Optimization of Arrival and Departure Routes in Terminal Maneuvering Area

Jun Zhou

MAIAA – Laboratoire de Mathématiques Appliquées, Informatique et Automatique pour l’Aérien
ENAC – École Nationale de l’Aviation Civile
UPS – Université Toulouse III - Paul Sabatier
Toulouse, France

International Conference on Research in Air Transportation (ICRAT) – Doctoral Symposium
31 May 2014
Outline

1. Context and problem description
2. Problem modeling
3. Solution approach
4. Simulation results
5. Conclusions and perspectives
Outline

1. Context and problem description
2. Problem modeling
3. Solution approach
4. Simulation results
5. Conclusions and perspectives
Air traffic growth

In 2012-2032, the forecast indicates that world economic growth (GDP) will be 3.2%, the number of airline passengers will grow at 4.1%, air traffic (RPK) will increase by 5.0%, and cargo traffic (RTK) will also grow by 5.0%.

Annual traffic growth is projected as follows:
- Middle East – Asia Pacific: 7.3%
- Within Latin America: 6.9%
- Within China: 6.9%
- Within Asia Pacific incl. China: 6.5%
- Europe – Asia Pacific: 5.5%
- North America – Latin America: 5.0%
- Africa – Europe: 4.8%
- Within/to CIS: 4.8%
- Europe – Latin America: 4.7%
- Transpacific: 4.5%
- Within Europe: 3.6%
- North Atlantic: 3.5%
- Within North America: 2.3%

Boeing Long-term Market Forecast

Current Market Outlook 2013–2032

Copyright © 2013 Boeing. All rights reserved.
Airports capacity

Airport is both the **starting** and **ending** point of air traffic.

Air traffic flow increases $\downarrow$

airports surrounding areas **capacity insufficiency**

### Eurocontrol long-term forecast 2012-2035

<table>
<thead>
<tr>
<th>Unaccommodated IFR Flights</th>
<th>Unaccommodated demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2035</td>
<td>Year 2035</td>
</tr>
<tr>
<td>1.9 million</td>
<td>12%</td>
</tr>
</tbody>
</table>
Terminal Maneuvering Area (TMA) (1/2)

A designated area of controlled airspace surrounding one or several airports
Designed to handle aircraft arriving to and departing from airports

TMA of the Paris region
TMA is one of the most complex types of airspace.
Standard Instrument Departure (SID) route:
A flight route followed by aircraft after takeoff from an airport.

Standard Terminal Arrival Route (STAR):
A route which connects the last enroute way-point to the Initial Approach Fix.
Currently, SID/STAR are designed manually based on the airport layout, existing Navaid infrastructures and nearby constraints.

This study considers the automation of SID/STAR design
- at a strategic level in 3D
- based on RNAV concept
Currently, SID/STAR are designed manually based on the airport layout, existing Navaid infrastructures and nearby constraints.

This study considers the automation of SID/STAR design
- at a strategic level in 3D
- based on RNAV concept

Optimization problem
Outline

1. Context and problem description
2. Problem modeling
3. Solution approach
4. Simulation results
5. Conclusions and perspectives
We consider TMA surrounding one airport, designed in a circular configuration centered on the airport.

- Two concentric circles $C_1$ and $C_2$, with radius $R_1$ and $R_2$, with altitude $H_1$ and $H_2$
- Entry/Exit points located on $C_1$
  \[ \mathcal{O} = \{ O_1, \ldots, O_{n_{in}}, O_{n_{in}+1}, \ldots, O_{n_{in}+n_{out}} \} \]
  - the first $n_{in}$ points are entry points
  - the remaining $n_{out}$ points are exit points
- Arrival/Departure points located on $C_2$
  \[ \mathcal{I} = \{ I_1, \ldots, I_{n_{arr}}, I_{n_{arr}+1}, \ldots, I_{n_{arr}+n_{dep}} \} \]
  - the first $n_{arr}$ points are arrival points
  - the remaining $n_{dep}$ points are departure points
Model parameters (2/2)

- Subset $\mathcal{K} \subseteq \mathcal{O} \times \mathcal{I}$, containing the pairs of points to be connected
- Total amount of flights $N$, arriving at and departing from the airport
- Proportion of flights associated with each pair of points
- Two parallel runways, available for all types of aircraft

- $\mathcal{O} = \{O_1, O_2, O_3, O_4\}$
- $\mathcal{I} = \{I_1, I_2, I_3\}$
- $\mathcal{K} = \{(O_1, I_1), (O_1, I_2), (O_2, I_2), (I_3, O_3), (I_3, O_4)\}$
Decision variables

Routes connecting points \((O_i, I_j) \in \mathbb{K}\):

\[
\gamma_{ij} : [0, 1] \rightarrow \mathbb{R}^3
\]

where

\[
\begin{align*}
\gamma_{ij}(0) &= O_i & \text{if } 1 \leq i \leq n_{in} \text{ and } 1 \leq j \leq n_{arr} \\
\gamma_{ij}(1) &= I_j \\
\gamma_{ij}(0) &= I_i & \text{if } n_{arr} + 1 \leq i \leq n_{arr} + n_{dep} \text{ and } n_{in} + 1 \leq j \leq n_{in} + n_{out} \\
\gamma_{ij}(1) &= O_j
\end{align*}
\]

\((\gamma_{ijx}, \gamma_{ijy}, \gamma_{ijz})\) are the components of \(\gamma_{ij}\) in axis \((x, y, z)\).
Constraints

Two main constraints are considered: forbidden areas and minimum separation.

Forbidden areas: mountains, cities, military areas, etc.

Minimum separation between routes:

Let $\Omega$ be a forbidden area.

