Optimization of ATM Scenarios Considering Overall and Single Costs

Matthias Bittner, Benjamin Fleischmann,
Maximilian Richter, Florian Holzapfel
Institute of Flight System Dynamics
Technische Universität München
Munich, Germany
m.bittner@tum.de

***Abstract*—Methods for the calculation of optimal trajectories for conflict free Air Traffic Management (ATM) scenarios are presented. In those scenarios multiple aircraft have to cross an airspace sector without violating the separation distance in an optimal way. The aircraft are modeled using two dimensional kinematic models, as this is sufficient for the motions regarded here. Optimal control theory and a direct collocation scheme as well as multi criteria optimization methods are used to solve the problems. The goal of the optimizations is twofold: on the one side the overall costs should be minimized and on the other side the fairness of the scenario should be maximized. In this context, fairness can be expressed as the distribution of costs between the different aircraft. The principle is demonstrated using two simulation examples.**

Keywords-ATM scenario optimization, conflict resolution, optimal control, multi-criteria optimization

I. INTRODUCTION

Several studies predict a further increase in air traffic in the skies above the overall world and especially above Europe. Even in the already lowered estimations of EUROCONTROL published in September 2013 [1] the number of IFR (instrument flight rules) flight movements in 2019 is expected to be 10.8 million which is 14% more than in 2012. In their long term forecast [2] a further growth is anticipated resulting in 40% more IFR flights in 2030 compared to the numbers for 2009. Besides this growth in air traffic, EUROCONTROL records more and more problems with the current air traffic management system. In the EUROCONTROL Performance Review Report 2010 [3], for example, it is stated that the number of en-route delays in 2010 was worse than any year before since 2001. This shows the big need for new technologies and tools for the air traffic management system supporting the controllers in their decision making.

Multiple aircraft are involved in every conflicting air traffic scenario. In most cases they are all operated by different airlines that are competing on the market. Thus, every airline is interested in not only a safe solution of the conflict but also a cheap solution for their aircraft. In some scenarios the resolution of a conflict may be almost fully paid by one aircraft, or at least a small portion of the aircraft involved, if the optimal solution for the overall costs is calculated. Therefore, in this work, air traffic scenarios are not only optimized for the overall costs but also for the individual costs of every aircraft and the fairness by means of cost distribution. The resulting problem can be formulated as a multi criteria optimization problem.

Various approaches to automatic conflict resolution have been suggested and investigated in the past. Zhao and Schultz [4], e.g., use optimal control methods to resolve conflicting trajectories for two aircraft. They minimize the deviations from the intended flight paths and finally enforce each aircraft back to their trajectory via terminal constraints. In [5] Frazzoli et.al. use semidefinite programming to resolve conflicted trajectories in the horizontal plane. In contrary to that, in [6] Hu et.al. consider three dimensional aircraft movement and present an algorithm to calculate optimal solutions for two-aircraft cases and an approximation for the multiple-aircraft case. In [7], once again three dimensional problems are tackled and methods from optimal control are used to calculate free-flight trajectories for multiple aircraft. Archibald et.al. in [8] describe a multiagent solution for solving conflicts based on satisficing game theory. In this theory a decision maker (in this case an aircraft) may sacrifice part of its achievement in case another decision maker can benefit from that. This way, the overall performance may be improved for the cost of several individuals. This problem formulation is close to the problem solved here but tackled quite differently. In [9] Visser describes a method of optimizing the trajectories for two aircraft in three dimensional space with respect to noise nuisance on ground. Vela et.al. in [10] present a conflict resolution algorithm that incorporates a controller work load model and is near-realtime capable. Once again, two dimensional scenarios are considered. In [11] Chaloulos et.al. present a decentralized model predictive control scheme (MPC) for hierarchical systems and use it to solve collision avoidance problems for unmanned aerial vehicles (UAVs). Besides the high level MPC approach, a low level controller is implemented that takes care of the internal dynamics of the aircraft. Here, a cooperative cost is used to ensure a certain level of fairness between individual and overall costs in the scenarios. The authors of [12] use a Mixed Integer Linear Program formulation to solve conflicts between arriving aircraft in a 4D approach scenario. Their objective is the minimization of the overall delay while keeping all inbound traffic conflict free. In [13] we calculated fuel minimal approach trajectories using a two stage method. In the first stage the optimal approaches for all considered aircraft have been calculated while neglecting eventually arising conflicts. Then, in the second stage, the solution of the first stage has been used as an initial guess for the fully constrained optimal control problem where the conflicts were resolved. The results of this work are partly based on the results that have been published at the AIAA conference 2014 [14] by our group.

