A Numerical Framework and Benchmark Case study for Muti-modal Fuel Efficient Aircraft Conflict Avoidance

Manuel Soler, Maryam Kamgarpour, and John Lygeros

***Abstract*— We formulate fuel optimal conflict free aircraft trajectory planning as a hybrid optimal control problem. The discrete modes of the hybrid system capture the air traffic procedures for conflict resolution, e. g., speed and turn advisories. In order to solve problems of realistic dimension arising from air traffic sector planning, we formulate a numerically tractable approach to solve the hybrid optimal control problem. The approach is based on introducing binary functions for each mode, relaxing the binary functions and including a penalty term on relaxation. The transformed and discretized problem is a nonlinear program. We use the approach on a benchmark case study for resolving conflict while minimizing fuel consumption of 8 Airbus 320 aircraft in a symmetrical set up.**

I. Introduction

Air Traffic Management (ATM) is responsible for safe, efficient and sustainable operation in civil aviation. Since its birth in the 1920s, the ATM system has evolved from its primitive form that consisted of a set of simple operation rules to its current version that is a complex network of management layers, communication, navigation, and surveillance subsystems. A paradigm shift in the current ATM system is being pursued in order to address the continuous growth of air traffic demand, skyrocketing fuel prices and growing concerns over the environmental impact of air transportation [18], [9].

Currently, ATM imposes certain trajectory restrictions, such as flying through a rigid airway structure, in order to guarantee safety and ease the task of air traffic controllers. Some of these restrictions result in non-minimal fuel consumptions and hence higher operative costs and emissions. The future ATM is to be built around the so called Trajectory Based Optimization (TBO) operational concept, which would allow aircraft more freedom to optimize their trajectories according to user's business interests. An important problem in implementing the TBO concept is designing trajectories that at the same time are optimal with respect to a cost function, and safe in the presence of hazardous weather and other aircraft. Moreover, it has been acknowledged that in the future ATM the human will be in the loop, and thus trajectories should also be *cognitive-friendly*. This ensures higher levels of efficiency and safety via automation, yet acounting for human's trust and acceptance. In this work, we propose an optimal control framework, consistent with operations of air traffic controllers, pilots and autopilots, for strategic (20 minutes to one hour) minimum fuel, deconflicted trajectory planning.

Aircraft conflict detection and resolution has been studied extensively. Please see [11] for an excellent survey. Some of the previous approaches include: Hybrid system reachability for designing provably safe maneuvers under worst-case trajectory prediction, for instance in [21]; conflict detection and resolution algorithms based on mixed integer linear program (MILP for short), for instance in [14], [1]; optimal control for minimum-time conflict free trajectories, for instance in [12]. In most previous work, due to the complexity of the problem, conflict resolution through speed, heading or altitude maneuvers are considered separately and an accurate fuel consumption model is not included.

We consider a hybrid optimal control modeling framework that can accurately capture aircraft dynamics [21], [5], [15]. In this framework, the discrete states represent the flight modes and operating procedures, modeled herein as different conflict resolution maneuvers. The continuous states describe the evolution of aircraft motion. Hence, the task of safe and optimal trajectory design can be formulated as an optimal control problem for a hybrid system subject to flight envelope and collision avoidance constraints.

Hybrid optimal control problems with a known mode sequence are referred to as multiphase optimal control problems in aerospace community. In [20], the multiphase optimal control has been formulated for optimizing trajectory of a single civilian aircraft, taking into account accurate nonlinear dynamics, which resulted in significantly reduced fuel consumption. The hierarchical hybrid optimal control approach of [6] was applied to single aircraft trajectory planning with accurate nonlinear dynamics and airspace constraints [10]. Recently, we addressed the fuel optimal conflict free trajectory planning as a multiphase mixed-integer optimal control problem (MIOCP for short) [16],[4]. Through discretization, we transcribed the MIOCP as a mixed-integer nonlinear programming problem (MINLP for short) [7], [3] and solved this via Branch & Bound techniques. This approach has two limitations which we will address in this paper. First, both the number of phases need to be fixed *a priori*; in other words, the number of advisories must be preset. Second, the MINLP becomes quickly intractable as the number of binary variables increases, i.e., the number of aircraft is very limited.

Motivated by realistic air traffic scenarios, we extend the previous work as follows: First, we formulate aircraft trajectory planning as a hybrid optimal control problem with two modes that account for speed and turn advisories.

M. Soler is with the Department of Bioengineering and Aerospace Engineering, U. Carlos III, Madrid, Spain, {masolera@ing.uc3m.es}.
M. Kamgarpour and J. Lygeros are with the Automatic Control Laboratory, Department of Electrical Engineering and Computer Sciences, ETH Zürich, {mkamgar,lygeros@control.ee.ethz.ch}.

The number of phases (advisories), mode sequence (type of advisories) and switching times (instant of advisory) are unknown. Second, we cast the hybrid optimal control problem into a classical optimal control problem through a relaxation of integer constraints and a penalty term on the relaxation. Third, we apply our formulation and solution approach to a benchmark case study with 8 aircraft in which we find minimum fuel, de-conflicted trajectories in a cognitive-friendly manner, i.e., fulfilling with ATC advisories.

