Optimization of Arrival and Departure Routes in Terminal Maneuvering Area

Jun ZHOU
ENAC, MAIAA, F-31055
Univ. de Toulouse, IMT, F-31400
Toulouse, France
junzhou@recherche.enac.fr

Sonia Cafieri, Daniel Delahaye, Mohammed Sbihi
ENAC, MAIAA, F-31055
Univ. de Toulouse, IMT, F-31400
Toulouse, France
sonia.cafieri@enac.fr
daniel.delahaye@enac.fr
mohammed.sbihi@enac.fr

Abstract—**Airport is both the starting and ending point of air traffic. The sharp increase in air traffic flow causes directly traffic congestion in Terminal Maneuvering Area (TMA) which affects the normal operation of the flights. Optimizing departure and arrival procedures is therefore crucial to regulate air traffic flow. This research focuses on generating 3D Standard Instrument Departure routes and Standard Terminal Arrival Routes in TMA at a strategic level. We propose an optimization approach to generate 3D routes avoiding obstacles and assuring a minimum separation between routes. The method combines Fast Marching Method and Simulated Annealing.**

Keywords-TMA; SID; STAR; Optimization; Fast Marching Method; Simulated Annealing

I. INTRODUCTION

Nowadays air transport in Europe and North America has basically reached saturation. The sharp increase in air traffic flow causes directly traffic congestion in Terminal Maneuvering Area (TMA), which affects the normal operation of the flights. TMA is designed to handle aircraft arriving to and departing from airports and perhaps is one of the most complex types of airspace.

Currently, the Standard Instrument Departure (SID) and Standard Terminal Arrival Route (STAR) are designed manually based on the airport layout, existing Navaid infrastructures and nearby constraints [1]. Therefore automatically designing departure and arrival routes at a strategic level in 3D is interesting as it may help to regulate air traffic flow.

Planning optimal aircraft routes is a rich and dynamic research area in Air Traffic Management (ATM). Routes design in TMA is however a specific problem for which to our knowledge there is not a rich literature. In [1] and [2], two problems closed to the one considered in the present paper are studied. In [1], for a given weather forecast, the author develops an integer programming approach to optimally choose terminal area arrival and departure fixes as well as sector boundaries. In [2], an optimization algorithm is proposed for aircraft routings that minimizes the noise impact in the residential communities surrounding the airport. The aim of this study is to give an optimization approach to automatically design 3D departure and arrival routes at a

strategic level taking into account some operational constraints.

This paper is organized as follows: Section II describes the problem and introduces the TMA model; Section III presents the proposed approach to solve the problem. Section IV gives some preliminary simulation results. Finally, Section V gives conclusions and future directions.

II. PROBLEM DESCRIPTION AND MODELIZATION

A. Problem Description

TMA is a designated area of controlled airspace surrounding one or several airports. It is designed to handle aircraft arriving to and departing from airports. It is one of the most complex types of airspace.

The SID is a flight route followed by aircraft after takeoff from an airport. The STAR is a route which connects the last enroute way-point to the Initial Approach Fix. These routes are specified by a sequence of waypoints. Each aircraft flying under Instrument Flight Rules (IFR) through the TMA must follow a SID when departing an airport, and a STAR when arriving. The design of SIDs and STARs has to take into account operational and environmental constraints, such as vertical and horizontal flow separation, noise restrictions, etc.

In the following, we propose a method to design automatically the SIDs and STARs for a given TMA configuration, characterized by a number of entry/exit points at the boundary of TMA, arrival/departure points around the runways, forbidden areas and some operational constraints.

B. Modelization

In this study, we only consider TMA surrounding one airport. The number and configuration of runways are known. We suppose that the runways have the sufficient distances and equipments for landing and taking off for all types of aircraft. Let N be the total amount of flights arriving at and departing from the airport. As TMA is generally designed in a circular configuration centered on the geographic coordinates of the airport, we assume it is composed of two concentric circles C_1 and C_2, with radius R_1 and R_2 respectively. Aircraft enter into or exit from TMA on several points located on C_1. More precisely, let $\mathbb{O} = \{O_1, \ldots, O_{n_{in}}, O_{n_{in}+1}, \ldots, O_{n_{in}+n_{out}}\}$ be the

set of entry and exit points, where the first n_{in} points are the entry points and the remaining n_{out} ones are exit points.

