Quantifying Trajectory Uncertainty Using a Sensitivity-Based Complexity Metric Component

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***Abstract*—Previous studies on complexity measures have focused on traffic density and the geometry of interacting flights to quantify the difficulty of a particular traffic situation. Within this paper, the strong correlation between trajectory uncertainty and controller workload is used to show that sector complexity is not primarily driven by geometric features, but rather by the quantity of uncertainty that a controller has to face. A complementary complexity component is introduced that can help quantify the amount of uncertainty and therefore improves the assessment of controller workload required to handle a given traffic situation. This complexity component is based on sensitivity analysis, where the magnitude of uncertainty is obtained by directly investigating the impact of parameter variations to the aircraft's predicted position ahead in time. Therefore, the quantification of uncertainty is based on the inherent dynamics of a given flight and its trajectory, rather than an assumed random error. Numerical results for simulations based on the BADA model and an aircraft intent formalization are presented to illustrate the potential benefit of quantifying traffic complexity by using sensitivity analysis. Furthermore, by following this approach, trajectory uncertainty from an air traffic controller's perspective for a particular flight may be quantified and estimations can be made as the predictability of trajectories improves with surveillance technologies and data sharing with the aircraft flight management system.**

Keywords-Trajectory Uncertainty, Complexity Metric, Sector Complexity, Controller Workload, Sensitivity Analysis

I. INTRODUCTION

In 2010, approximately 5% of all flights in out of the top 30 airports in Europe were held on the ground to manage en route congestion, see [1]. To meet future traffic demands, airspace capacity must be increased. Reducing controller workload through automation is a key component of both SESAR (*Single European Sky ATM Research*) and NextGen (next generation ATM program by the FAA) strategies to achieve the required gain. Measurement of sector complexity is a means of assessing and differentiating controller workload. Sector complexity metrics are mainly used to assess airspace design and to project workload when planning sector openings and closings. Previous works focus on geometric traffic characteristics as a measure for complexity and controller workload. Sample traffic characteristics include the number of aircraft, aircraft density, number of climbing and descending flights, speed variation and the proximity of projected trajectories. There is a large body of research on geometric based complexity studies, including the Eurocontrol Performance Review Unit's study titled *Complexity Metrics for ANSP Benchmarking Analysis* [1], NASA studies on *Airspace Complexity and its Application in Air Traffic Management* [2], and the FAA report on the *Relationship of Sector Activity and Sector Complexity to Air Traffic Controller Taskload* [3]. The SESAR Joint Undertaking workgroup on airspace complexity (project *4.7.1 Complexity Management in En-Route*) has summarized much of the research on complexity in their recent report titled *Consolidation of Previous* (Complexity) *Studies*, [4]. A comprehensive overview of various complexity studies can be found in [5]. Other methods of assessing controller workload include measurement of specific tasks performed, based on the Eurocontrol CAPAN Model, [6], or as in [7].

This paper introduces the strong correlation between trajectory uncertainty, complexity and controller workload. It is shown that sector complexity is not primarily driven by geometric features, as portrayed in most of the previous research, but rather by the underlying quantity of uncertainty that a controller must manage. In today's Air Traffic Management (ATM) system, the controller's limited data on aircraft intent is a key driver of uncertainty. While previous research has focused, for example, on increased complexity caused by climbing and descending flights compared to level flights. The authors' contend that it is uncertainty associated with the future positions of the climbing and descending aircraft that drives increased complexity. Why is this important? It is important because much of today's focus on 4D trajectories is an effort to reduce uncertainty. As we move to improved exchanges of aircraft intent with the controller's ground automation, we will need complexity measures that address the improvements in predicted trajectories. The role of uncertainty as a key driver of complexity and workload needs to be adequately addressed.

The implementation of the medium term conflict detection (MTCD) as a part of the VAFORIT system at DFS' Karlsruhe Center showed that during the deployment of system, Karlsruhe UAC controllers strongly believed that trajectory uncertainty drove their workload even when no measurable action on their part was required. In the early 2000s, controller experiences were similar during the deployment of the FAA's User Request Evaluation tool (URET), which included MTCD. Controller feedback from these two independent system implementations illustrates the relationship between uncertainty and controller workload. Task based workload measures, like Eurocontrol's CAPAN, will clearly play a key role in evaluating future ATM enhancements and can be used to further calibrate the proposed complexity measure framework. It must be noted that in its current form, the proposed complexity framework is intended as a complimentary factor to existing complexity measures.

The structure of the paper is as follows: In section II, the relationship between trajectory uncertainty and perceived sector complexity is elaborated. In section III, the proposed uncertainty-based metric component is introduced, for which section IV will show some simulation results. Finally, the paper is concluded in section V, where further research topics are suggested as well.