This work is organized as follows: In Section II we define the hybrid optimal control problem and describe how it models the aircraft trajectory planning problem. In Section III we describe our solution method. In Section IV we formulate the aircraft fuel optimization as a constrained hybrid optimal control problem. In Section V we present the benchmark case study. In Section VI we summarize and discuss future work.

II. Hybrid Optimal Control

Aircraft flight dynamics can be described by a switched dynamical system, that is, a dynamical system with multiple modes of operations. These modes might denote for instance climb/descend maneuvers, acceleration/deceleration maneuvers, and turn maneuvers. Each flight mode is characterized by a different set of dynamic equations and constraints. The objective in optimization based trajectory planning is to find feasible aircraft trajectory, while minimizing a desired cost function such as fuel or time of flight. In the following paragraphs, we set up the mathematical framework for addressing this problem.

We consider a switched system described by a set of differential equations

$$\dot{x}(t) = f_q(x(t), u(t)),\ q \in Q := \{1, 2, \ldots, n_q\}, \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$ represents the continuous states, $f_q : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$, and n_q represents the number of discrete modes. The input $u(t)$ is in a compact set $U \subset \mathbb{R}^{n_u}$. For aircraft flight, x denotes the dynamic states of the aircraft and u denotes the control variables.

A switching sequence σ is defined as the timed sequence of active dynamical systems, or modes, as follows:

$$\sigma = [(t_0, q_0), (t_1, q_1), \ldots, (t_N, q_N)], \quad (2)$$

where $N + 1$ represents the number of phases, $t_0 \leq t_1 \leq \cdots \leq t_N \leq t_{N+1}$ are the switching times, and $q_i \in Q$ for $i = 0, 1, \ldots, N$. The pair (t_i, q_i) for $1 \leq i \leq N$ indicates that at time t_i the dynamics change from mode q_{i-1} to q_i. Thus, in the time interval $[t_i, t_{i+1})$, referred to as the i-th phase, the state evolution is governed by the vector field f_{q_i}. As an illustration, an aircraft might be flying at constant speed and constant heading and then switch at t_1 to an accelerating/decelerating mode. The switch would correspond to a speed advisory.

The states and inputs must fulfill constraints for each mode $q \in Q$, compactly represented as

$$h_q(x(t), u(t)) \leq 0, \quad (3)$$

where in the above $h : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_h}$. These constraints are used to capture the flight envelope, operational constraints, collision avoidance constraints, or hazardous weather avoidance requirements.

The hybrid optimal control problem is as follows: Given an initial condition $x(0)$, find a switching sequence σ and an input $u : [0, t_f] \to U$, that fulfill dynamics (1), constraints (3) and minimize a cost function $J(\sigma, u)$. That is, solve the following hybrid optimal control problem:

$$\begin{aligned}
\min_{\sigma,u} \quad & J(\sigma, u) := \phi(x(t_f)) + \textstyle\sum_{i=0}^{N} \int_{t_i}^{t_{i+1}} L_{q_i}(x(t), u(t))dt \\
\text{s.t.} \quad & \dot{x}(t) = f_{q_i}(x(t), u(t)),\ i \in [t_i, t_{i+1}], \\
& h_{q_i}(x(t), u(t)) \leq 0,\ i \in [t_i, t_{i+1}],\ i = 0, \ldots, N.
\end{aligned}$$

In the above, ϕ is referred to as the Mayer term, denoting a final cost, and the integral term in J is referred to as Lagrange term, denoting a running cost. The final cost can be used to quantify the deviation from a desired final state (reach a waypoint, reach destination at a given time), while the running cost can denote costs accumulated during the flight such as fuel consumption. The initial time t_0 is given while the final time $t_{N+1} := t_f$ is an optimization variable. For well-defined problem, we assume $\forall q \in Q$ the functions f_q, h_q, ϕ, and L_q are Lipschitz and differentiable and their derivatives are also Lipschitz in their arguments.

The hybrid optimal control problem defined above is challenging for two reasons. First, the unknown number of modes, switching sequence and switching times result in a non-classical optimal control problem. Second, in realistic air traffic scenarios, multiple aircraft are involved and are coupled through collision avoidance constraints. Thus, the states, inputs and constraints are of high dimension. Moreover, the dynamics and constraints are in general non-convex.

III. Solution Approach

We describe how to obtain a numerical solution to the formulated hybrid optimal control problem. The Maximum Principle and the Dynamic Programming approach in theory allow for consideration of the switching sequence through defining an extended input function $\bar{u} = (u, \alpha) : [0, t_f] \to U \times Q$, where the integer-valued α would capture the mode sequence and switching times. The difficulty in using these methods lies in the high dimensionality of state and constraints for realistic aircraft planning. For numerical tractability, our goal is to cast the hybrid optimal control problem as a nonlinear optimization program (NLP) and use existing advanced solvers for NLP to obtain a solution.

Our approach is summarized as follows: First, we introduce binary control functions for each mode to formulate the hybrid optimal control problem as a MIOCP [17]. Next, we relax the binary functions. In contrast to [17] in which bounds on the relaxed solution were derived, we include a penalty function on the relaxation to obtain a classical optimal control problem whose solution could approach that of the original problem as the penalty weight increases. Finally, we apply a collocation discretization rule [8] to convert the equations of the system into constraints and thus formulate an NLP. We now describe each step.