The rest of this paper is organized as follows: In the next section the optimization problem to be solved will be stated including the simulation model used, the separation constraint, the combination of multiple aircraft into one problem as well as the regarded cost functions and a definition of fairness. In section III some methods for the solution of the resulting multi criteria optimization problem are presented. Afterwards, in section IV the numerical results created by these methods are described and interpreted. Finally, in V some conclusions are drawn and a short outlook is given.

II. Optimization Problem

The conflict resolution inside an ATM scenario is formulated as an aircraft trajectory optimization problem that belongs to the mathematical class of optimal control problems. Within the next subsections the considered problem statement is given.

A. General Multi Aircraft Trajectory Optimization Problem

In general, a trajectory optimization problem involving N aircraft can be stated as follows (derived from the formulation in [15]):

Determine the optimal control histories

$$\boldsymbol{u}_{i,opt}(t) \in \mathbb{R}^{m_i}, \qquad i = 1, \dots, N \tag{1}$$

and the corresponding optimal state histories

$$\boldsymbol{x}_{i,opt}(t) \in \mathbb{R}^{n_i}, \qquad i = 1, \dots, N \tag{2}$$

that minimize the Bolza cost functional

$$J = \Phi\left(\boldsymbol{x}_i(t_f), t_f\right) + \int_{t_0}^{t_f} \mathcal{L}(\boldsymbol{x}_i(t), \boldsymbol{u}_i(t), t)dt \tag{3}$$

subject to the state dynamics

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{f}_i(\boldsymbol{x}_i(t), \boldsymbol{u}_i(t), t_i), \qquad i = 1, \dots, N \tag{4}$$

the initial and final boundary conditions

$$\begin{aligned} \boldsymbol{\Psi}_0(x_i(t_0), t_0) &= \boldsymbol{0}, \quad \boldsymbol{\Psi_0} \in \mathbb{R}^p \\ \boldsymbol{\Psi}_f\left(x_i(t_f), t_f\right) &= \boldsymbol{0}, \quad \boldsymbol{\Psi_f} \in \mathbb{R}^q \end{aligned} \tag{5}$$

and the equality and inequality path constraints

$$\begin{aligned} &\boldsymbol{C}_{eq}(\boldsymbol{x_i}(t), \boldsymbol{u}_i(t), t) = \boldsymbol{0}, \\ &\qquad \boldsymbol{C}_{eq} \in \mathbb{R}^r, \quad i = 1, \dots, N \\ &\boldsymbol{C}_{in}(\boldsymbol{x}_i(t), \boldsymbol{u}_i(t), t) \le \boldsymbol{0}, \\ &\qquad \boldsymbol{C}_{in} \in \mathbb{R}^s, \quad i = 1, \dots, N \end{aligned} \tag{6}$$

Due to the complexity of the models and the constraints this problem cannot be solved by directly evaluating the optimality conditions (i.e. using indirect methods). So, a direct trapezoidal collocation scheme [15] is applied and the resulting parameter optimization problem is solved numerically using off-the-shelf software. For that purpose, besides SNOPT [16], which is mainly used in the examples, also IPOPT [17], WORHP [18] or others can be used. Afterwards, the results for the discretized problem are interpolated to get an estimate of the solution to the original problem.