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II. UNCERTAINTY AND COMPLEXITY IN AIR TRAFFIC CONTROL

In this section, primary drivers of uncertainty in Air Traffic Control are investigated. From these considerations, it may be concluded how and why the drivers of uncertainty also drive controller workload. Furthermore, possible enhancements of tools like medium term conflict detection by increasing trajectory information sharing between ground automation systems and the aircraft FMS are also discussed, [8], [9]. These improvements are major components of NextGen and SESAR, see [10,11]. Complexity measures that address trajectory uncertainty in assessing controller workload should play an important role in the evaluation of full data link functionality. It may be concluded that by incorporating trajectory uncertainty in a complexity metric explicitly, the understanding of complexity is raised, hence improving the efficiency of any countermeasure applied to the situation.

When asking controllers for their perception of complexity, they usually agree that most level flights contribute less to workload than flights climbing and descending. This is reflected by common controller procedures to prefer horizontal vectoring over level changes for level flights in en-route scenarios, see [12]. Most controllers vector aircraft to resolve conflicts as opposed to using altitude, because it is faster and easier to control. Vectors are preferred due to variable aircraft performance and response times in climb and descend. This finding is supported by many current complexity measures which have separate factors for climbing and descending flights. Experience from the implementation of VAFORIT at DFS further suggests that complexity is less driven by geometry and more by the uncertainties in the trajectories. In earlier VAFORIT releases, controller's turned off the MTCD feature mainly due to the over conservative projection of conflicts due to the trajectory uncertainties. Non-level flights are more complex to handle because they have much less predictable trajectories. Controllers are not able to make firm judgments as to whether an aircraft will reach a desired requested altitude and automation systems do not have key data on aircraft weight, thrust, and configuration to improve decision making in the vertical realm [13].

Uncertainty in trajectories increases when a controller requests a change in altitude or a vector for conflict. In today's environment, neither the controller nor the automation system knows exactly when the turn or change in altitude will be made. In general, key drivers of trajectory uncertainty can include: **Trajectory Data Quality**, i.e. the accuracy and availability of raw data (aircraft state, including weight, controls or weather), **Model Quality**, i.e. the model's ability to capture and to process all necessary information, and **Operational Procedures**, i.e. the procedures that determine aircraft trajectories via clearances assigned by traffic controllers and their respective execution. Furthermore, airline and customer preferences can greatly affect trajectory uncertainty and influence actual operations.

Trajectory Data Quality and Operational Procedures are investigated below in more detail to motivate the proposition of focusing on trajectory uncertainty as a measure for airspace complexity.

A. Trajectory Data Quality

In [14], it is pointed out how increased data reliability will enhance the efficiency of future automated support tools. This can be understood when it is observed that every trajectory prediction is based on a certain variable vector $p_0 \in \mathbb{R}^{n_p}$. These variables include fundamental parameters like aircraft weight, its velocity, thrust, flap settings or exact heading. In current automation processes, most of these data parameters are not accurate, but estimates are available. Thus, the prediction of aircraft trajectories is based on estimates

$$p \approx p_0,$$

where p_0 are the true aircraft parameters. This estimation of parameters p is usually obtained by data fitment and is then inserted into a generic aircraft performance (BADA) that is used to compute a trajectory prediction. Figure 1 shows varying trajectories for different take-off weights (TOW).

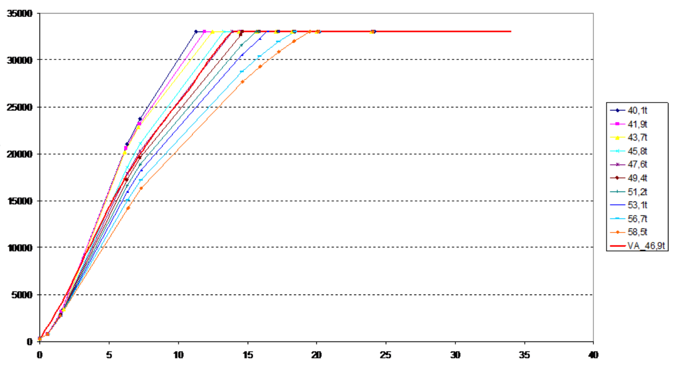

Figure 1: Flight profile variation for B737-800 aircraft with different TOWs, compared to VAFORIT prediction (VA curve).

Due to the inaccuracy of parameters, the resulting trajectories are highly uncertain and often miss-match with actual flight trajectories. This is addressed by studies like [15], [16], [17], and [18]. In the latter, six different sources of trajectory prediction errors have been evaluated, and top-of-descent errors as well as cruise speed errors can have large effect on accuracy of predictions. These issues regarding current automation processes have been addressed in several recent studies, including [14], [15], [18], and [19]. It is shown that there is a large variation in performance, especially concerning climb and descent flight phases. Most significantly, in [18], it is highlighted that the climb rate dramatically depends on the aircrafts' mass. Current ground automation has no knowledge of actual aircraft mass, leading to uncertainty in the trajectory prediction.

In [20], it was shown how FANS can improve aircraft derived data for better trajectory prediction. A desired complexity metric will therefore incorporate expected changes to these aspects and relate perceived complexity to data quality.