A. *Formulation as* MIOCP

To determine the mode sequence, we introduce a binary function $w_q : [0, t_f] \to \{0, 1\}$ for each mode $q = 1, \ldots, n_q$. Let $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \{0,1\}^{n_q}$ be defined as $f = \sum_{q=1}^{n_q} w_q f_q$. By adding the constraint $\|w(t)\|_1 = \sum_{q=1}^{n_q} w_q(t) = 1$, we ensure there is one active mode at each time t and so the dynamical system is well-defined. Similarly, define $h = \sum_{q=1}^{n_q} w_q h_q$ and $L = \sum_{q=1}^{n_q} w_q L_q$. One can define the switching sequence σ, based on $w(t) = [w_1, \ldots, w_{n_q}]$. For example, suppose $w_i(t^-) = 1$ for $q = i$ and $w_j(t^+) = 1$ for $q = j$. Then, a switch from mode i to j occurs at time t. Again, as an illustration, this switch would represent an advisory (if mode j would be for instance an accelerating/decelerating mode, it would be a speed advisory).

The hybrid optimal control problem with the new control variables, dynamics, constraints and cost function can be written as a Mixed Integer Optimal Control Problem MIOCP:

$$\begin{aligned} \min_{w,u} \quad & J(w,u) = \phi(x(t_f)) + \textstyle\int_{t_0}^{t_f} L(x(t),u(t),w(t))dt \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t),u(t),w(t)), \\ & h(x(t),u(t),w(t)) \leq 0, \\ & w_q(t) \in \{0,1\},\ q = 1,\ldots,n_q \\ & \textstyle\sum_{q=1}^{n_q} w_q(t) = 1,\ \forall t \in [t_0, t_f]. \end{aligned} \tag{MIOCP}$$

One could solve the above problem by discretizing the dynamics to formulate a MINLP as in [19]. This results in n_q binary decision variables for each discretization step and thus, the problem becomes intractable for more than a few modes.

B. *Relaxation of binary constraints through a penalty term*

First, we relax $w_q(t)$ by requiring it to belong to $[0, 1]$ instead of $\{0, 1\}$. Then, we define $\beta_q : [t_0, t_f] \to [-1, 1]$ for $q = 1, \ldots, n_q$, as a vector of auxiliary optimization variables, with $\beta_q(t) = 2w_q(t) - 1$. We define a penalty cost as $\beta_q \mapsto l(|\beta_q|)$, where $l(1) = 1$ and $l(|x|) \to \infty$ as $|x| \to 0$ for all $x \in [-1, 1]$. Here, we consider $l(x) = \frac{C}{x^d}$, where $C \in \mathbb{R}_+$ and $d \in \{\mathbb{N}_{+even}\}$ are constant design parameters. With the relaxation and the penalty term, we formulate a classical optimal control problem as:

$$\begin{aligned} \min_{w,\beta,u} \quad & J(w,\beta,u) = \phi(x(t_f)) \\ & + \textstyle\int_{t_0}^{t_f} \Big(L(x(t),u(t),w(t)) + C \sum_{q=1}^{n_q} \frac{1}{|\beta_q(t)|^d}\Big)dt \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t),u(t),w(t)), \\ & h(x(t),u(t),w(t)) \leq 0, \\ & \beta_q(t) \in [-1,1],\ q = 1,\ldots,n_q, \\ & w_q(t) = \tfrac{1}{2}(1 - \beta_q(t)),\ q = 1,\ldots,n_q \\ & \textstyle\sum_{q=1}^{n_q} w_q(t) = 1,\ \forall t \in [0, t_f]. \end{aligned} \tag{R-MIOCP}$$

The control variables in the transformed problem are the input $u(t)$, the auxiliary inputs $\beta_q(t)$, the switching law $w_q(t)$ for $t \in [t_0, t_f]$, $q = 1, \ldots, n_q$ and the final time t_f. While integer constraints are not explicitly added, in practice, the penalty term ensures that for sufficiently large d, the optimized solution would approach $|\beta_q(t)| = 1$ and consequently, $w_q(t) \in \{0, 1\}$. That is, the relaxed solution becomes a feasible solution for the original problem.

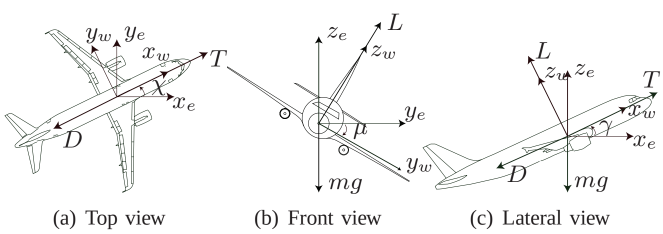
The above formulation is a standard constrained optimal control problem. Due to non-convexity of dynamics and constraints and high dimensions of state, inputs and constraints, we resort to numerical solution via discretization. With a time discretization of dynamics, for example through Trapezoidal, or Simpson rule, the optimal control problem can be converted to a nonlinear program (NLP for short). This method is called direct collocation [8]. It has been widely used for solving optimal control problems in aircraft and aerospace applications due to its computational efficiency [2], [19]. Let the number of discretization steps be denoted by n_s The number of optimization variables would be $n_s(n_u + 2 \times n_q)$. The number of constraints would be $n_s(n_x + n_h + 2 \times (n_q + 1))$. Thus, the optimization variables increase linearly as the dimension of states, input, constraints or modes increase.

