Optimal Metering Policies for Optimized Profile Descent Operations at Airports

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***Abstract*— Optimized Profile Descent (OPD) is an arrival procedure for the Next Generation Air Transportation System (NextGen), which has been demonstrated to effectively decrease noise, emissions and fuel costs. Reference [1] identifies optimal sequencing and spacing policies for OPD operations under uncertainty and shows that the use of these policies at given metering points is expected to result in potential savings of 10-15% in fuel consumption over the current OPD practice. In this paper, we develop models to further increase such potential savings through optimal metering policies, which include identification of the optimal number and locations for such metering points. We present an algorithmic framework based on implementations of a stochastic dynamic program and a nonlinear stochastic integer program to identify best metering point configurations, and present some numerical results based on actual traffic information at a major U.S. airport.**

Keywords- runway operations; optimized profile descent; continuous descent arrival; stochastic dynamic programming

I. Introduction

Optimized profile descent (OPD), which is also referred to as the continuous descent arrival or continuous descent approach (CDA), is a distinct arrival procedure proposed for aircraft landings at airports, which has been demonstrated to effectively decrease noise, emissions and fuel costs. Different from the conventional stair-step procedure, OPD flights descend continuously from the top of descent (TOD) and attempt to reduce level stay, as shown in Fig. 1. The main advantage of OPD is that, compared to an aircraft that uses the conventional approach, an OPD flight will stay at a higher altitude for longer time which in turn will reduce noise, emissions and fuel burn. However, the management of OPD flights is more difficult for a controller due to the reduction in level segments - especially given the uncertainty in aircraft trajectories. Such management is performed through a set of metering points, where spacing between flights is adjusted as necessary to ensure safety and efficiency during the approach to the runway. Reference [1] has identified optimal spacing policies for OPD operations under uncertainty and shows that the use of these policies is expected to result in potential savings of 10-15% in fuel consumption over the current practice. On the other hand, it is clear that the number and location of metering points greatly affect the variance of the uncertainty in flight trajectories, and thus have a significant role in defining the realized maneuvering costs. To this end, in this paper we seek answers to the following research questions: what is the optimal number of OPD metering points, and what are their optimal locations such that all relevant costs are minimized, while maximizing runway utilization? We develop an algorithmic framework to answer these questions, in which multiple stochastic model implementations are utilized. Some preliminary results are also presented that quantify the potential savings that can be achieved through the proposed optimal metering policies.

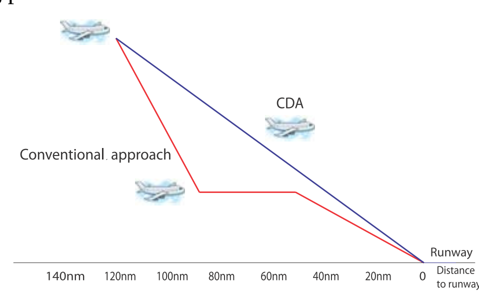

Figure 1. Comparison of OPD with the conventional stair-step approach

II. BACKGROUND AND GENERAL FRAMEWORK

A. Background

The existing analyses and literature on metering point locations are limited. Reference [2] points out that convective weather results in inaccuracy in traffic management tools, reducing the effectiveness of decisions at metering points. Thus, they integrate weather information within a traffic management advisor and provide some proactive suggestions on decisions at metering points under adverse weather. Similar to that paper, we aim at exploring decision support issues at metering points. We also emphasize that weather conditions might bring trajectory deviations and provide a stochastic formulation which captures the uncertainty brought by weather and pilot performance. The concept of using a set of metering points to monitor and adjust aircraft spacing during OPD, which also forms the basis for our framework, was first discussed by [3]. In that paper, the authors propose a cueing system where a sequence of altitude/speed checkpoints was added to recommend flap schedules to pilots. A simulator based experiment suggests significant benefits of having such metering points to OPD operations from the perspective of both controllers and pilots. While [3] attempts to find the number of metering points to use based on their survey data, they do not explore any optimization based approaches. Overall, to the best of our knowledge, our paper is the first that aims to identify the optimal locations of metering points. In addition, our analyses constitute one of the few stochastic optimization models for OPD procedures.

B. Decision Framework

In current practice, there exist several way points that can be used for guidance and direction purposes along the trajectory during an OPD procedure. Some of the way points are used as metering points, where the aircraft is controlled so that the runway spacing is ensured at desired levels or the lower end air traffic conflicts can be resolved. However, the locations of these way points are mostly based on expert opinions or general conventions, and do not result from optimization procedures. It is possible that fuel savings can be obtained by optimally selecting the number and locations of these control points. Moreover, given that the existing way point locations at airports are basically virtual locations in air, modification of these locations does not require a huge infrastructure effort or cost.