Similarly, aircraft arrive to or departure from airport through several points located on C_2. Suppose that $\mathbb{I} = \{I_1, \dots, I_{n_{arr}}, I_{n_{arr}+1}, \dots, I_{n_{arr}+n_{dep}}\}$ is the set of arrival and departure points, where the first n_{arr} points are the arrival points and the remaining n_{dep} ones are departure points. An example of TMA is shown in Fig. 1 with $\mathbb{O} = \{O_1, O_2, O_3, O_4\}$ and $\mathbb{I} = \{I_1, I_2\}$.

The routes that we want to design connect some points on C_1 to some other points on C_2. Let $\mathbb{K} \subset \mathbb{O} \times \mathbb{I}$ be the subset which contains the pairs of points to be connected on C_1 and C_2. Furthermore, we suppose that the proportion of flights on each route is given.

The design of SIDs and STARs can be done in an *optimal* way, with respect to a given criterion, such as minimizing the total distance flown by all flights. Therefore, the problem can be expressed as an optimization problem. In the following, we give the main elements of our optimization model.

Given $(i, j) \in \mathbb{K}$, a route connecting points O_i and I_j can be defined as a function:

$$\gamma_{ij}: [0; 1] \rightarrow \mathbb{R}^3 \qquad (1)$$

where $\gamma_{ij}(0)$ represents the starting point and $\gamma_{ij}(1)$ is the ending point. There are two possible cases: route γ_{ij} either starts from an entry point on C_1 and ends at an arrival point on C_2; or it starts from a departing point on C_2 and ends at an exit point on C_1. This can be expressed by the following equations:

$$\left.\begin{array}{l} \gamma_{ij}(0) = O_i \\ \gamma_{ij}(1) = I_j \end{array}\right\} \text{ if } 1 \leq i \leq n_{in} \text{ and } 1 \leq j \leq n_{arr} \quad (2)$$

$$\left.\begin{array}{l} \gamma_{ij}(0) = I_i \\ \gamma_{ij}(1) = O_j \end{array}\right\} \text{ if } n_{arr} + 1 \leq i \leq n_{arr} + n_{dep}$$

$$\text{and } n_{in} + 1 \leq j \leq n_{in} + n_{out}. \quad (3)$$

We denote the components of γ_{ij} in axis (x, y, z) by $(\gamma_{ijx}, \gamma_{ijy}, \gamma_{ijz})$ respectively.

In this study, we consider two main constraints: the *forbidden areas* and the *minimum separation*. In TMA some special areas exist, where an aircraft is not allowed to fly, for instance geographical obstacles, big cities, military areas, etc. We refer to these areas as "forbidden areas". Furthermore, in air traffic control, aircraft have to be maintained at a minimum separation distance to reduce the risk of collision. The minimum vertical separation is $1000\,ft$ in TMA for all aircraft. Generally, the minimum horizontal separation in TMA is equal to $3NM$. We consider this separation equal to $6NM$, in order to take into account the deviation of aircraft from the pre-designed route. The aircraft protection zone is shown in Fig. 2.

The separation constraints are expressed as follows, $\forall (i, j), (k, l) \in \mathbb{K}, \; \forall (\mu_1, \mu_2) \in [0; 1]$
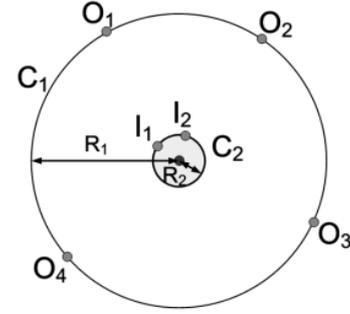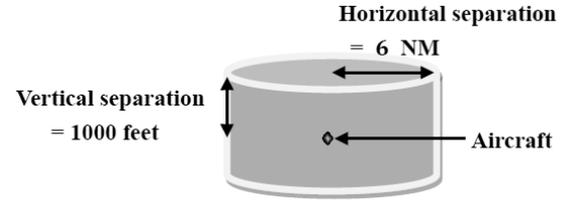

Figure 1. Example of TMA

Figure 2: Aircraft protection zone cylinder in 3D

$$\sqrt{\left(\gamma_{ijx}(\mu_1) - \gamma_{klx}(\mu_2)\right)^2 + \left(\gamma_{ijy}(\mu_1) - \gamma_{kly}(\mu_2)\right)^2} \geq \delta\,NM$$

$$(4)$$

$$|\gamma_{ijz}(\mu_1) - \gamma_{klz}(\mu_2)| \geq 1000\,ft. \qquad (5)$$

We minimize the total distance flown by all the flights during a certain period.

$$L = \sum_{(i,j)\in\mathbb{K}} w_{ij}\,N\,l_{ij} \qquad (6)$$

where l_{ij} is the length of route γ_{ij} and w_{ij} is the proportion of flights on route γ_{ij}.