B. *Operational Procedures*

Additional trajectory uncertainty stems from the execution of operational instructions or *clearances*. One of the major factors in this group is the timing of execution of controller requested changes in heading or flight level. With current procedures, a controller will not know when exactly an aircraft will start its climb to a new altitude or turn towards a new heading. Both aspects increase the perceived complexity during the trajectory change due to the large amount of possible aircraft positions. Controllers compensate for this uncertainty by instructing pilots to reach an assigned altitude before a certain point. Even after a pilot has confirmed instructions, controllers may continue to experience workload monitoring progress.

Controller intent also adds to trajectory uncertainty. In [21], it is pointed out that actions taken by controller to de-conflict or to synchronize traffic is one of the major factors of uncertainty. The actions are carried out by individual persons and the specific techniques greatly vary per controller or controller team. The conclusion is that uncertainties in controller intent are by far the largest source of error in flight profile prediction.

Figure 2 and Figure 3 show the large variation of flight profiles and rates of climb and descent. These figures highlight trajectory uncertainty for the same aircraft type. Without better information on intent and/or aircraft characteristics, conflict detection tools may have limited value. In fact, controllers may have better information on the characteristics of individual airlines than the automation system, thereby increasing workload.

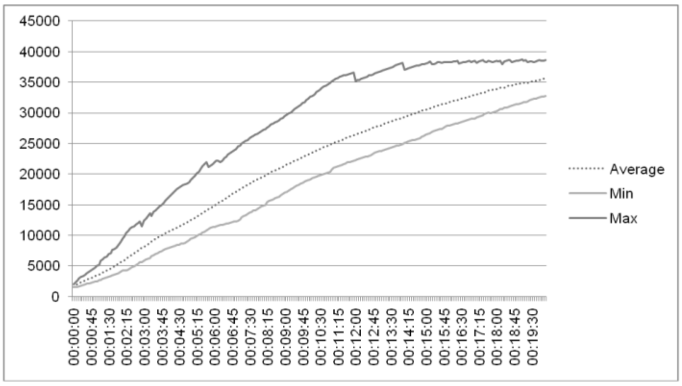

Figure 2: Flight profile variation for B737-800 aircraft.

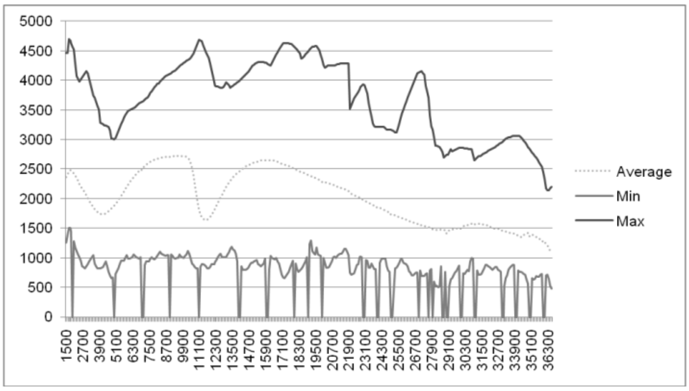

Figure 3: ROCD with respect to flight level, variation for B737-800 aircraft.

III. Proposed Complexity Metric Framework

In this section, the proposed methodology for assessing uncertainty for a particular flight is introduced. Some theoretical preliminaries have to be addressed before the actual complexity component is presented.

Preliminaries

In order to compute an aircraft trajectory ahead in time, it is simulated according to the aircraft flight mechanics described by a set differential equations and the initial value given for the initial instant of time t_0 as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{p}) \;\; ; \;\; \boldsymbol{x}(t_0) = \boldsymbol{x_0}(\boldsymbol{p}). \qquad (1)$$

By applying numerical integration, the aircraft's state vector $\boldsymbol{x}$ at time t is predicted as

$$\boldsymbol{x}(t) = \boldsymbol{x}(t_0, \boldsymbol{p}, t) \in \mathbb{R}^{n_x}. \qquad (2)$$

The vector $\boldsymbol{p} = (p_1 \quad p_2 \quad \cdots \quad p_{n_p})^T \in \mathbb{R}^{n_p}$ is a set of parameters which are not precisely known and thus impact the uncertainty of the trajectory projection. The parameters may be described by certain deviation $\boldsymbol{\Delta p} \in \mathbb{R}^{n_p}$ of the nominal parameters $\overline{\boldsymbol{p}} \in \mathbb{R}^{n_p}$ so that

$$\boldsymbol{p} = \overline{\boldsymbol{p}} + \boldsymbol{\Delta p}. \qquad (3)$$

As mentioned above, this vector could incorporate traditional parameters such as weight or environmental factors just as well as operational factors, like the instant of time when the aircraft starts a maneuver.