Hence, the general decision process that we consider in this paper can be described as follows: The decision maker, i.e. the air traffic control authority, initially decides on the number and locations of metering points for a given airport. This is a one-time decision and applies to all flights, although the locations can be adjusted during the day if practically feasible. From an implementation perspective, when an aircraft reaches a metering point, the distance from the aircraft it trails is observed, and any spacing adjustment decisions are made by the controller. The process can continue for each existing metering point until the flight lands at the runway. This framework can be represented through a multi-stage decision structure, where the number and location decisions are made first, followed by a series of spacing adjustment decisions at the selected metering point locations after observations on stochastic spacing realizations are made. However, the fact that the number of metering points is a decision by itself, and that it also determines the number of decision epochs, implies a complex endogenous structure which is difficult to model in a tractable way. Hence, as part of our analyses, we decouple the decisions for the number and locations of the metering points, and develop an algorithmic procedure that would determine the best configuration to use to minimize associated costs.

C. Algorithmic Setup

As part of the decision process, we assume that the arrival rate and distribution of aircraft types are known for a given airport. Similarly, the TOD location and thus its distance from the runway are also known. In addition, the information on trajectory uncertainty is assumed to be available in the form of a probability distribution as described by [1]. The costs of maneuvering during different phases of flight and utilization of runway are also assumed to be predefined as described in Section III. The overall goal is to find the number and corresponding locations of metering points so that the resulting fuel burn and runway utilization costs are minimized.

An overview of the algorithmic procedure we use to achieve this is shown in Fig. 2 through two distinct phases.

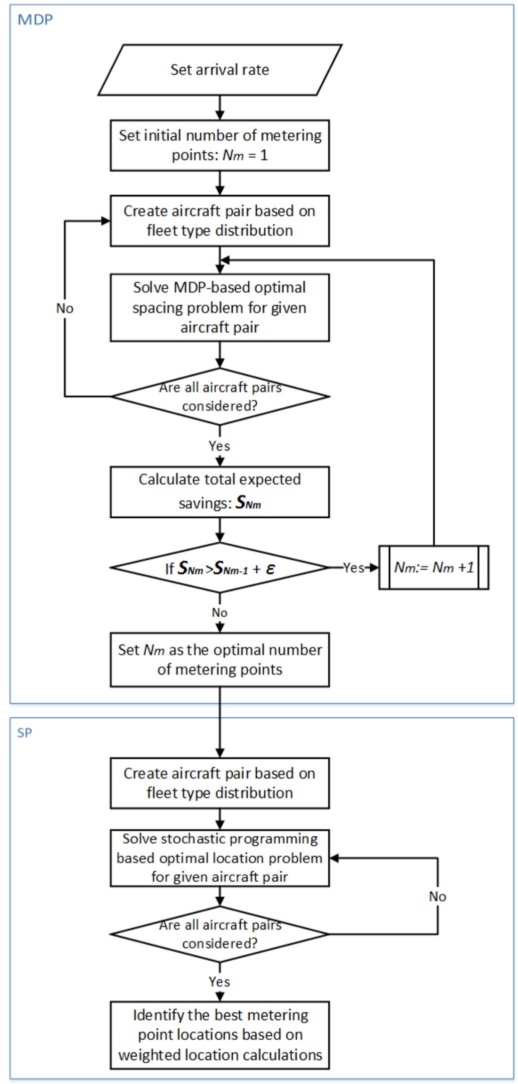

Figure 2. Algorithm to find ideal number and locations of metering points

In the first phase, we iteratively search for the estimated optimal number of metering points through a Markov decision process (MDP) model as described in [1]. In that paper, we develop a stochastic dynamic programming model to obtain optimal sequencing and spacing policies for arriving aircraft so that fuel burn, environmental and runway utilization costs are minimized. The corresponding optimal savings are also reported based on a fixed number of metering points. In this paper, we obtain the optimal savings iteratively for different numbers of metering points by initially assuming equal spacings in between. The optimal number of metering points is obtained when the marginal savings are sufficiently negligible as a larger number of points is considered. In each iteration of the first phase of the algorithm, we solve the MDP model for the given aircraft mix by considering all possible pairs of aircraft types, and obtain the expected savings corresponding to that number of metering points. We stop after identifying a sufficiently 'good' number of metering points, and use that as input for the second phase of the algorithm.