B. Aircraft Simulation Model

When considering air traffic management scenarios the influences of the inherent aircraft dynamics are negligible as the considered maneuvers are relatively slow and far away from the aircraft's flight envelopes. Additionally, no wind influence is incorporated here. Thus, the aircraft dynamics can be modeled using pure kinematic relations. Moreover, the problems investigated here are only two-dimensional as they do not contain level changes. The formulation used would easily allow an extension to three dimensions, but this will lead to a significant increase in computational burden. Consequently, the position equations of motion in a locally fixed Navigation Frame for each aircraft $i = 1,2, \dots, N$ are given by (7), where N is the total number of aircraft considered. The coordinates x_i and y_i therein denote the position of the ith aircraft in the Navigation Frame.

$$\begin{aligned} \dot{x}_i &= V_{K,i} \cdot \cos \chi_{K,i} \\ \dot{y}_i &= V_{K,i} \cdot \sin \chi_{K,i} \end{aligned} \tag{7}$$

These models are controlled by their respective velocity $V_{K,i}$ and their course angle $\chi_{K,i}$. This leads to the following state and control vectors $\mathbf{X}_i$ and $\mathbf{U}_i$ for each aircraft:

$$\begin{aligned} \mathbf{X}_i &= [x_i, y_i]^{\mathrm{T}} \\ \mathbf{U}_i &= \left[V_{K,i}, \chi_{K,i}\right]^T \end{aligned} \tag{8}$$

C. Separation Constraint

In order to maintain safety of all involved aircraft a separation path constraint is added to the scenarios. This constraint has to be fulfilled along the whole trajectories of all aircraft. The minimum separation is given as

$$d_{min} = 5 \text{ NM} = 9260 \text{ m}. \tag{9}$$

As only the horizontal motion of aircraft is considered here, the distance between two aircraft can be calculated by

$$d = \sqrt{(\mathrm{x_i} - x_j)^{\,2} + (\mathrm{y_i} - y_j)^{\,2}} \tag{10}$$

In order to avoid the square root, the distance constraint can be reformulated and implemented as

$$d_{\min}^2 \le (\mathrm{x_i} - x_j)^{\,2} + (\mathrm{y}_i - y_j)^{\,2} \tag{11}$$

which is numerically more stable.

In case of a scenario containing N aircraft N_d pairs of distance constraints have to be fulfilled, where N_d is the number of possible aircraft combinations and evaluates to

$$N_d = \binom{N}{2} = \frac{N \cdot (N-1)}{2}. \tag{12}$$

D. Combining Multiple Aircraft in one Problem

Not all aircraft involved in an ATM scenario always remain inside the considered airspace for the same time. Some of them might enter earlier and also leave earlier while others remain in the considered airspace for a shorter or a longer time. To be able to compare the positions of all aircraft involved in the optimization scenario anyway, they all have to be simulated in one common time domain. Besides the rather inflexible idea of partitioning the whole scenario into multiple phases where in each phase another combination of aircraft is active, in [14] a method to fade-out the dynamics has been proposed and will be used here.

Therein, the aircraft dynamics are extended by two fading factors. One is used to activate the dynamics at the beginning of the trajectory based on time and the other one is used to fade out the dynamics at the final position based on the aircraft location. As in the examples shown here all aircraft start at the same point in time, only the fade-out part based on the position is required:

$$\dot{\mathbf{X}}_i = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \end{pmatrix} = \begin{pmatrix} V_{K,i} \cdot \cos \chi_{K,i} \\ V_{K,i} \cdot \sin \chi_{K,i} \end{pmatrix} \cdot \delta_{\mathrm{x}} \cdot \delta_{\mathrm{y}} \tag{13}$$

The two fading factors can be calculated according to

$$\delta_x = \frac{1}{2}\tanh\left(a \cdot |x - x_f|\right) + \frac{1}{2} \qquad (14)$$

$$\delta_y = \frac{1}{2}\tanh\left(a \cdot |y - y_f|\right) + \frac{1}{2} \qquad (15)$$

with x_f and y_f being the final position of the aircraft. Depending on the actual flight direction, one of the two factors has to be set to one. The parameter a used in (14) and (15) controls the steepness of fading, where values too high result in very high gradients which is numerically problematic and values too low result in a very long and unrealistic fading. In the examples considered here, a value of

$$a = 1{,}0799 \cdot 10^{-2} \cdot 1/m \qquad (16)$$

turned out to be a good compromise after testing values for $a \in [10^{-3}..1]$. It has to be mentioned that with this formulation the final boundary conditions cannot be given as equality constraints anymore but have to be specified as small ranges. This does not affect the solutions in practice.

