Optimal Metering Policies for Optimized Profile Descent Operations at Airports

Heng Chen and Senay Solak
Isenberg School of Management
University of Massachusetts Amherst

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Outline

- Motivation
- Problem Framework and Algorithmic Design
- Stochastic Programming Model
- Convexification and Lagrangian Decomposition
- Numerical Implementation and Conclusion
Motivation: Optimized Profile Descent

- Capacity limits in airport; significance of fuel and environmental costs
- Optimized Profile Descent (OPD) helps improve efficiency in these areas
Motivation: Optimized Profile Descent

- High trajectory flight results in decreased noise levels
- Reduced thrust (near idle thrust) during descent results in less fuel burn cost and emissions
- Flight tests suggest 30% reduction in noise and emissions; 25-50 gallons reduction in fuel consumption
Motivation: Current Practice

Required separation at metering fixes achieved by:

Conventional approach:
1. Vectoring, holding
2. Speed control

OPD:
1. Speed control

The locations of these metering points are mostly based on expert opinions or general conventions.
Motivation: Proposed OPD Policies

- OPD capability added to around 30 airports in U.S., 50 airports in Europe

- Many airlines are using or collaborating in development of OPD procedures

- Chen and Solak (2014) provided a stochastic dynamic framework to identify spacing and sequencing rules for OPD
Motivation: Proposed OPD Policies

If $s_t$ is the observed spacing at metering point $t$ between two flights, an optimal target spacing change $\Delta_t$ at metering $t$ for the trailing aircraft is given as $\Delta_t = m_t s_t + n_t$, with parameters precalculated, e.g.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Fuel</th>
<th>Sustainability</th>
<th>Total</th>
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<tbody>
<tr>
<td></td>
<td>$m_t^F$</td>
<td>$n_t^F$</td>
<td>$m_t^S$</td>
</tr>
<tr>
<td>1</td>
<td>-0.64</td>
<td>4.11</td>
<td>-0.70</td>
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<tr>
<td>2</td>
<td>-0.37</td>
<td>2.75</td>
<td>-0.47</td>
</tr>
<tr>
<td>3</td>
<td>-0.33</td>
<td>1.50</td>
<td>-0.60</td>
</tr>
<tr>
<td>4</td>
<td>-0.08</td>
<td>1.68</td>
<td>-0.35</td>
</tr>
<tr>
<td>5</td>
<td>-0.04</td>
<td>1.71</td>
<td>-0.31</td>
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</table>

B738 trailing A320
Motivation: Proposed OPD Policies

- If OPD is fully implemented (at the same rate as LAX)
  - Potential annual environmental savings: $5 million
  - Potential annual fuel burn savings: $24 million

<table>
<thead>
<tr>
<th>Airport Code</th>
<th>Location</th>
<th>Estimated Daily OPD Flights</th>
<th>Annual Environmental Savings($)</th>
<th>Annual Fuel Burn Savings($)</th>
<th>Annual Total Saving($)</th>
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<tbody>
<tr>
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<td>753,912</td>
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<td>PHL</td>
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<td>PHX</td>
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<td>229</td>
<td>376,951</td>
<td>1,826,117</td>
<td>2,203,068</td>
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<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>3,022</strong></td>
<td><strong>$4,964,359</strong></td>
<td><strong>$24,049,561</strong></td>
<td><strong>$29,013,919</strong></td>
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Problem Framework: Research Questions

- Currently no specific method being used to determine number and locations of metering points for OPD.
- Cost structure and trajectory variance are functions of distances between metering points.
- Hence: Are there values in optimizing metering point locations?
- **What are the optimal number and locations of metering points?**
The number and locations of metering points are determined, which will apply to all arriving aircraft.

Stochastic decision problem due to random trajectory deviations
Problem Framework

- A multi-stage decision structure:
  - First, the number and location decisions are made.
  - Then, spacing adjustment at the selected metering point locations based on observed spacings are made.

- Objective: Minimize expected costs associated with maneuvering and runway utilization

- Challenge: The ideal location for each type of aircraft will be different; need to account for all types of aircraft in identified solutions
Problem Framework

- **Input**
  - Arrival rate
  - Flight mixes
  - Location of top of descent
  - Distribution of trajectory deviation
  - Cost structure: fuel burn, runway utilization, cost of violation of minimal spacing required

- **Output:**
  - Strategic: number and locations of metering points
  - Tactical: the spacing adjustments for each arriving aircraft
Problem Framework

- **Multi phase algorithmic framework**
  - Sequential use of Markov Decision Processes (MDP) and Stochastic Programming (SP)
  - Phase I: Find the ideal number of metering points
  - Phase II: Based on Phase I solutions, identify the optimal locations

- **Endogenous structure:**
  - The number of metering points determines number of epochs for spacing change in the model.
  - The locations determine the dynamics of trajectory deviation.
Algorithmic Design: Phase I - Optimal Number of Metering Points

**Idea:** Iteratively search for the estimated optimal number of metering points using MDP model of Chen and Solak (2014)

**Assumption:** Equal spacings in between

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Based on the spreadsheet tools, savings for each pair of aircraft are obtained.