In the second phase, we use the given number of metering points and solve a multi-stage stochastic program (SP) to identify the optimal locations for these points with the objective of cost minimization. The key constraints in the model involve the dynamics of the spacing changes between adjacent metering points, which also involve stochastic parameters defining trajectory uncertainty. As part of the implementation, we again consider all aircraft pairs, and solve the SP model for each aircraft pair with random initial spacing values. After the optimal locations of metering points for each aircraft pair are obtained, the ideal locations are calculated using weights based on the probability of each aircraft pair. In the following sections, we describe the SP model in detail.

III. SP Model for Optimal Metering Location

For a given OPD implementation, suppose the distance between the TOD and the runway is denoted as L, while flights arrive at the airport following a Poisson distribution with rate λ. When an aircraft approaches the TOD, the initial spacing between that flight and the one preceding it is measured as s_0. Furthermore, we assume that there are N metering points located along the trajectory, where the first and last ones are the TOD and the runway, respectively. Each metering point is indexed as t, where $t = 1,2,\dots,N$. We further define the location of metering point *t* as y_t, where $y_1 = 0$ and $y_N = L$ by definition. The distance between adjacent metering points is defined as d_t, i.e. $d_t = y_{t+1} - y_t$. When a trailing aircraft reaches metering point t, the spacing from the leading aircraft is measured and denoted as s_t. Then, a target spacing change for the next metering point, Δ_t, is issued to the pilot. Due to safety concerns and technical limitations, there are lower and upper bounds for Δ_t, denoted as $\underline{\Delta}_t$ and $\overline{\Delta}_t$, respectively. The realized spacing at metering point *t+1* follows a normal distribution based on the spacing at the previous metering point and the issued spacing change command, i.e., $s_{t+1} \sim N(\mu_{t+1}, \sigma_{t+1})$, where $\mu_{t+1} = \Delta_t + p_t s_t + q_t d_t + r_t$ and $\sigma_{t+1} = \eta_t d_t + \zeta_t$. p_t, q_t, r_t, η_t and ζ_t are coefficients to express the mean and standard deviation of the realized spacing as described by [1]. Due to wake vortex effects, the air traffic control authority requires a minimal separation requirement at the runway for any given pair of aircraft, denoted as $\underline{s}_N$.

Based on this setup, the model utilizes three types of cost components as part of the objective function definition. These are fuel burn costs, costs for violation of minimum spacing requirements, and runway utilization costs. *Fuel burn costs* relate to the fuel consumption during the landing process, and are separately defined for the cruise and descent phases of a flight during the landing process [4]. Letting $z_t = d_t + \Delta_t$, the cruise stage fuel burn cost can be defined as:

$$f_{cr}(y_t, \Delta_t) = c_0(c_4 + c_2 y_t)^{4.26}(z_t + c_1 z_t^2 / d_t) + c_3 \frac{1}{(c_4+c_2 y_t)^{4.26} z_t^2}\left(\frac{d_t^4}{z_t} + c_1 d_t^3\right) \quad (1)$$

for $t = 1, \dots, N_c$, where N_c is the number of metering points in cruise stage, and c_i, *i=0,1,…,12* are constants defined by [4], some of which are utilized in the relationships below. The descent fuel burn cost, on the other hand, can be defined as $f_d(y_t, \Delta_t) = \min\{f_{nom}, f_{min}\}$, where for $t = N_c, \dots, N$:

$$f_{nom}(y_t, \Delta_t) = (c_9 + c_{10} y_t) d_t^2 / z_t \quad (2)$$

$$f_{min}(y_t, \Delta_t) = c_{11}\left(\frac{d_t^2}{z_t} + c_{12} d_t\right)(c_5 + c_6 y_t + c_7 y_t^2 + c_8 y_t^3) \quad (3)$$

Costs for violation of minimum spacing are used to ensure spacing change commands such that the risk of separation requirement violation is minimized. This cost, defined as $f_c(s_t)$, is evaluated based on the huge cost of aircraft colliding with each other and the probability of a collision given a spacing s_t as defined by [5]. The cost function is defined for all the metering points as:

$$f_c(s_t) = 950080 \exp[(-1.0412 s_t - 0.5806) log 10]. \quad (4)$$

Runway utilization costs are determined by the difference between final realized spacing and minimal separation at runway, and can be approximated in a linear fashion as $f_r(s_N) = 72.3(s_N - \underline{s}_N)$, as defined by [6].