In theory, there can be a switching at each discretization time step, although we did not observe many switches in practice as will be later shown in our case study section. To avoid too many switches, one could include a penalty term in the cost function which would be proportional to $|\beta(k) - \beta(k-1)|$, where k denotes a discretization step.

IV. AIRCRAFT FUEL OPTIMIZATION AS A HYBRID OPTIMAL CONTROL PROBLEM

A variable-mass 3 degree of freedom aircraft model with parameters based on BADA 3.6 [13] has been considered. The equations of motion over a spherical flat-earth are:

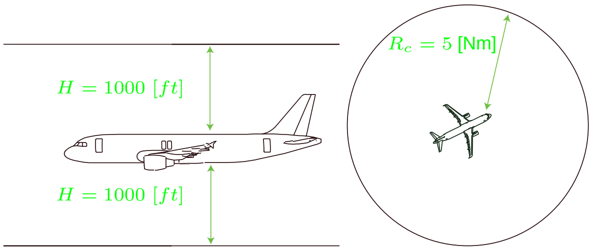
$$\frac{d}{dt}\begin{bmatrix} V \\ \chi \\ \gamma \\ \lambda \\ \theta \\ h_e \\ m \end{bmatrix} = \begin{bmatrix} \frac{T(t) - D(h_e(t),V(t),C_L(t)) - m(t)\cdot g\cdot \sin\gamma(t)}{m(t)} \\ \frac{L(h_e(t),V(t),C_L(t))\cdot\sin\mu(t)}{m(t)\cdot V(t)\cdot\cos\gamma(t)} \\ \frac{L(h_e(t),V(t),C_L(t))\cdot\cos\mu(t) - m(t)\cdot g\cdot\cos\gamma(t)}{m(t)\cdot V(t)} \\ \frac{V(t)\cdot\cos\gamma(t)\cdot\cos\chi(t)}{R\cdot\cos\theta(t)} \\ \frac{V(t)\cdot\cos\gamma(t)\cdot\sin\chi(t)}{R} \\ V(t)\cdot\sin\gamma(t) \\ -T(t)\cdot\eta(V(t)) \end{bmatrix}. \tag{4}$$



(a) Top view (b) Front view (c) Lateral view

Fig. 1. Aircraft state and forces

The states are: the 3D position (longitude, latitude, altitude), i.e, λ, θ, and h_e, respectively; the true airspeed, V; the heading angle, χ; the flight path angle, γ; and the aircraft mass, m. The bank angle μ, the engine thrust T, and the coefficient of lift C_L are the control inputs, that is, $u(t) = (T(t), \mu(t), C_L(t))$. Lift $L = C_L S\hat{q}$ and drag $D = C_D S\hat{q}$ are the components of the aerodynamic force, with S being



(a) Vertical Separation (b) Horizontal Separation

Fig. 2. Minimum required separation.

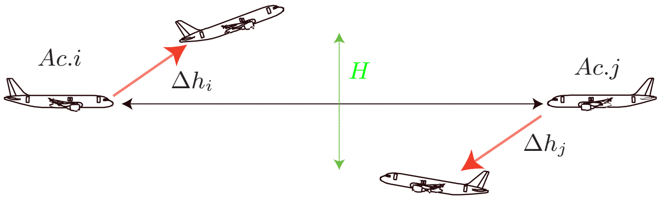


Fig. 3. Vertical advisories through climb/descend maneuver.

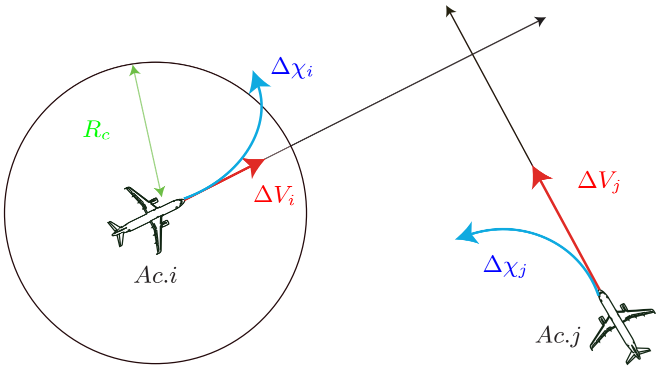


Fig. 4. Horizontal advisories through a turn maneuver.

the reference wing surface area and $\hat{q} = \frac{1}{2}\rho V^2$ the dynamic pressure. Parabolic drag polar, $C_D = C_{D0} + KC_L^2$, and International Standard Atmosphere (ISA) are considered. The aircraft 2D position is approximated as $x_e = \lambda(R+h_e)\cos\theta$ and $y_e = (R+h_e)\theta$, with R being the radius of Earth. η represent the specific fuel flow, which is a function of the airspeed.