$\forall (O_i, I_j) \in K, \forall \mu \in [0, 1]$

$\gamma_{ij}(\mu) \notin \Omega$

$\forall (O_i, I_j), (O_k, I_l) \in K, \forall (\mu_1, \mu_2) \in [0, 1]$

$\sqrt{(\gamma_{ijx}(\mu_1) - \gamma_{klx}(\mu_2))^2 + (\gamma_{ijy}(\mu_1) - \gamma_{kly}(\mu_2))^2} \geq 6\text{NM},$

$|\gamma_{ijz}(\mu_1) - \gamma_{klz}(\mu_2)| \geq 1200\text{ft}$
Minimizing the total distance flown by all flights during a certain period.

\[ L = \sum_{(i,j)} w_{ij} N \ l_{ij} \]

where

- \( l_{ij} \) is the length of route \( \gamma_{ij} \)
- \( w_{ij} \) is the proportion of flights on route \( \gamma_{ij} \)
1. Context and problem description
2. Problem modeling
3. Solution approach
4. Simulation results
5. Conclusions and perspectives
The problem is solved in three steps.

1. Compute an individual route by Fast Marching Method (FMM) and Gradient Descent method, where we take into consideration the forbidden areas.
Solution approach

The problem is solved in three steps.

1. Compute an individual route by **Fast Marching Method (FMM)** and **Gradient Descent** method, where we take into consideration the **forbidden areas**.

2. Given a fixed order of route designing, compute sequentially the routes taking into account the **minimum separation** constraints.
The problem is solved in **three steps**.

1. Compute an individual route by **Fast Marching Method (FMM)** and **Gradient Descent** method, where we take into consideration the forbidden areas.

2. Given a fixed order of route designing, compute sequentially the routes taking into account the minimum separation constraints.

3. Find an order minimizing the objective function by applying **Simulated Annealing (SA)**.
1\textsuperscript{st} step: designing one route (1/3)

Given \((O_i, I_j) \in \mathbb{K}\), searching for an optimal route by seeking the minimal travel time route from \(\gamma_{ij}(0)\) to \(\gamma_{ij}(1)\).

The minimal-time optimal trajectory problem can be modelled by a \textbf{wave front propagation} problem. (J. A. Sethian, 1999, adapted by B. Girardet, 2012)

\[ ||\nabla u(x)|| F(x) = 1 \]

where \(u(x)\) is the time at which the front reaches the point \(x\).
1\textsuperscript{st} step: designing one route (2/3)

Choosing the front propagation speed at point \( x \)

\[
F(x) = (1 - \alpha(x)) F
\]

where

- \( F \) is a constant value
- \( \alpha(x) \in [0, 1[ \)
  - \( \alpha(x) = 0.99 \) in forbidden area
  - \( \alpha(x) = 0 \) in free area
1\textsuperscript{st} step: designing one route (3/3)

- Solving the wave front propagation problem in isotropic case (no wind)
- Obtaining the minimum time to reach any point in space starting from $\gamma_{ij}(0)$
- Generating the route

Fast Marching Method (FMM)

Gradient Descent
- starting from $\gamma_{ij}(1)$
- moving towards $\gamma_{ij}(0)$
- taking steps proportional to $-\nabla u$
2nd step: generating all routes with fixed order

Fix an order, computing sequentially the routes:

Once a route is computed, this route and its protection zone are considered as additional forbidden area constraints for the remaining routes, by selecting an adapted propagation speed $F(x)$. 

Example
3rd step: getting an optimal order

The total distance may be reduced by changing the order of route designing.

Simulated Annealing is a stochastic global optimization method, emulating the physical process of metal annealing.

At each iteration, a new order is generated:

Repeat the 2nd step with the new order and compute the total distance.
Outline

1. Context and problem description
2. Problem modeling
3. Solution approach
4. Simulation results
5. Conclusions and perspectives
two parallel runways $A$ and $B$

- $R_1 = 100\, km$ and $R_2 = 10\, km$
- $H_1 = 25000\, ft$ and $H_2 = 4000\, ft$
- $n_{in} = 4$, $n_{out} = 4$, $n_{arr} = 2$, $n_{dep} = 2$
- three forbidden areas
One pair \((O_1, I_1)\)

The axes \(x, y, z\) have different scales; the range of axes \(x\) and \(y\) is \([0; 225km]\), the one of axis \(z\) is \([0; 8.5km]\).
Result for fixed ordered routes

8 pairs, 3 obstacles:

the black routes are the STARs and the gray ones are the SIDs
Result for optimal order

- Relative reduction $\Delta = ((L_{init} - L_{min})/L_{min}$ is equal to 9.7%.
- Total execution time is around 1 hour.
Outline

1. Context and problem description
2. Problem modeling
3. Solution approach
4. Simulation results
5. Conclusions and perspectives
Conclusions

- A methodology to generate **automatically** SIDs and STARs in TMA at a strategic level
- Taking into account:
  - **forbidden areas** constraints
  - **minimum separation** between routes

- The generated routes are **continuous** and **smooth**, thus they are available for **Continuous Descent Operations (CDO)**.
A more complete and complex modeling

1. **Design procedure:**
   - first, design a route in the horizontal plane
   - afterwards, evaluate it in the vertical axis

2. **Form of the route** ⇒ impact on the model
   - line segments + arcs of circles
   - Bézier curves
   - B-spline curves
3. Other constraints:

- route curvature
- runway capacities
- metroplex TMA

4. Multi-objectives:

- minimizing fuel consumption
- noise abatement
5. Route redesigning at tactical level: weather events

Other optimization methods:

- Modeling the problem as a Mixed Integer Programming problem
- Applying deterministic or stochastic methods
- A hybrid optimization method more adapted to our problem
The end

Thank you!