Taking the state vector from (2), the following linearization can be made:

$$\boldsymbol{x}(t; \overline{\boldsymbol{p}} + \boldsymbol{\Delta p}) \approx \boldsymbol{x}(t; \overline{\boldsymbol{p}}) + \boldsymbol{S}(t) \cdot \boldsymbol{\Delta p}\,, \qquad (4)$$
$$\boldsymbol{x}(t_0) = \boldsymbol{x_0}(\overline{\boldsymbol{p}} + \boldsymbol{\Delta p}) \approx \boldsymbol{x_0}(\overline{\boldsymbol{p}}) + \boldsymbol{S_0}(t) \cdot \boldsymbol{\Delta p}\,,$$

with the sensitivity matrix $\boldsymbol{S}(t)$ denoting the derivatives of the states $\mathbf{x}$ with respect to the parameter set $\boldsymbol{p}$

$$\boldsymbol{S}(t) = \frac{\partial \boldsymbol{x}(t)}{\partial \boldsymbol{p}} \in \mathbb{R}^{n_x \times n_p}. \qquad (5)$$

It is assumed that the sensitivity matrix itself is independent from the parameters $\boldsymbol{p}$ which is reasonable when nonlinearities in the differential equation system are cancelled out. This is usually the case in stationary flight. In addition to that, if the deviations $\boldsymbol{\Delta p}$ are reasonably small, the approximation error are most likely to be within the required limits [22]. The computation of the sensitivity matrix (6) can be done on-line by augmenting the dynamic system by

$$\dot{\boldsymbol{S}} = \boldsymbol{f}'_{\boldsymbol{x}}(t, \boldsymbol{x}(t), \boldsymbol{p}) * \boldsymbol{S}(t) + \boldsymbol{f}'_{\boldsymbol{p}}(t, \boldsymbol{x}(t), \boldsymbol{p}) \quad \text{with} \qquad (6)$$
$$\boldsymbol{S}(t_0) = \boldsymbol{S_0} = \frac{\partial x_0(\boldsymbol{p})}{\partial \boldsymbol{p}},$$

and is only valid under the assumption that $\boldsymbol{f}$ is continuous with continuous partial derivatives w.r.t. $\boldsymbol{x}$ and $\boldsymbol{p}$. Further details on mathematical background of sensitivity analysis can be found in [22] and [23].

Probabilistic Airspace Occupancy

Consider n_A aircraft flying in the airspace region of interest $S \subset \mathbb{R}^3$. Then, the predicted future aircraft position $\boldsymbol{x}^{A_i}(t; \boldsymbol{p}) \in S$ of the aircraft $i = 1,2,\ldots,n_A$ can be determined for a certain instant of time $t \in T$ within the regarded look-ahead time horizon $T = [t_0, t_f]$. Assuming that the deviations of the nominal sensitivity parameters $\boldsymbol{\Delta p}$ to be zero mean normal distributed with a covariance matrix $\boldsymbol{\Sigma}(\boldsymbol{\Delta p}) \in \mathbb{R}^{n_p \times n_p}$, the covariance matrix of the aircraft states $\boldsymbol{\Sigma}(\boldsymbol{x}(t)) \in \mathbb{R}^{n_x \times n_x}$ can be determined by linear transformation [24] as

$$\boldsymbol{\Sigma}(\mathbf{x}(t)) = \boldsymbol{S}(t) \cdot \boldsymbol{\Sigma}(\boldsymbol{\Delta p}) \cdot \boldsymbol{S}(t)^{\boldsymbol{T}}. \qquad (7)$$

Thus, the probability density function $\theta_i(\boldsymbol{q}, t)$ for aircraft A_i at an arbitrary point $\boldsymbol{q} \in S$ can be defined as a three dimensional multivariate normal distribution with zero mean

$$\theta_i(\boldsymbol{q}, t) = \frac{1}{(2\pi)^{\frac{3}{2}} \cdot |\boldsymbol{\Sigma}(\mathbf{x}^{\mathrm{A_i}}(\mathrm{t}))|^{\frac{1}{2}}} \cdot e^{-\frac{1}{2}\left(\boldsymbol{q} \cdot \boldsymbol{\Sigma}\left(\mathbf{x}^{\mathrm{A_i}}(\mathrm{t})\right)^{-1} \boldsymbol{q}^T\right)}. \qquad (8)$$

The contour lines of constant values of the three dimensional probability density function θ_i from equation (8) represent ellipsoids centered at the nominal aircraft position $\boldsymbol{x}^{A_i}(t, \overline{\boldsymbol{p}})$ of aircraft A_i. It is proposed to use these ellipsoids to illustrate the uncertainty area around a nominal aircraft position for a specific instant of time [25]. The main axes of the ellipsoid are proportional to the unit eigenvectors of the covariance matrix of the predicted aircraft position $\boldsymbol{\Sigma}\left(\boldsymbol{x}^{A_i}(t)\right)$ scaled by its eigenvalues respectively. Furthermore, the eigenvectors are always perpendicular due to the definition of the covariance matrix being positive definite and symmetric (see Figure 4 below).

