for large values of Δw . The way in which the transition from one region to the other is performed depends on the value of the CI. For instance, for $CI = 0$, the optimum altitude coincides with the tropopause between $\Delta w = 25$ and 29 kt, and for $CI = 1$ kg/s the altitude jumps from 35650 ft to 36530 ft at approximately $\Delta w = 17.5$ kt. A more detailed view of this transition is shown in Fig. 8 for CI ranging from 0 to 0.5 kg/s.

The variation of the altitude with the CI depends on the wind-shear parameter (see Fig. 7). For negative and small positive values of Δw , one has that the optimum altitude decreases as the CI increases; but for large positive values of Δw , when the optimum altitude is located in the stratosphere, the behavior is reversed: the optimum altitude increases as the CI increases. The two different behaviors are better observed in Fig. 9 where the evolution of the optimum altitude with the CI is represented for positive values of the wind-shear parameter: for small values of Δw , h_c^* decreases, and for large values of Δw , h_c^* increases. As one can also see, for intermediate values of Δw the optimum altitude can be in the troposphere or in the stratosphere, depending on the value of the CI. For instance, for

$\Delta w = 17.5$ kt the optimum altitude is in the stratosphere for $CI \in [0.89, 1.77]$ kg/s and in the troposphere in any other case.

These results show that the wind shear does have an important effect on the optimum cruise altitude.

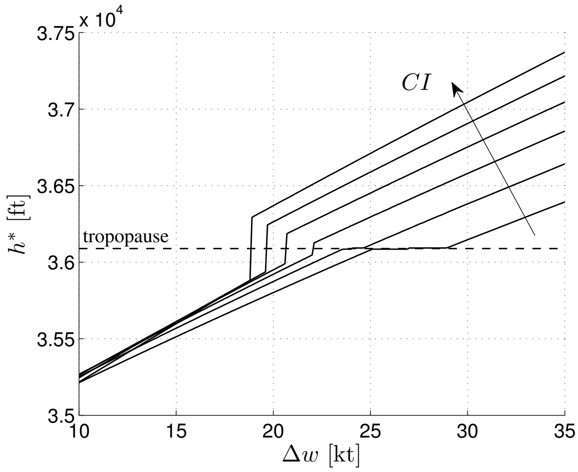

Figure 8. Optimum altitude vs wind shear for CI = 0,0.1,0.2,0.3,0.4, 0.5 kg/s, and $\overline{w} = 30$ kt.

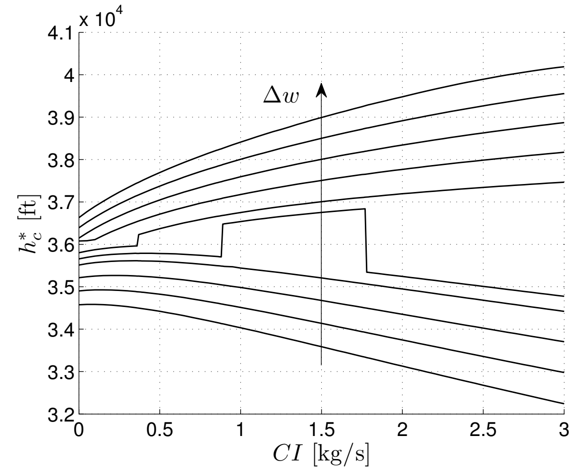

Figure 9. Optimum altitude vs cost index for $\Delta w = 0,5,10,15,17.5,20,25,30,35,40$ kt, and $\overline{w} = 30$ kt.

The evolution of the optimum Mach number is shown in Fig. 10. One can see that, in general, it slightly increases for $\Delta w < 0$ and decreases for $\Delta w > 0$. The little jumps about $\Delta w = 17.5$ kt match with the altitude jumps just mentioned. The variation of the Mach number with the wind shear is small and comparable to the variation with the average wind. For instance, for $CI = 0$, one has that the difference between the maximum and minimum value of the Mach number is just 0.011, and for $CI = 3$ kg/s is 0.016.

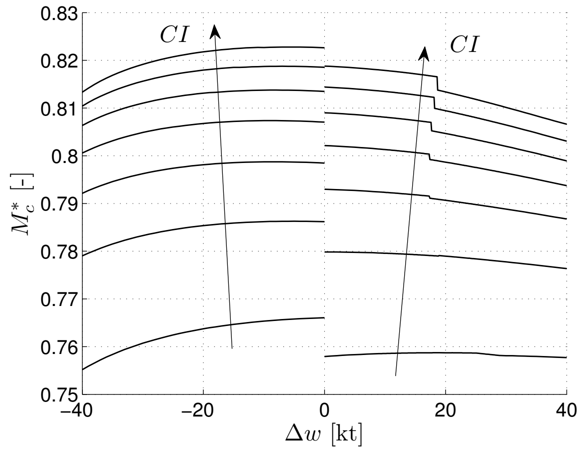

Figure 10. Optimum Mach number vs wind shear for CI = 0,0.5,1,1.5,2,2.5, 3 kg/s; $\overline{w} = -30$ kt for HW, $\overline{w} = 30$ kt for TW.

The global properties m_F, t_f, and DOC are shown in Figs. 11, 12, and 13, respectively. For a given CI, they are considerably larger for $\Delta w < 0$, because in that case one has headwinds, as opposed to the tailwinds one has for $\Delta w > 0$. As before, m_F and DOC increase, and t_f decreases as the CI increases.