A. Constraints

1) Flight Envelop: Flight envelop constraints reflect the physical limits of the aircraft due to, for instance, structural limitations, engine power, and aerodynamic characteristics. There might also be constraints due to operational limits, such as the maximum operational altitude. BADA performance limitations model and parameters [13] are used:

$$\begin{array}{ll} 0 \le h(t) \le \min[h_{M0}, h_u(t)], & \gamma_{min} \le \gamma(t) \le \gamma_{max}, \\ M(t) \le M_{M0}, & m_{min} \le m(t) \le m_{max}, \\ \dot{V}(t) \le \bar{a}_l, & C_v V_s(t) \le V(t) \le V_{Mo}, \\ \dot{\gamma}(t)V(t) \le \bar{a}_n, & 0 \le C_L(t) \le C_{L_{max}}, \\ T_{min}(t) \le T(t) \le T_{max}(t), & \mu(t) \le \bar{\mu}. \end{array}$$

Above, h_{M0} denotes the theoretical ceiling of the aircraft and $h_u(t)$ denotes the maximum operational altitude (it is a function of temperature and mass); M_{M_0} denotes the maximum operating Mach number (to operate within a subsonic regime), being $M(t)$ the Mach number; $V_s(t)$ denotes the stall speed (C_v is a safety coefficient) and V_{M_0} denotes the maximum operating calibrated (CAS) airspeed; $\bar{a}_n$ and $\bar{a}_l$ denote the maximum normal and longitudinal accelerations, respectively; T_{min} and T_{max} denote the minimum and maximum available thrust, respectively. $\bar{\mu}$ denotes the maximum bank angle due to structural limitations.

2) Aircraft Collision Constraint: Two aircrafts are required to be separated by a distance of R_c nautical miles in the horizontal plane or H feet in altitude, as shown in Fig. 2. Let (x^i, y^i, h^i) denotes the Cartesian position of aircraft $i = 1, 2$. The collision avoidance constraint is written as

$$\|[x^1, y^1] - [x^2, y^2]\|_2 \ge R_c \ \vee \ |h^1 - h^2| \ge H.$$

Note that both of the airspace constraints are nonconvex.

B. Flight Modes

In the en-route portion of flight, aircraft fly straight line segments connecting waypoints. To avoid conflict, the aircraft may be required to deviate from their nominal paths. In terms of air traffic control, these deviations are characterized by maneuvers which may consist of heading, speed, or altitude changes. In Figures 3 and 4 an example of such advisories for avoiding collision, consistent with current operations, are shown. We consider flight maneuvers as modes of the switched system. A 3D flight plan can be subdivided into a sequence of modes pertaining to flights in a vertical or horizontal plane. We characterize the maneuvers by three modes of *control speed* (mode 1), *control heading* (mode 2), and *control altitude* (mode 3). These maneuvers are routinely used in current air traffic control practice since they are easily communicated to the pilots and are easily implemented by autopilots [21].

1) Control speed: The aircraft flies with variable speed but constant heading. The bank angle μ is set to zero. The engine thrust is the input, that is, $u(t) = T(t)$. This mode is an accelerating/deceleration mode, when it gets activated a speed advisory occurs.

2) Control heading: The speed is held constant while the heading can change. The input is μ, that is $u(t) = \mu(t)$. This mode is a turning mode (at constant airspeed), when it gets activated a turn advisory occurs.

In above two modes γ, $\dot{\gamma}$, and $\dot{h}$ are set to zero. Thus, the following algebraic constraint is present: $L\cos\mu = mg$.

3) Control altitude: We consider vertical climb/descent, so that the bank angle μ is set to zero. Without loss of generality, we let $\chi = 0$. The thrust and the lift coefficient are the inputs, that is, $u(t) = (T(t), C_L(t))$. This mode is a climbing/descent mode, when it gets activated a vertical advisory occurs.

TABLE I

BOUNDARY VALUES.

	Initial values					Final values	
	m [kg]	V [m/s]	λ_e°	θ_e°	χ [deg]	λ_e°	θ_e°
Ac. 1	55000	220	6	46.5	0	10	46.5
Ac. 2	55000	220	10	46.5	180	6	46.5
Ac. 3	55000	220	8	44.5	90	8	48.5
Ac. 4	55000	220	8	48.5	270	8	44.5
Ac. 5	55000	220	$8-2\cdot\sqrt{2}/2$	$46.5-2\cdot\sqrt{2}/2$	45	$8+2\cdot\sqrt{2}/2$	$46.5+2\cdot\sqrt{2}/2$
Ac. 6	55000	220	$8+2\cdot\sqrt{2}/2$	$46.5+2\cdot\sqrt{2}/2$	225	$8-2\cdot\sqrt{2}/2$	$46.5-2\cdot\sqrt{2}/2$
Ac. 7	55000	220	$8+2\cdot\sqrt{2}/2$	$46.5-2\cdot\sqrt{2}/2$	135	$8-2\cdot\sqrt{2}/2$	$46.5+2\cdot\sqrt{2}/2$
Ac. 8	55000	220	$8-2\cdot\sqrt{2}/2$	$46.5+2\cdot\sqrt{2}/2$	315	$8+2\cdot\sqrt{2}/2$	$46.5-2\cdot\sqrt{2}/2$

V. BENCHMARK CASE STUDY

A. Problem set up

A benchmark case study with 8 aircraft ($n_{air} = 8$) is presented to test the effectiveness of the approach. We restrict the problem to the horizontal plane at $h = 11000$ [m], allowing only horizontal advisories (speed and/or turn advisories). The benchmark is standard in conflict avoidance problems, configuring a symmetrical layout as illustrated in Figure 5.