A. Model Formulation

In this section we describe our multi-stage SP model where the locations of the metering points and spacing adjustment decisions are determined. The uncertainty in the framework is modeled through the variance of realized spacing. More specially, as mentioned in [1], the standard deviation of spacing is assumed to be a linear expression of the distance between metering points, i.e. $\eta_t d_t + \xi_t$, where d_t is the distance between adjacent metering points, while η_t and ξ_t are coefficients. Since d_t is known and constant, after the location decision is made, possible realizations of the spacing at the metering point *t* are dependent on the random values of η_t and ξ_t. Let Ψ be the set of vectors of all possible realizable values of η_t and ξ_t at different metering points. We refer to each possible realization $\psi \in \Psi$ as a scenario with a corresponding probability p_ψ. Overall, the model aims to minimize the sum of the fuel, safety and utilization costs while determining the metering location and allocation decisions. The fuel cost function depends on metering locations and spacing adjustments while the runway utilization costs are described through $f_r(s_N^\psi)$. Given this, we formulate our described model as follows. Function (5) refers to the objective function where the expectation of all costs is minimized. Constraints (6) define bounds for the spacing adjustments while constraints (7) are used to define the spacing between aircraft at each metering point. Constraints (8) describe the sequence of metering points, while constraints (9) represent nonanticipativity.

$$\min E\left[\sum_{t=1}^{N_c} f_{cr}(\Delta_t^\psi, y_t) + \sum_{t=N_c}^{N} f_d(\Delta_t^\psi, y_t) + \sum_{t=1}^{N} f_t(s_t^\psi) + f_r(s_N^\psi)\right] \quad (5)$$

$$\text{s.t} \quad \underline{\Delta}_t \le \Delta_t^\psi \le \overline{\Delta}_t \qquad \forall t \quad (6)$$

$$s_{t+1}^\psi = \Delta_t^\psi + p_t s_t^\psi + q_t d_t + r_t + q_t^\psi \eta_t d_t + \xi_t^\psi \forall t, \psi \quad (7)$$

$$y_t + d_t = y_{t+1} \qquad \forall t, t \ne N \quad (8)$$

$$\Delta_t^\psi = \Delta_t^{\psi'}, s_t^\psi = s_t^{\psi'} \qquad \forall t, \quad \psi \quad \psi', \mathrm{R}_{\psi\psi'}^t = 1 \quad (9)$$

B. Convex Representation of the Model

For the above stochastic formulation, we note that all the constraints are linear. Thus, the objective function (5) determines the convexity of the problem. The fuel burn cost functions f_{cr}, f_{nom} and f_{min} are nonconvex due to inclusion of products of multiple variables, and thus the optimization problem is nonconvex. Our approach to deal with this issue involves transforming these expressions through bilinear terms, which are then approximated through piecewise linear terms.

For the cruise stage fuel cost functions (1), we define $P_t = (c_4 + c_2 y_t)^{4.26}$, $Q_t = z_t + c_1 z_t^2 / d_t$, $R_t = \frac{1}{(c_4+c_2 y_t)^{4.26} z_t^2}$ and $V_t = (\frac{d_t^4}{z_t} + c_1 d_t^3)$. We can show P_t, Q_t, R_t and V_t are convex terms. Thus, the cruise fuel burn cost function can be written with two bilinear terms as $f_{cr} = c_0 P_t Q_t + c_3 R_t V_t$. For the descent stage fuel cost functions (2), similarly, we define $X_t = d_2/z_t + c_{12} d_t$, $W_t = c_5 + c_6 y_t + c_7 y_t^2 + c_8 y_t^3$, $F_t = c_9 + c_{10} y_t$ and $G_t = d_t^2/z_t$. Thus, the descent stage fuel burn cost functions can be written as $f_d = \min\{F_t G_t, c_{11} X_t W_t\}$.