E. Cost Functions and Fairness

For the sake of simplicity the costs generated by each flight are approximated by the flight time required to pass through the given sector. As no fast manoeuvers are performed, no height changes are considered and the velocities are strictly limited here, this assumption is reasonable. To be able to calculate the flight time even when considering the fading-out dynamics from above, the time t_i for each aircraft is added as an additional state:

$$\dot{\mathbf{X}}_i = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{pmatrix} = \begin{pmatrix} V_{K,i} \cdot \cos\chi_{K,i} \\ V_{K,i} \cdot \sin\chi_{K,i} \\ 1 \end{pmatrix} \cdot \delta_x \cdot \delta_y \qquad (17)$$

This way, it is ensured that the time an aircraft requires to pass through the sector also stops when its motion stops. Then, the overall cost of one scenario results as

$$t_{sum} = \sum_{i=1}^{N} t_{i,final} \qquad (18)$$

Here, other cost functions like operational cost, fuel consumption, emissions or a combination thereof can also be integrated into the simulation.

Within the scope of this work, fairness means a decent distribution of costs to all aircraft inside a scenario. As not all aircraft have the same distance to cross inside the considered sector, the absolute costs for crossing the sector are not a suitable means of comparison. Instead, a relative cost change is used for which the costs for each aircraft crossing the sector while neglecting any other aircraft are calculated as a reference. Then, the relative increases of costs for resolving the conflicts, compared to the reference, are calculated from

$$c_i = \frac{t_{i,final} - t_{i,final,min}}{t_{i,final,min}} \cdot 100\% \qquad (19)$$

where $t_{i,final}$ is the crossing time of the aircraft i in the scenario with resolved conflicts and $t_{i,final,min}$ is the minimum crossing time of the aircraft i if all other aircraft were neglected. A scenario is treated as fair if these relative increases of costs are similar. Considering numbers for the cost increases, see section IV.

Hence, the goal of the optimization is to minimize every single cost increase c_i. Thus, the problem can be interpreted as a multi criteria optimization problem with the cost function vector containing one entry for each of the N participants

$$\mathbf{J} = \begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix}. \qquad (20)$$

From a statistical point of view, the mean c_m and the biased variance c_{var} of these cost increases can be evaluated as

$$c_m = \frac{1}{N}\sum_{i=1}^{N} c_i \qquad (21)$$

and

$$c_{var} = \frac{1}{N}\sum_{i=1}^{N} (c_i - c_m)^2. \qquad (22)$$

The variance of the relative cost increases of all aircraft may be considered as an alternative means of fairness which leads to another multi criteria optimization problem with the cost function vector

$$\mathbf{J} = \begin{pmatrix} c_{sum} \\ c_{var} \end{pmatrix} = \begin{pmatrix} \sum c_i \\ \frac{1}{N}\sum (c_i - c_m)^2 \end{pmatrix}. \qquad (23)$$

In this formulation, the minimization of the sum c_{sum} is equivalent to the minimization of the mean c_m and the minimization of the variance c_{var} is equivalent to the minimization of the standard deviation c_{std}.

III. Multi Criteria Optimization Methods

A lot of techniques exist for the solution of multi criteria optimization problems. Kaya, e.g., gives an overview of some methods applied to optimal control problems in [19]. Therein, different scalarization schemes are presented and compared.

In [20] Lin categorizes multi objective optimization problems into min-norm and min-max problems and discusses several solution strategies for both types. Miettinen [21] also gives an overview over several methods but classifies them by the way the decision maker is involved in the process. He mainly focuses on interactive methods, meaning that the decision maker actively influences the optimization process during runtime.

Here, the problem is viewed in two different ways as stated in (20) and (23). For those two formulations different solution algorithms are applied and therefore presented below.

A. Minimization of a weighted sum of costs

An obvious choice when trying to optimize the presented ATM scenarios is a weighted sum of the single costs:

$$J_{Sum} = \sum_{i=1}^{N} w_i \cdot c_i \qquad (24)$$

When considering the relative cost increases for the different aircraft, all weights can be chosen as

$$w_i = 1 \qquad \forall i \in 1..N \qquad (25)$$

as they are comparable anyway. As can be clearly seen, this optimization will result in the minimum overall costs without considering fairness. From a multi criteria optimization problem point of view, fixing the weights will result in one point in the approximation of the Pareto front.