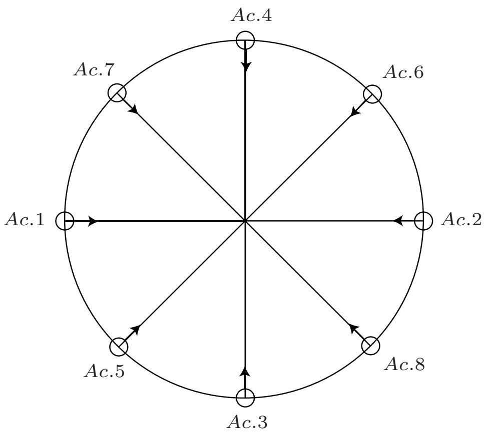


Fig. 5. Benchmark case study. The labeled points are initial position of the aircraft on the horizontal plane. The original flight plan of aircraft is straight-line flight.

The 8 aircraft are modeled as BADA 3.6 A320 [13]. Initial and final conditions are set to preserve the symmetry, their values can be found in Table I. The original flight along straight-line as indicated in this setup would result in 27 conflicts (measured independently by pairs).

The problem is thus formulated as a multiphase mixed-integer optimal control problem as described in Section II. Binary control functions $w_q(t)$ are relaxed to nonlinear variables and bounded between -1 and 1 as already described in Section III-B. A discretization grid with a total of $n_s = 50$ sample points per aircraft has been used ($\sum_1^8 n_s = 400$ in total). The penalty term in the objective function of Problem (R-MIOCP) is

$$C \sum_{q=1}^{n_q} \frac{1}{|\beta_q(t)|^d},$$

where for this problem we chose $C = 1$ and $d = 4$.

TABLE II

NUMERICAL RESULTS.

	Fuel consum. [kg]	Time [s]	num. advisories	num. conflicts
Ac. 1	748.9	1420.4	7	0
Ac. 2	763.8	1431.8	6	0
Ac. 3	1098.4	2076.1	5	0
Ac. 4	1105	2053.7	5	0
Ac. 5	972.7	1860	4	0
Ac. 6	966	1754.7	3	0
Ac. 7	948.2	1792.2	1	0
Ac. 8	966.7	1772.6	2	0

B. Numerical results and discussions

The resulting large-scale NLP problem, which had 4420 variables, 3208 equality constraints, and 4176 inequality constraints, has been solved using IPOPT [22]. The total computational time on a Mac OS X 2.56 GHz laptop with 4 GB RAM was 6426.7 [s]. It is important to notice that the initial guess should be a feasible solution, which in this case implies conflict free trajectories, in order for the program to converge.

The number of switchings (number of advisories), the corresponding sequence of active modes (sequence and type of advisories), and the control input for each mode are provided by the solution to the NLP. Table II shows the consumptions, flight times, number of advisories, and number of conflicts for the different aircraft. Notice that all conflicts have been eliminated due a to number of advisories per aircraft that ranges from 1 (Ac. 7) to 7 (Ac. 1). The number of advisories results from the number of switchings of the binary control functions $w(t)$, which are illustrated in Figure 7. In this particular case study, since there are two modes, a single binary variable $w(t) \in \{0, 1\}$ for each aircraft is sufficient to capture switching between the two modes. Here, $w(t) = 1$ corresponds to a control speed mode, and thus the switching between $w(t^*) = 0$ and $w(t^*) = 1$ at time t^* corresponds to a speed advisory. On the contrary, $w(t) = 0$ corresponds to a control heading mode, and thus the switching between $w(t^*) = 1$ and $w(t^*) = 0$ at time t^* corresponds to a turn advisory.

The resulting optimal paths are illustrated in Figure 6. Airspeed, heading angle, mass, bank angle, and thrust profiles are depicted in Figure 8. From these figures, one can observe high variability of path and airspeed deviations to avoid the conflicts. Note that without considering conflicts, while aircraft would fly different heading and position, all aircraft would fly the same trajectory in terms of speed,

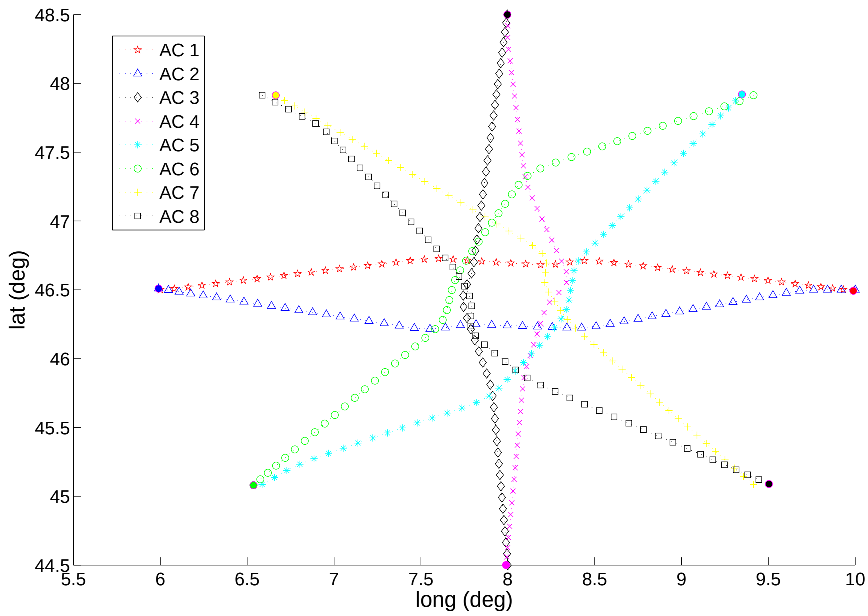


Fig. 6. Optimal paths.