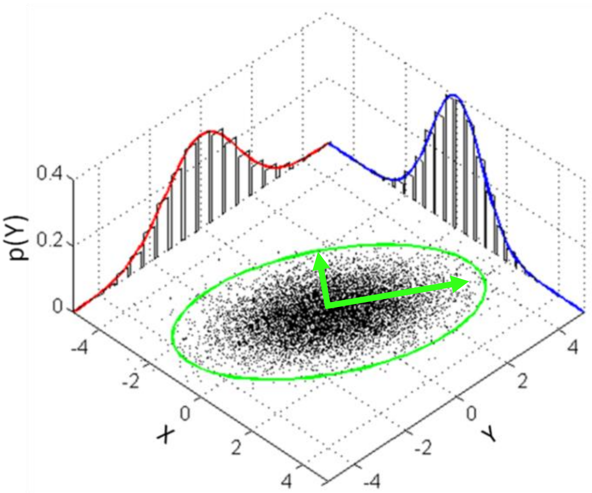

Figure 4: Example of a 2D multivariate normal distribution with eigenvectors of covariance matrix, [26].

The probability of the occupancy of a certain airspace volume can then be calculated with a spatial integral over the specific region. As it is introduced in [27], this region may be defined as an ellipsoid $\mathcal{M}(\boldsymbol{c}) \subseteq S$ centered at $\boldsymbol{c} \in S$ as

$$\mathcal{M}(\boldsymbol{c}) = \{\widehat{\boldsymbol{x}} \in \mathbb{R}^3 : (\widehat{\boldsymbol{x}} - \boldsymbol{c})^T \boldsymbol{M} (\widehat{\boldsymbol{x}} - \boldsymbol{c}) \leq 1\}\,, \qquad (9)$$

where $\boldsymbol{M} \in \mathbb{R}^{3\times3}$ is a diagonal matrix given by

$$\boldsymbol{M} = diag\left(\frac{1}{r_h^2}, \frac{1}{r_h^2}, \frac{1}{r_v^2}\right) \tag{10}$$

with $r_h \geq r_v > 0$ defining the size of the ellipsoid in the horizontal plane and in the vertical direction. For the proposed application of complexity assessment, the horizontal proximity is usually considered as more critical than the vertical one and if $r_h = r_v$, all directions are weighted the same reducing the ellipsoid to a sphere.

For each aircraft A_i, the cumulated probability $\pi^{A_i}(\boldsymbol{c}, t)$ that aircraft A_i is located within $\mathcal{M}(\boldsymbol{c})$ at an instant of time $t \in T$ can be computed as

$$\pi^{A_i}(\boldsymbol{c}, t) = \iiint_{\mathcal{M}(\boldsymbol{c})} \theta_i([x\ y\ z], t)\ dx\ dy\ dz. \tag{11}$$

Referring to the definition in [28], the first order probabilistic occupancy measure $\gamma_1(\boldsymbol{c}, t) \in [0,1]$ at position $\boldsymbol{c} \in S$ for the time instant $t \in T$ is then denoted as

$$\gamma_1(\boldsymbol{c}, t) = P(\boldsymbol{x}^{\boldsymbol{A_i}}(t) \in \mathcal{M}(\boldsymbol{c}) \mid for\ t \in T,\ i \in \{1,2,..,n_A\}) \tag{12}$$

and represents the probability of at least one aircraft being inside the ellipsoid $\mathcal{M}(\boldsymbol{c})$ at the instant of time $t \in T$. It is zero when none of the existing aircraft enters the ellipsoid $\mathcal{M}(\boldsymbol{c})$ at time t and reaches the maximum value when at least one aircraft is located within $\mathcal{M}(\boldsymbol{c})$ with certainty.

Similarly, the second order probabilistic occupancy measure $\gamma_2(\boldsymbol{c}, t) \in [0,1]$ at position $\boldsymbol{c} \in S$ for an instant of time $t \in T$ can be defined as

$$\gamma_2(\boldsymbol{c}, t) = P(\boldsymbol{x}^{\boldsymbol{A_i}}(t)\ and\ \boldsymbol{x}^{\boldsymbol{A_k}}(t) \in \mathcal{M}(\boldsymbol{c}),\ for\ t \in T\ and\ i \neq k \in \{1,2,..,n_A\}). \tag{13}$$

Note that $\gamma_2(\boldsymbol{c}, t) = 0$ means that there will be at most a single aircraft inside the ellipsoid $\mathcal{M}(\boldsymbol{c})$ at instant of time $t \in T$. On the other hand, if $\gamma_2(\boldsymbol{c}, t) = 1$, then at least two aircraft will enter the ellipsoid $\mathcal{M}(\boldsymbol{c})$ at time instant t.