If the addition of one more metering fix does not add value, stop.
When the number of metering points is known, a multi-stage stochastic programming model can be built.
Stochastic Programming Model

- With the number of metering points fixed, the location problem is a multistage decision model.

- The decision timeline for the SP

  \( \psi \in \Psi \), with probability \( p_\psi \)
SP: Transition Dynamics

- Transition dynamics $P(s_{t+1}|s_t, \Delta_t)$ modeling deviation in trajectories
- Calculation motivated by the analysis of Ren (2007)
  - Follows normal distribution
  - Mean and variance are functions of current spacing, target spacing and distance between metering points
  - Defined as
    - $N(\mu_{t+1}, \sigma_{t+1})$ where
    - $\mu_{t+1} = \Delta_t + s_t + g_t (s_t, d_t)$
      - $\Delta_t + p_t s_t + q_t d_t + r_t$;
    - $\sigma_{t+1} = h_t(d_t) = \eta_t d_t + \xi_t$;
    - $d_t$ is the distance between metering points $t$ and $t+1$

Ref: (Ren 2007)
SP: Cost Structure

- Fuel burn cost
  - Cost of maneuvering to achieve target spacing change $\Delta t$ at next metering point for current spacing
  - Different parameters for each aircraft type and flight level

- Two different flight phases (BADA)
  - Cruise fuel burn cost
    \[
    f_{cr}(y_t, \Delta t) = c_0(c_4 + c_2 y_t)^{4.26}(z_t + c_1 z_t^2 / d_t) + c_3 \frac{1}{(c_4 + c_2 y_t)^{4.26} z_t^2}(d_t^4 / z_t + c_1 d_t^3)
    \]
  - Descent fuel burn cost
    \[
    f_d(y_t, \Delta t) = \max\{f_{nom}, f_{min}\},
    f_{nom}(y_t, \Delta t) = (c_9 + c_{10} y_t)d_t^2 / z_t
    f_{min}(y_t, \Delta t) = c_{11}(d_t^2 / z_t + c_{12} d_t)(c_5 + c_6 y_t + c_7 y_t^2 + c_8 y_t^3)
    \]

Where $z_t = d_t + \Delta t$, and $y_t$ is the location of metering point $t$. 
SP: Cost Structure

- Costs for violation of minimum spacing
  - Evaluated based on the probability of a collision (Blom et al. 2011)

\[ f_c(s_t) = 950080 \exp\left[(-1.0412s_t - 0.5806)\log_{10}\right]. \]

- Final spacing costs based on utilization of runway and determined according to differences from minimum required spacing levels at runway
  - As calculated by Solveling et al. (2010)

\[ f_r(s_N) = 72.3(s_N - s_N) \]
SP Formulation

\[
\begin{align*}
\min E\left[\sum_{t=1}^{N_c} f_{cr}(\Delta_t, y_t) + \sum_{t=N_c}^{N} f_d(\Delta_t, y_t) + \sum_{t=1}^{N} f_c(s_t) + f_r(s_N)\right] \\
\text{s.t} \quad & \Delta_t \leq \Delta_t' \leq \Delta_t \quad \forall t \\
& s_{t+1}^\psi = \Delta_t^\psi + p_t s_t^\psi + q_t d_t + r_t + q_t^\psi \eta_t d_t + \xi_t \psi' \forall t, \psi \\
& y_t + d_t = y_{t+1} \quad \forall t, t \neq N \\
& \Delta_t' = \Delta_t, s_t^\psi = s_t' \quad \forall t, \psi, \psi', R_{\psi,\psi'}^t = 1
\end{align*}
\]

- Nonlinear Nonconvex Multistage Stochastic Program
  - The fuel burn cost functions $f_{cr}, f_{nom}$ and $f_{min}$ are nonconvex

- Constraint (7) represents the dynamics due to spacing change at each metering point.
SP: Convex Representation

- Objectives can be written using several bilinear terms

- Let $P_t = (c_4 + c_2 y_t)^{4.26}$, $Q_t = z_t + c_1 z_t^2 / d_t$, $R_t = \frac{1}{(c_4 + c_2 y_t)^{4.26} z_t^2}$ and $V_t = (\frac{d_t^4}{z_t} + c_1 d_t^3)$, then, cruise stage cost can be written as: $f_{cr} = c_0 P_t Q_t + c_3 R_t V_t$
  - $P_t, Q_t, R_t, V_t$ are convex functions

- Let $X_t = d_2/z_t + c_{12} d_t$, $W_t = c_5 + c_6 y_t + c_7 y_t^2 + c_8 y_t^3$, $F_t = c_9 + c_{10} y_t$ and $G_t = d_t^2/z_t$. Thus, the descent stage fuel burn cost functions can be written as: $f_d = \max\{F_t G_t, c_{11} X_t W_t\}$
SP: Convex Representation