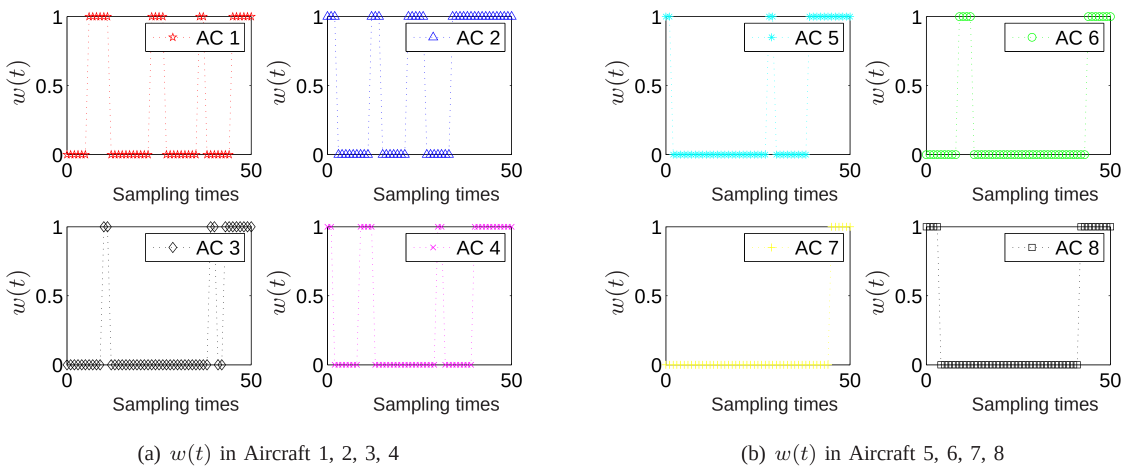


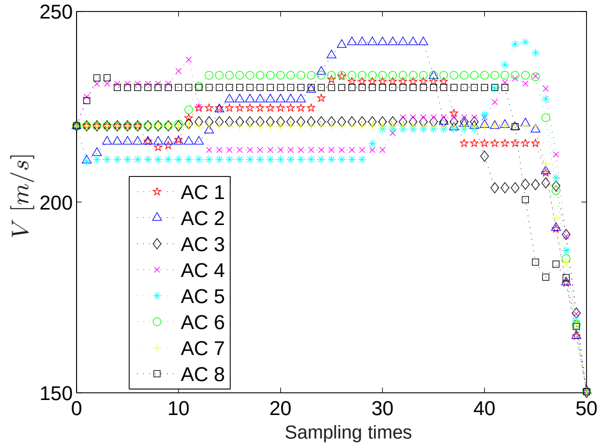
Fig. 7. Binary control functions $w(t)$ as a function of the number of samples. Note that $w(t)$ represent herein to the control speed mode.

consumption, thrust, bank angle, and time. Aircraft 1 and 2 present less deviations from their nominal paths and, even though their speeds are relatively high, as they are advised twice to speed up and then to slow down, their consumption is lower because of less distance flown. Ac. 3 and Ac. 4 have significantly higher fuel consumption and flight times. This is due to the fact that, specially at the beginning, they are enforced to fly at much higher speed that the other aircraft. Aircraft 3 maintains this relatively high speed for most of its flight path, whereas Ac. 4 is allowed to slow down but at the very end it is advised to speed up again. Similar behavior is observed for Ac. 5, Ac. 6, Ac. 7, and Ac. 8: their paths are in general longer to avoid conflicts. Overall, one can observe a series of cooperative strategies to meet the requirements of the problem and at the same time minimize fuel consumption.

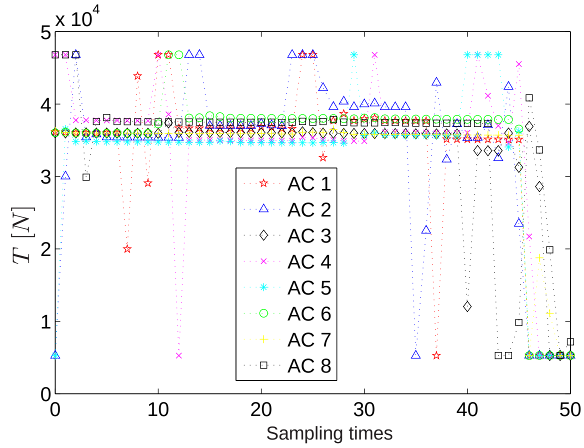
C. *Limitations and Future work*

The above presented work corresponds to a preliminary study that should be further explored and extended. Ongoing and future work includes:

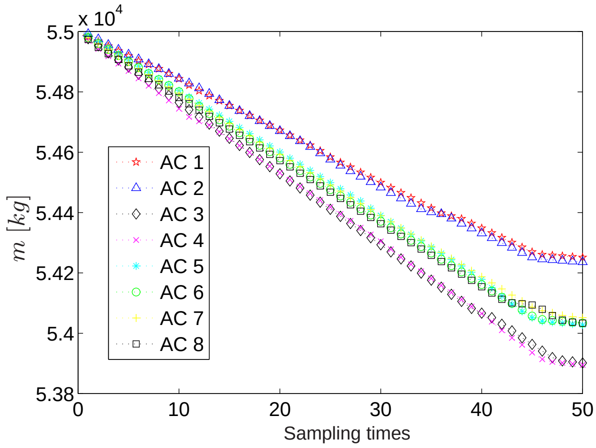
- Implement the algorithm in a real air traffic control (ATC) sector, considering first a single FL and then extending it considering a vertical structure.
- Introduce arrival time constraints at the exit waypoints in order not to disrupt the air traffic system downwards due to aircraft flying ahead/behind schedule.
- Include wind forecasts into aircraft dynamics and take into account the uncertainty in the forecast and its



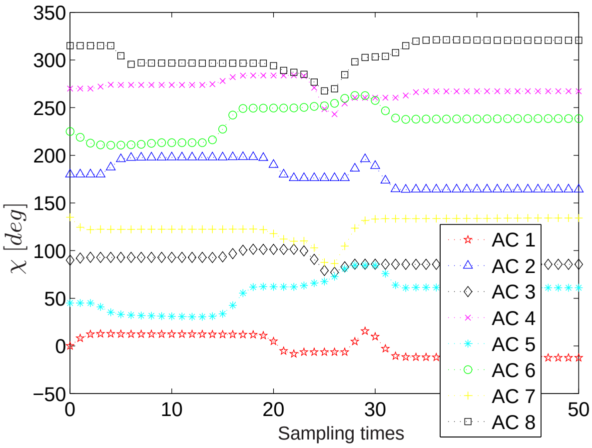
(a) $V(t)$



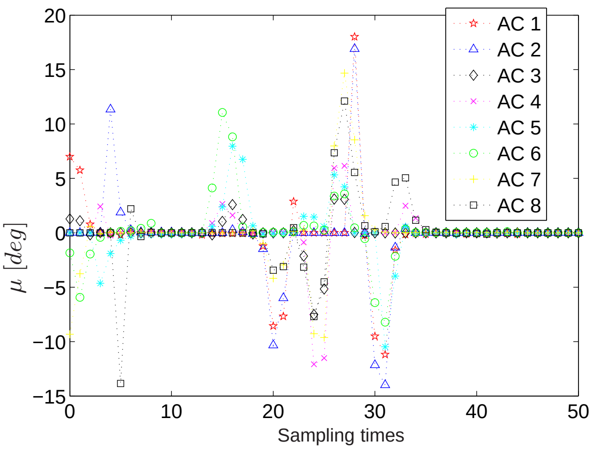
(b) $T(t)$



(c) $m(t)$



(d) $\chi(t)$



(e) $\mu(t)$

Fig. 8. State and control variables at sampling times

propagation along the trajectories.

- Drastic reduction of the computational time in order to be able to run the algorithm in real time.
- Impose constraints in the maximum number of advisories.
- Analyzing Pareto optimality of the solution to ensure fair advisories for each aircraft.
- Increase the number of samples in order to improve the conflict detection sensitivity of the algorithm.
- Comparison of fuel savings resulting from the algorithm with respect to current conflict resolution procedures.
- Trust and acceptance test by pilots and air traffic controllers.

Note that a real ATC sector, restricted to a single flight level, would never have such a density of aircraft and conflicts to be solved. Airways are structured so that the traffic flow follow certain patterns, e.g., West-East and South-North corridors in even flight levels; East-West and North-South corridors in odd flight levels, and thus conflicts are expected to occur at cross-junctions only by one aircraft pair. Also, the typical size of an ATC sector is roughly half of this scenario, which means we can either reduce the number of samples to improve the CPU time, yet maintain it to improve the accuracy of the solution. Proving optimality will be certainly tough, but running a comparison with current ATM would help in providing a measure of potential improvements in terms of fuel savings. Finally, for further development and real implementation (either as an on-board Flight Management System (FMS) trajectory planning, or as earth-based ATC decision support tool), human factors should be considered since human trust and acceptance must be granted.

VI. Conclusions

We formulated fuel optimal conflict free aircraft trajectory planning as a hybrid optimal control problem. This formulation allowed for inclusion of accurate aircraft dynamic models and conflict resolution maneuvers that are consistent with air traffic control procedures. We developed an algorithm for solving the hybrid optimal control problem through transforming it to a nonlinear program. Our approach was illustrated with a case study with accurate civilian aircraft model, and realistic number of aircraft.

This benchmark case study indicated that the optimal control algorithm proposed is numerically tractable to address trajectory planning in realistic scenarios. However, in order to have an online implementation, we are working on decreasing the computational time. In addition, currently, we are implementing the method for conflict avoidance based on data from an air traffic sector in Spain.

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