If the different sets of sensitivity parameters affecting the future positions of each aircraft A_i are assumed to be independent, then the first order and second order probabilistic occupancy measures can be calculated as by

$$\gamma_1(\boldsymbol{c}, t) = 1 - \prod_{j=1}^{n_A}\left(1 - \pi^{A_j}(\boldsymbol{c}, t)\right) \text{ and} \tag{14}$$

$$\gamma_2(\boldsymbol{c}, t) = \gamma_1(\boldsymbol{c}, t) - \sum_{h=1}^{n_A}\left(\pi^{A_h}(\boldsymbol{c}, t) \cdot \prod_{j=1, j\neq h}^{n_A}\left(1 - \pi^{A_j}(\boldsymbol{c}, t)\right)\right). \tag{15}$$

Concerning the independency of the sensitivity parameters, the correlations caused by wind effects might be significant, especially for aircraft flying close to each other [29], [30]. Note that in case of any non-negligible correlations, the above expressions represent only approximations of the probabilistic occupancy measures γ_1 and γ_2.

By varying the center $\boldsymbol{c}$ of the ellipsoid $\mathcal{M}(\boldsymbol{c})$ over S, one can define the first order and second order probabilistic occupancy maps of the airspace region S at instant of time $t \in T$ as follows (24):

$$\Gamma_1(\cdot, t): \boldsymbol{c} \in S \rightarrow \gamma_1(\boldsymbol{c}, t) \text{ and} \tag{16}$$

$$\Gamma_2(\cdot, t): \boldsymbol{c} \in S \rightarrow \gamma_2(\boldsymbol{c}, t). \tag{17}$$

It can be seen that the value of Γ_1 at any point $\boldsymbol{c} \in S$ is bigger or equal to the Γ_2 map, since the corresponding events are nested.

Complexity Measure

For measuring air traffic complexity, controllers would perceive an air traffic scenario as less complex if the aircraft information available is highly accurate in comparison to a scenario with uncertain information. Hence, it is self-evident to use the above presented airspace occupancy calculations to incorporate trajectory uncertainty in air traffic complexity computation. Various methods for computing a scalar complexity values are proposed in literature and are reviewed at this point.

Taking into account the geometry of the uncertainty ellipsoids, in [25], it is proposed to sum up their volumes to obtain a measure of the general predictability of the future air traffic situation. It is further suggested to use the overlap of the uncertainty ellipsoids to capture interaction between aircraft and quantify possible future controller action. Both metrics are easily interpretable and can be computed on-line at low computational effort but may not sufficiently represent the actual complexity or workload of the overall scenario.

In [27], the integration ellipsoid (9) is scaled so a certain threshold probability of two aircraft coming close to each other is reached. A critical scaling factor is determined by moving the center $\boldsymbol{c}$ of the integration ellipsoid through the considered airspace S taking into account the whole time horizon T. This method allows to measure air traffic complexity by investigating the most critical (future) interaction of currently present aircraft.

The drawback of neglecting all other (less) critical proximities of aircraft is addressed in [22] and it is proposed to cluster the 4D occupancy map from (17). Each cluster $C_i \subseteq S \times T$ with $i = 1,2,..,n_C$ is identified by a connected region with a spatial as well as a temporal expansion where the second order probabilistic occupancy γ_2 from (15) is bigger than a specified threshold value $\gamma_{2,T}$. Then, the n_C clusters can be evaluated separately and summed up to a total complexity measure $C_{cluster}$ by

$$C_{cluster} = \sum_{i=1}^{n_C} \sup_{(\boldsymbol{c},t)\in C_i} \gamma_2(\boldsymbol{c},t)\,, \qquad (18)$$

with

$$\gamma_2(\boldsymbol{c},t) > \gamma_{2,T} \quad \forall(\boldsymbol{c},t) \in C_i\,, i = 1,2,..,n_C. \qquad (19)$$

The maximum second order occupancy γ_2 of an identified cluster represents the criticality of the whole conflict zone which could often be eliminated entirely by one single controller intervention. This complexity measure also takes into account the actual amount of potential conflict zones which is a major driver of the current controller workload. In fact, this measure is very basic and by also considering the intensity or the expansion of each cluster, a higher sophisticated evaluation would be possible. For instance, conflict zones of a long temporal expansion might be caused by a pair of aircraft with a low intersection angle whereas a high and short peak rather is a sign of a 90 degree intersection.

Summing up, it is not the intention of the authors to propose a concrete complexity metric at this point rather than giving a short overview of possible (scalar) air traffic evaluations. Depending on the purpose, an appropriate metric should be selected and tuned carefully. The covariances of the parameters may be seen as weights to adapt the metric on its foreseen application area. In practice, the challenge is to find a balance between the rate of false alerts, the level of confidence in conflict detection and the loss in ATM efficiency [31]. Hence, the weighting process can be seen as a multi-objective optimization which has to be repeated as the boundary conditions change. This could be, for instance, when new technologies and procedures are implemented to improve trajectory information [25]. For a better understanding, in the next section the proposed complexity evaluations are applied on an example air traffic scenario where the influence of a selected set of properly weighted sensitivity parameters is investigated.