For tailwinds, the performance index DOC decreases as Δw increases; the larger the wind shear, the better. For headwinds, however, DOC increases as $|\Delta w|$ increases; the larger the wind shear (in modulus), the worse. As an example, for $CI = 1.5$ kg/s one has the following results: for tailwinds, when Δw increases from 0 to 40 kt, the decrease in DOC is 1933 kg; and for headwinds, when $|\Delta w|$ increases from 0 to 40 kt, the increase in DOC is 1773 kg.

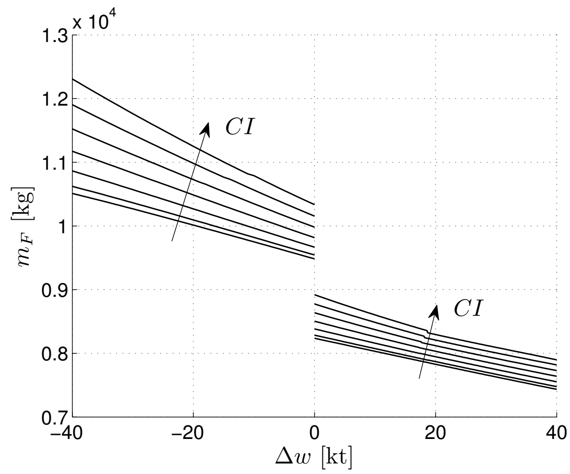

Figure 11. Fuel consumption vs wind shear for CI = 0,0.5,1,1.5,2,2.5,3 kg/s; $\overline{w} = -30$ kt for HW, $\overline{w} = 30$ kt for TW.

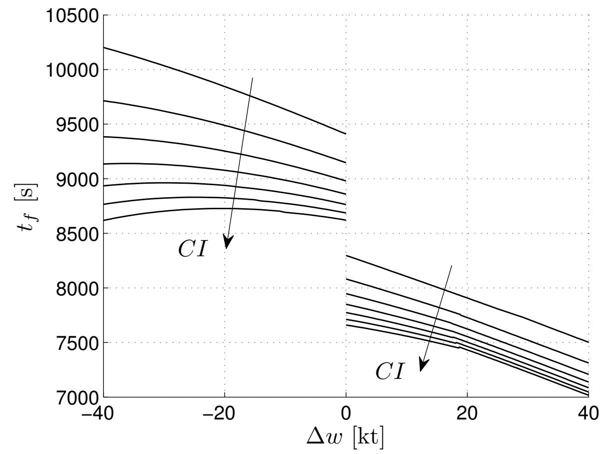

Figure 12. Flight time vs wind shear for CI = 0,0.5,1,1.5,2,2.5,3 kg/s; $\bar{w} = -30$ kt for HW, $\bar{w} = 30$ kt for TW.

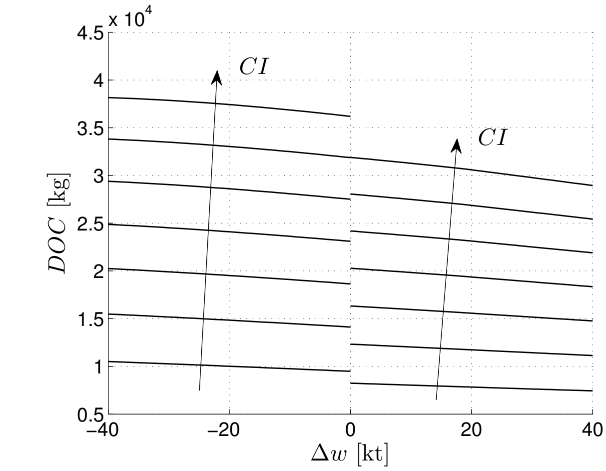

Figure 13. DOC vs wind shear for CI = 0,0.5,1,1.5,2,2.5,3 kg/s; $\bar{w} = -30$ kt for HW, $\bar{w} = 30$ kt for TW.

IV. CONCLUSIONS

An analysis of the effects of average wind and wind shear on optimal aircraft cruise trajectories has been presented, which is based on the use of a predefined trajectory pattern and parametric optimization. The trajectory pattern is formed by five segments commonly flown by airlines following ATC rules. The case of linear wind profiles has been considered.

The effect of the average wind on the optimal procedure (altitude and Mach number) has been shown to be small, and much larger its effect on the global properties (fuel consumption, flight time, and cost), as expected for the difference between tailwinds and headwinds.

The results have shown that the wind shear has a strong effect on the optimum cruise altitude. In particular, it has been found that, depending on the value of the wind shear, the optimal cruise can take place either in the troposphere or in the stratosphere; in general, for tailwinds, the larger the wind shear, the higher the optimal cruise, and, for headwinds, the larger the wind shear (in modulus), the lower the optimal cruise. It has also been shown that the behavior of the optimum altitude as a function of the cost index is opposite in the troposphere and in the stratosphere, namely, the optimum altitude decreases with the cost index in the troposphere, whereas it increases in the stratosphere. On the contrary, the effect of the wind shear on the optimum cruise Mach has been shown to be quite small.

Finally, the effect of the wind shear on the optimal performance has been also shown to be important: for tailwinds, the larger the wind shear, the better the performance; on the contrary, for headwinds, the larger the wind shear (in modulus), the worse the performance.

The analysis of cruise flight with larger range, with several cruise steps (stepped climb cruise), is left for future work. It is expected that for large, positive values of the wind shear (for tailwinds) the whole optimal cruise take place in the stratosphere; for lower values, one may have the optimal cruise starting in the troposphere and ending in the stratosphere; and for negative values (for headwinds) one may expect that the whole optimal cruise take place in the troposphere.

The analysis of other types of wind profiles and of cases that include wind variation along the trajectory are also left for future work.

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