III. SOLUTION APPROACHES

To deal with the difficulty of the problem, we propose a three-steps solution approach.

a. Compute an individual route by Fast Marching Method (FMM) and Gradient Descent method, where we take into consideration the forbidden areas.

b. Given a fixed order of the routes, compute sequentially the routes taking into account the minimum separation constraints.

c. Find an order minimizing (6) by applying Simulated Annealing (SA).

A. Designing one route

Given $(i, j) \in \mathbb{K}$, we first compute a route yielding the minimal travel time from $\gamma_{ij}(0)$ to $\gamma_{ij}(1)$, see (2) and (3). The minimal-time optimal trajectory problem corresponds to a wave front propagation problem [3], [4]. In the isotropic case where the wave propagation speed does not depend on the direction, the equation that describes the evolution of the front is in the following form, known as the Eikonal equation:

$$\|\nabla u(x)\|F(x) = 1 \qquad (7)$$

supplemented by the boundary condition

$$u\left(\gamma_{ij}(0)\right) = 0 \qquad (8)$$

where x is the front position (corresponding to the aircraft position); $u(x)$ represents the time at which the front reaches the point x and $F(x)$ is the speed at the point x. In order to take into account the constraints, we associate a different propagation speed to points belonging to forbidden areas and to points located elsewhere, by expressing the speed at point x as follows:

$$F(x) = (1 - \alpha(x)) \cdot F \qquad (9)$$

with $\alpha(x) \in [0; 1)$ and F a constant value. In strictly forbidden area we consider $\alpha(x) = 0.99$ and elsewhere $\alpha(x) = 0$.

To solve the wave front propagation problem in the isotropic case, we apply the FMM developed by Sethian in [5]. The general idea of FMM is that the front propagates towards the points that it reaches at a minimum time. At the end of the evolution, we get the minimum time to reach any point in space starting from $\gamma_{ij}(0)$. Then the Gradient Descent method is used to generate the route. It starts from the ending point $\gamma_{ij}(1)$ and moves towards the starting point $\gamma_{ij}(0)$ by taking steps proportional to the opposite of the gradient of $u(x)$ at the current point x.

B. Generating all routes

Given an order of the routes, we compute sequentially routes as explained in the first step. In order to satisfy the minimum separation constraints, once a route is computed, this route and its protection zone are considered as additional forbidden area constraints for the remaining routes.

C. Getting an optimal order

By changing the order of routes, the length of each route is modified, because the routes computed previously have an influence on the shapes of the following routes. The number of possible choices for route order is $k!$, where $k = card(\mathbb{K})$, that can be rapidly increase with k. To get an optimal order, we apply a Simulated Annealing that is a stochastic global optimization method. It is conveniently applied to large scale problems.

Simulated Annealing works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is "frozen", a minimum energy configuration is obtained. For our application, the state space is the permutation group of the k routes and a solution is an element of this group, representing a selected order of routes. The algorithm starts with a random solution, then at each iteration a new solution is generated by selecting randomly two positions in the previous solution and inversing the order of the sub sequencing between them.

IV. SIMULATION RESULTS

In this section we present the results obtained at each step *A, B, C* in Section III.

We work with an airport with two parallel runways A and B. Their orientations are $(12L, 30R)$ and $(12R, 30L)$ respectively. The orientation $12L$ of runway A and the orientation $12R$ of runway B are only used for takeoff; the other two sides are only used for landing. The coordinate of the center O of the two circles C_1 and C_2 is $(100\,km, 100km, 0\,ft)$. Fig. 3 shows this configuration.

Moreover, $R_1 = 100\,km$ and $R_2 = 10\,km$; Entry/exit points are $\{O_1, O_2, O_3, O_4\}$ and $\{O_5, O_6, O_7, O_8\}$ respectively. Arrival/departure points are $\{I_1, I_2\}$ and $\{I_3, I_4\}$ respectively. The z-coordinates of points $O_i, i \in \{1, \dots, 8\}$ and $I_i, i \in \{1, \dots, 4\}$ are $25000ft$ and $4000ft$ respectively. Their *x,y*-coordinates are presented in Table.1. The number of flight is $N = 20000$; The pairs to be connected are $\mathbb{K} = \{(O_1, I_1), (O_2, I_1), (O_3, I_2), (O_4, I_2), (I_3, O_5), (I_3, O_8), (I_4, O_7), (I_4, O_6)\}$. The proportion of flight on each route is $5\%, 20\%, 15\%, 10\%, 5\%, 15\%, 25\%$ and 5% respectively.