IV. Simulation Results

In this last section, the simulation results of a vertical air traffic scenario are presented applying the above presented complexity evaluation. Three aircraft flying on top of each are instructed to perform a flight level change of $\Delta h = 6000ft$ with a predefined rate of descent $ROCD_{req} = -2000\frac{ft}{min}$ at a constant true airspeed $V_{req} = 420kt\,(TAS)$. From the air traffic controller's point of view, the instant of time when the pilot begins descending is of high importance. As soon as the rate of descent of the lower flying aircraft is confirmed, it is possible to give the descent clearance for the above flying aircraft. Additionally, taking into account also the accuracy of the radar and the tolerances in vertical and horizontal speed, it is evident that special controller attention is needed for such a scenario. For further information of the choice of the numerical values of the parameters, see [22] and please note that Table 1 could be extended easily.

Table 1. Selected Sensitivity Parameters and their standard deviations used in simulation.

sensitivity parameter	**standard deviation σ**
initial north position x_0	$50\,m$
initial east position y_0	$50\,m$
initial altitude z_0	$5\,m$
kinematic velocity V_{req}	$4\,kt$
vertical velocity $ROCD_{req}$	$\frac{200}{6}\,\frac{ft}{min}$
starting time of descending t_{begin}	$\frac{80}{6}\,s$

For simulation, an Airbus A320 point mass model with coefficients provided by the Base of Aircraft Data (BADA) by Eurocontrol [32] is used. The atmosphere is modeled as defined in DIN ISO 2533, known as the International Standard Atmosphere (ISA). For the sake of simplicity, wind influences are neglected and the aerodynamic side force is set to zero. Besides, a non-rotating, flat earth is assumed.

When a controller transmits a clearance, it is necessary to translate it to an explicit set of aircraft intents for generating the aircraft input commands. For flight trajectory prediction, a precise knowledge of planned maneuvers is required and thus, some formal language for data-link is necessary. Therefore, the *Aircraft Intent Description Language* (AIDL) developed by Boeing Research & Technology Europe [33] is used in the simulation framework for the unambiguous definition of aircraft trajectories.

Furthermore, the size of the integration ellipsoid (9) to compute the occupancy map is defined by a vertical radius $r_v = \frac{1000}{2}\,ft$ and a horizontal radius $r_h = \frac{3}{2}\,nm$. These radius represent the reduced vertical separation minimum (below FL290), and the horizontal radar separation minimum (at a distance less than 40 miles from the radar antenna), [34]. Thus, when the probability is greater than zero that two aircraft are within the integration ellipsoid at the same instant of time (15), a certain risk of violating the separation minima exits and thus affecting controller workload significantly. Additionally, the same values of r_v and r_h are used to weight the horizontal and vertical directions for computing the volumes of the uncertainty ellipsoids.

Besides, for computing the underlying occupancy map (17), a discretization of the 4D space is necessary, aiming for a compromise between computational effort and required precision.

In Figure 5 below, the altitude in feet [ft] is plotted over the distance (here: east direction) in nautical miles [nm]. Please note that for a better visibility, the horizontal and the vertical axes are not scaled the same way and therefore the glide slope is not a true-to-scale representation. The drawn ellipses around the nominal aircraft position represent the 3σ uncertainty ellipsoids (99.73% coverage) projected on the z-y plane the aircraft are flying in. The gray-scale background depicts the second order occupancy map computed by equation (17) at discrete points. In general, the way discretization of the 4D space is of high importance due to the fact that the clustering process may fail as a matter of bad gridding. It is very important to find an appropriate compromise between computational effort and required precision which is also addressed in [22]. Here, a fixed grid size of 500m (horizontal) and 25m (vertical) are chosen. In the plot below, two potential conflict zones can be identified and by summing up their maximum probability respectively, the cluster complexity $C_{cluster} = 0.003784$ is computed (considering only this single time frame). The value differs from zero and thus indicates that a certain probability of violating the (vertical) separation minimum exists 190 seconds ahead in time due to parameter uncertainties.

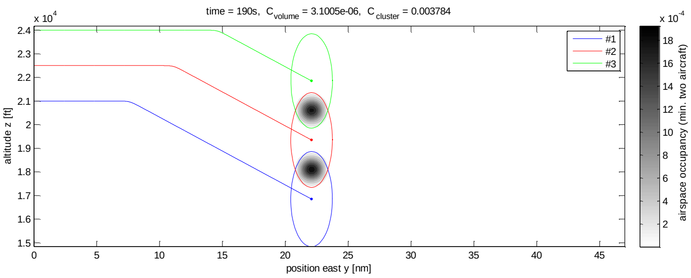

Figure 5: Example simulation of vertical separation 3σ uncertainty ellipses 190s ahead

After successfully changing the flight level in the prediction, all three aircraft continue with horizontal flight and the vertical uncertainty shrinks to the resolution of the radar as it can be seen in Figure 6. Thus, it is assumed that even if the vertical speed varies during descent, all three aircraft are reaching the instructed flight level with certainty. Furthermore, in longitudinal direction the uncertainty ellipsoid is still continuously increasing due to the velocity sensitivity parameter V_{req}. The complexity value $C_{cluster}$ vanishes because the threshold value $\gamma_{2,T}$ (19) is not reached again after the flight level change. The volume complexity C_{volume} also decreases by two powers of ten which is reasonable comparing the sizes of the ellipses in Figure 6 and Figure 7.