- Approximation of bilinear terms by piecewise linearization; e.g. for $P_t Q_t^\psi$

\[
\sum_{ix, iy} \lambda_1^{(ix, iy, t, \psi)} = 1 \quad \forall t, \psi
\]

\[
P_t = \sum_{ix, iy} (P_{min} + (P_{max} - P_{min})(ix - 1)/Nx) \lambda_1^{(ix, iy, t, \psi)} \quad \forall t, \psi
\]

\[
Q_t^\psi = \sum_{ix, iy} (Q_{min} + (Q_{max} - Q_{min})(iy - 1)/Ny) \lambda_1^{(ix, iy, t, \psi)} \quad \forall t, \psi
\]

\[
P_t Q_t^\psi = \sum_{ix, iy} [P_{min} + (P_{max} - P_{min})(ix - 1)/Nx]
\]

\[
\cdot [Q_{min} + (Q_{max} - Q_{min})(iy - 1)/Ny] \lambda_1^{(ix, iy, t, \psi)} \quad \forall t, \psi
\]

\[
\alpha_1^{(t, ix, iy, \psi)} = \sum_{iy} \lambda_1^{(ix, iy, t, \psi)} , \quad \beta_1^{(t, ix, iy, \psi)} = \sum_{ix} \lambda_1^{(ix, iy, t, \psi)} \quad \forall t, ix, iy, \psi
\]

- Two dimensional grid where the axes are over $P_t$ and $Q_t$

- Other bilinear terms are similarly approximated.
SP: Lagrangian Decomposition

- Difficult to solve directly when the number of metering points is greater than five. \((2^5 \text{ scenarios}, 100 \text{ pairs})\)

- A Lagrangian function is generated by adding a set of constraints to the objective function.

\[
L(d, z, \Delta, \lambda) = g_N(\hat{d}_t, z_t) + \sum_t \sum_{\psi, \psi' | \mathbb{R}^t_{\psi \psi'} = 1} \lambda_{t}^{\psi \psi'} (\Delta_t^{\psi} - \Delta_t^{\psi'})
\]

- This allows for decomposition of the original problem into sub-problems for each scenario

\[
L(d, z, \Delta, \lambda) = \sum_{\psi} L_{\psi}(d, z, \Delta, \lambda)
\]
SP: Lagrangian Decomposition

Algorithm:

- **Step 1:** For $\psi = 1, \ldots, |\Psi|$ : Solve the decomposed minimization problem for each scenario. Let $V_\psi$ and $\Delta_\psi$ be the objective value and solution for each sub-problem, respectively.
- **Step 2:** Let $V_L = \sum_{\psi} V_\psi$, which is the optimal solution for the Lagrangian dual.
- **Step 3:** Let $\psi_{fx} = \arg\min_{\psi} \{V_\psi\}$. If there are multiple, select the one with the smallest index. Let $\Delta_{fx} = \Delta_{\psi_{fx}}$, which corresponds to the scenarios that yield the smallest objective value.
- **Step 4:** Let $\Delta_\psi = \Delta_{fx}$; compute the corresponding objective values $V_{U\psi}$ for each sub-problem. Let $V_U = \sum_{\psi} V_{U\psi}$.
- **Step 5:** If $\frac{|V_U - V_L|}{V_L} \leq \epsilon$, stop; else, update the multiplier in the Lagrangian function and go to Step 1.
Implementation: Case Study

- Hartsfield-Jackson Atlanta International Airport (ATL) is selected as a representative major airport

- The distance from TOD to runway is assumed to be 150 nm

- Three different arrival rates, namely 20, 30 and 40 flights/hr are considered.

- Flight mixes are generated based on historical statistics of U.S. flights in 2012.
Phase I: Increase the number until the marginal savings are sufficiently small (<1%).

Optimal number of metering points: 8
- 20 flights/hr: 7 metering points
- 30 and 40 flights/hr: 8 metering points
Implementation: Case Study

- **Phase II**: Given 8 metering points, 256 (2^8) scenarios

<table>
<thead>
<tr>
<th>y_t</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.2</td>
<td>18.1</td>
<td>28.6</td>
<td>40.3</td>
<td>49.7</td>
<td>87.8</td>
<td>127.4</td>
<td>150</td>
</tr>
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</table>

- The first 5 metering points are more closely distributed; the remaining 3 have larger distances.

- Higher altitude traffic control is more beneficial to OPD.

- This configuration results in an increased savings of around $23/flight, when compared with the current configuration. These savings imply a potential value of $3.8 million/year at ATL, given the assumption that adding a metering point incurs no direct costs.
Conclusions and Future Work

- Identified the optimal number and locations for metering points under an OPD setting

- Adds to the values of OPD implementation; can be extended to other metering procedures other than OPD

- Sensitivity of these saving values over different airport setups are being analyzed

- Both general and specific insights for airports can be derived
Thank you…

Heng Chen
heng@som.umass.edu