A. Designing one route

In this step, we firstly generate a route connecting the pair (O_1, I_1). Fig. 4 presents the simulation results. Three obstacles are taken into consideration. The area located in the center of an obstacle in dark gray is the surrounding area that can be flown over considering a penalty. The simulation result shows that the generated route is smooth and avoids the forbidden area.

B. Generating all routes

The simulation result is shown in Fig. 5 where the black routes are the STARs and the gray ones are the SIDs. The obstacles are unchanged.
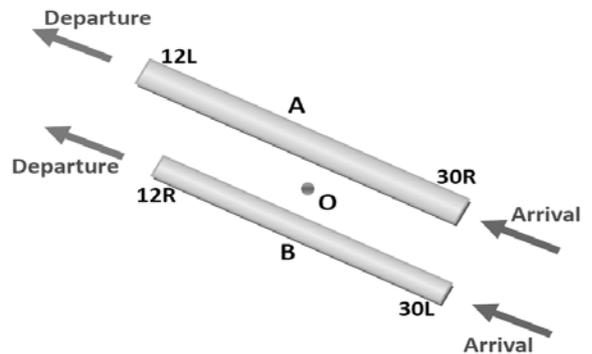

Figure 3: Runway configuration

TABLE I.   COORDINATES IN AXIS x AND y

	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8
x (km)	100	200	120	0	140	180	40	20
y (km)	200	100	2	100	194	40	20	160

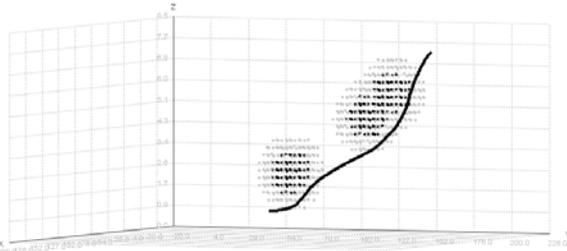|	I_1	I_2	I_3	I_4
x (km)	110	100	100	90
y (km)	100	90	110	100

Figure 4: Designing one route *(the axes x, y, z have different scales; the range of axes x and y is [0; 225km], the one of axis z is [0; 8.5km])*
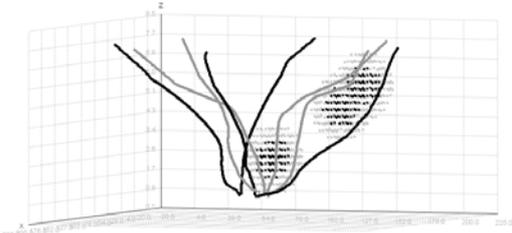

Figure 5: Generating all routes *(the axes x, y, z are represented in the same way as in Fig.4)*

C. Getting an optimal order

The simulation starts from a randomly generated order, with the initial total distance $L_{init} = 2204141 \ km$. By applying Simulated Annealing, we find the minimum total distance $L_{min} = 2008012 \ km$. The relative reduction $\Delta = (L_{init} - L_{min})/L_{min}$ is equal to 9.7%.

V. CONCLUSION AND PERSPECTIVES

In this paper, we introduce a methodology to generate automatically SIDs and STARs in TMA at a strategic level, considering forbidden areas and minimum separation constraints.

In future work, in order to get closer to the operational context, we plan to consider some other constraints in TMA, for instance the route curvature, noise restrictions, runway capacities, etc. To deal with the more complex problem, the study will focus on mathematical modelization aspects as well as on the development of efficient determinist and stochastic optimization methods. Furthermore, we will address route design at tactical level taking into account weather events. Some routes can be blocked and have to be dynamically redesigned in order to address efficiently the demand.

REFERENCES

[1] D. M. Pfeil, "Optimization of airport terminal-area air traffic operations under uncertain weather conditions", M.S.Thesis, SSM, MIT, Cambridge, 2011.

[2] H. G. Visser and R. A. A. Wijnen, "Optimization of noise abatement arrival trajectories", Journal of Aircraft, 38 (2001), pp. 620–627.

[3] B. Girardet, L. Lapasset, D. Delahaye, C. Rabut, and Y. Brenier, "Generating optimal aircraft trajectories with respect to weather conditions", ISIATM, Toulouse, 2013.

[4] D. Bertsekas, "Dynamic programming and optimal control", vol 1, Athena Scientific Belmont, 1995.

[5] J.A. Sethian. "Fast Marching Methods". SIAM review, 41(2):199–235, 1999.