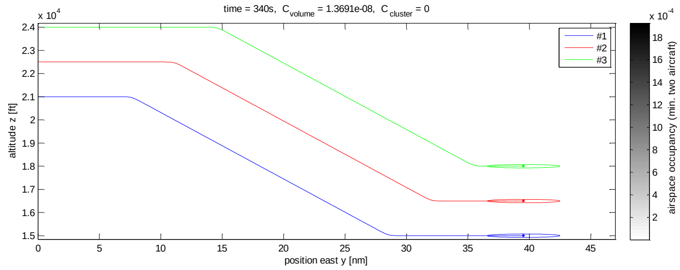

Figure 6: Example simulation of vertical separation 3σ uncertainty ellipses 340s ahead

For a better comparability, in Figure 7, the sensitivities with respect to t_{begin} and $ROCD_{req}$ are multiplied with their standard deviation $\sigma_{t_{begin}} = \frac{80}{6}s$ and $\sigma_{ROCD_{req}} = \frac{200}{6}\frac{ft}{min}$ respectively. Thus, it can be seen that the vertical expansion of the uncertainty ellipsoid is basically based on the variation of the descent starting time t_{begin} due to its large standard deviation. Further, it is noticeable that a deviation of t_{begin} introduces some constant sensitivity during descent whereas a variation in velocity (vertical as well as horizontal) leads to an increasing uncertainty over time due to the integration of the differential equations (1).

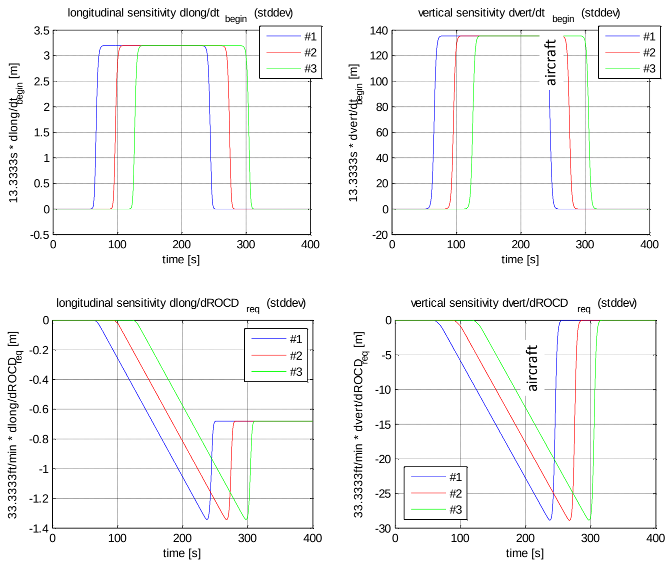

Figure 7: Longitudinal and vertical sensitivities with respect to t_{begin} and $ROCD_{req}$ multiplied with their standard deviation respectively

The vertical separation example shows how uncertainty during the vertical maneuver is quantified, and that there is a significant area of uncertainty for each of the descending aircraft. After approaching at the desired flight level, however, the prediction of the aircraft's trajectory can be performed at a high level of confidence again. As long as the aircraft within the regarded sector are flying sufficiently far from one another, the level of uncertainty in trajectory prediction does not affect controller workload dramatically.

The scenario presented shows how the proposed framework may be used to assess the inherent uncertainty of a trajectory, hence enabling the incorporation of uncertainty driven complexity into existing complexity metrics.

V. Conclusion and Next Steps

This paper presents a novel way of quantifying airspace trajectory uncertainty as a complementary complexity metric component. The fundamental argument for this is the relationship between trajectory uncertainty and perceived controller workload and sector complexity.

The proposed assessment is based on the inherent sensitivity of an aircraft's trajectory and represents a realistic measure for its uncertainty. The probabilistic airspace occupancy by an aircraft with respect to certain parameters is used to illustrate airspace complexity and to project controller workload within a sector.

By means of sensitivity analysis, this uncertainty is quantified incorporating the system dynamics of the respective aircraft and the flight trajectory which is defined by the intent of the pilot as well as the intent of the air traffic controller. Different scalar complexity measures have been discussed and demonstrated in a simulation example.

The proposed methodology may be used to quantify workload, as well as be applied to conflict detection and resolution, given that overlapping uncertainty ellipsoids reveal information about possible conflicts that need to be solved. The unique value of this approach is that workload will change in time as trajectory uncertainty decreases with surveillance technology and aircraft to controller data sharing. Further research is recommended to adapt this proposed approach to controller needs and to identify an efficient implementation into a realistic operational context.

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