We now show using the terms $P_t^\psi Q_t^\psi$ how the bilinear terms are approximated by piecewise linear functions. We approximate the bilinear terms over a two dimensional grid where the axes are over P_t and Q_t. We discretize P_t and Q_t into Nx and Ny intervals respectively to form the grid. Furthermore, we introduce the auxiliary variables $\lambda_1^{\{ix,iy,t,\psi\}}, ix = 1, \ldots, N_x, iy = 1, \ldots, N_y$ and two SOS2 variables $\alpha_1^{\{ix,t,\psi\}}$ and $\beta_1^{\{ix,iy,t,\psi\}}$. $P_{\min}$, $P_{\max}$, Q_{min} and Q_{max} are the lower and upper bounds for P_t and Q_t^ψ, respectively. Thus, we can approximate the bilinear terms with the following set of constraints, which we denote as PQ^ψ.

$$\sum_{ix,iy} \lambda_1^{\{ix,iy,t,\psi\}} = 1 \qquad \forall t,\psi \tag{10}$$

$$P_t = \sum_{ix,iy} (P_{min} + (P_{max} - P_{min})(ix-1)/Nx) \lambda_1^{\{ix,iy,t,\psi\}} \forall t,\psi \tag{11}$$

$$Q_t^\psi = \sum_{ix,iy} (Q_{min} + (Q_{max} - Q_{min})(iy-1)/Ny) \lambda_1^{\{ix,iy,t,\psi\}} \forall t,\psi \tag{12}$$

$$P_t Q_t^\psi = \sum_{ix,iy} [P_{min} + (P_{max} - P_{min})(ix-1)/Nx] \cdot [Q_{min} + (Q_{max} - Q_{min})(iy-1)/Ny] \lambda_1^{\{ix,iy,t,\psi\}} \forall t,\psi \tag{13}$$

$$\alpha_1^{\{t,ix,\psi\}} = \sum_{iy} \lambda_1^{\{ix,iy,t,\psi\}}, \beta_1^{\{t,iy,\psi\}} = \sum_{ix} \lambda_1^{\{ix,iy,t,\psi\}} \qquad \forall t, ix, iy, \psi \tag{14}$$

Similarly, we can approximate the other bilinear terms, $R_t^\psi V_t^\psi$, $F_t^\psi G_t^\psi$ and $X_t^\psi W_t^\psi$ with these piecewise linear expressions. We thus obtain similar sets of constraints, which are defined as RV^ψ , FG^ψ and XW^ψ, where $\psi \in \Psi$.

IV. Numerical Results and Practical Implications

Hartsfield-Jackson Atlanta International Airport (ATL) is selected as a representative major airport, and the OPD implementations at ATL, as described in [7], are utilized for the simulation setup. For analysis purposes, three different arrival rates, namely 20, 30 and 40 flights/hr are considered. For each arrival rate, the flight arrival times are simulated in a one-hour interval. Ten major types of aircraft are used, where their statistical distribution is obtained from historical data. First come first serve policy is used as the sequencing rule.

As described in Section II and Fig. 2, given a fixed arrival rate, we increase the number of metering points and solve the corresponding MDP model until the marginal savings are sufficiently small, e.g. less than 1% in our setup.

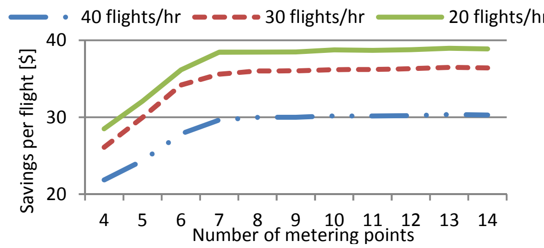

Figure 3. Savings under different arrival rates using MDP implementations

Table I. Ideal locations of 8 metering points (in units of nm from TOD)

	1	2	3	4	5	6	7	8
y_t	10.2	18.1	28.6	40.3	49.7	87.8	127.4	150

The cost savings per flight for each arrival rate under different numbers of metering points are shown in Fig. 3. The results indicate that when the arrival rates are 30 and 40 flights/hr, the optimal number is 8, while 7 metering points are sufficient to achieve the maximum savings for the case of 20 flights/hr. For the overall setup, we assume that 8 metering points represent the ideal configuration, and implement the SP model accordingly. The optimal locations generated are shown in Table I. As we can see, the first five metering points are more closely distributed and the distances between them are around 10 nm. The remaining ones have larger distances from each other. This implies that higher levels of traffic control are more beneficial at higher flight levels. This configuration results in an increased savings of around $23/flight, when compared with the current configuration. These savings imply a potential value of $3.8 million/year at ATL.

V. Conclusions

In this paper, we develop models improving the efficiency of OPD procedures through optimal metering policies, which include identification of the optimal number and locations for such metering points. Initial numerical analyses indicate that use of such optimal configurations would result in important savings for airlines. As future research, the sensitivity of these saving values over different airport setups can be analyzed, and both general and specific insights for airports can be derived.

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