B. Minimization of a p-Norm

To increase fairness more weight can be put on bigger cost increases and less weight on smaller ones by using a p-Norm

$$\|c\|_p = \left(\sum_{i=1}^{N} |c_i|^p\right)^{1/p} \quad , \qquad p > 1 \tag{26}$$

As the scenario with the conflicts is more constraint than the reference scenario, the cost functions can only be equal to or greater than the reference costs. Thus, all cost changes are positive:

$$c_i \geq 0 \quad , \quad i = 1, \ldots, N \tag{27}$$

With (27), (26) becomes

$$\|c\|_p = \left(\sum_{i=1}^{N} c_i^p\right)^{1/p} . \tag{28}$$

The optimization of a p-Norm within the context of multi criteria optimization has also been studied in [20] and [21].

C. Minimum Distance to Target Optimization

Considering the problem formulation in (20) and including the wish to have a result that is as fair as possible, a minimum target optimization, as it is e.g. discussed in [20], can be used. In contrary to the previous formulations where the cost function values are directly minimized, now the square of the deviation from a user defined target cost is used as the criterion. With c_T being this target cost, the optimization criterion becomes

$$J_T = \sum_{i=1}^{N} (c_i - c_T)^2. \tag{29}$$

To be able to perform a minimum target optimization, a value for c_T is required, whose estimation is quite challenging for ATM scenarios. Thus, an optimization minimizing the overall costs is performed here, before the minimum optimal cost value (regarding one aircraft) is scaled by k_T and used as target:

$$c_T = \min c_i \cdot k_T \tag{30}$$

The factor k_T can be tuned to shift the weight between overall costs and fairness inside a scenario and therefore can be used to estimate points close to the Pareto front of the problem – at least when chosen arbitrarily. It has to be mentioned that in this formulation, deviations from the target value to lower cost values are penalized equally as deviations to higher cost values.

D. Min-Max Optimization

Another way of considering the distribution of the individual cost values is by minimizing the maximum thereof. This scalarization can be derived from the p-norm approach by making p infinitely large:

$$\|c\|_\infty = \lim_{p\to\infty} \left(\sum_{i=1}^{N} c_i^p\right)^{1/p} = \max_i(c_i) \tag{31}$$

The resulting formulation is close to the Tchebychev scalarization that is suggested for multi criteria optimal control problems in [19], but with all weights set to one. In the implementation of this scheme an additional parameter c_{max} is introduced and minimized while a supplementary constraint ensures that all individual costs remain below this parameter. A problem that will arise when using this solution strategy is the fact that only the maximum cost function is minimized. All other cost functions may stay above their respective optimal values (but below the maximum cost function value, of course) and will not be further pushed down by the optimization algorithm. To tackle this problem the min-max approach can be extended to a limited minimum sum optimization that is presented in the next section.

E. Limited Minimum Sum Optimization

When using the minimum sum optimization, the overall costs become minimal without considering the fairness between the different participants. When using the min-max optimization the opposite holds. In this approach the aforementioned techniques are combined. First, a min-max optimization is performed, before the resulting maximum individual cost is used as a limit for all individual costs in the problem:

$$c_i \leq c_{\max} \quad , \quad i = 1, \ldots, N. \tag{32}$$

Afterwards, a minimization of the overall costs is performed to lower the costs for all participants that are not at their optimum yet. This method can be further extended by tightening the upper limit for any individual cost based on a parameter k_c:

$$c_j \leq k_c \cdot \max c_i, \quad k_c \geq 1, \qquad \forall i, j \in .1..N \tag{33}$$

By tuning k_c, a reduction in the overall costs might be achieved and the weighting of the overall costs to fairness may be shifted. Consequently, for different values of k_c an approximation of a Pareto front can be calculated.

F. Minimization of Mean and / or Variance

When considering the second formulation of the problem as stated in (23) another weighted sum formulation can be derived:

$$\begin{aligned} J_{mv} &= w_m \cdot \left(\frac{J_{Sum}}{N}\right)^2 + w_{var} \cdot J_{var} \\ &= w_m \cdot \bar{c}^2 + w_{var} \cdot \frac{1}{N} \sum_{i=1}^{N} (c_i - \bar{c})^2. \end{aligned} \tag{34}$$

Here, w_m and w_{var} are the weights for the two terms of the sum and have to be chosen arbitrarily. To be able to achieve a meaningful comparison between both values, the mean has been squared – as the variance also is of second order.

One possibility to choose the weights is setting one of them to zero and the other to one. When doing so, either a minimization of the mean, which is equal to the minimization of the sum presented before, or a minimization of the variance results. The latter has also been implemented in the test scenarios. To ensure the quality of the solution by means of overall costs, an additional condition has been added to the calculations that limits the sum and therefore also the mean of the costs

$$c_{sum} \leq c_{sum,min} \cdot k_T \qquad k_T \geq 1 \tag{35}$$

where $c_{sum,min}$ is calculated by minimizing the overall costs. This condition ensures a maximum in the summed cost and therefore also a maximum in the mean. Different values for the scaling k_T have been used in the examples leading to different optimal points.

Another possibility for the weights would be to set both to one, which leads to an equivalent weighting of the mean and the variance but in some cases might not be numerically favorable. Thus, the weights might also be calculated based on a previously run minimization of the mean (indicated by the asterisk *). Then both parts of the sum can be weighted such that they lie within the same numerical range by choosing

$$w_m = \frac{1}{(\bar{c}^*)^2} = \frac{1}{\left(\frac{1}{N}\sum_{i=1}^{N} c_i^*\right)^2} \tag{36}$$

and

$$w_{var} = \frac{1}{var(c_i^*)^2} = \frac{1}{\left(\frac{1}{N}\sum_{i=1}^{N}(c_i^* - \bar{c}^*)^2\right)^2}. \tag{37}$$

The calculations based on the two latter formulations only lead to one point each on the approximated Pareto front. Anyway, choosing different combinations of w_m and w_{var} makes it possible to calculate multiple points.

IV. Numerical Results

In this section, the presented models are used to solve two example scenarios and the results are shown and interpreted. As mentioned before, all scenarios are discretized using a trapezoidal collocation scheme while the numerical optimization is performed using SNOPT or WORHP. All problems have been normalized (scaled) and solved to a feasibility and optimality tolerance of 10^{-6}. As the solutions to the optimal control problems have been calculated, the overall state and control histories are known. Anyway, they are mostly not shown here in detail as they do not contain valuable information for the analysis.

It has to be mentioned that the increases due to the detours seem to be very small. Anyway, it is important to see them in relation to each other as these relations are a measure of fairness.

A. Scenarios and cost minimal results (unfair)

In these subsections the scenarios are introduced and the results for the minimization of the overall costs are shown.

1) Scenario 1

The first scenario consists of four aircraft that are positioned pairwise opposite to each other. The setup can be seen in Figure 1. The aircraft starting points are marked by triangles, their destinations by crosses. The intersection of the trajectories is located at the origin of the locally fixed frame; the starting positions are 40 NM and 45 NM out and the final positions are 60 NM and 65 NM out. All boundary points are either located on the x-axis or on the y-axis, as can be seen in the figure.

Additionally, Figure 1 shows the solution for this scenario when minimizing the overall costs. It can clearly be seen, that aircraft 3 and 4 have to fly a noticeable detour while aircraft 1 and 2 are hardly affected by the solution of the conflict. Hence, in this scenario the cost minimal solution is not desirable for all parties. The numerical results for the relative increases in the flight time – compared to the conflict free scenario – and the total time for each aircraft can be seen in Table 1. The cost increases have a mean of $c_m = 0.465$ % and a variance of $c_{var} = 0.115$ % resulting in a standard deviation of $c_{std} = 0.339$ %.

TABLE I. Cost function values for scenario 1

Aircraft Number	Total Flight time	Cost increase
1	1066 s	0.170 %
2	1172 s	0.106 %
3	1074 s	0.917 %
4	1178 s	0.668 %

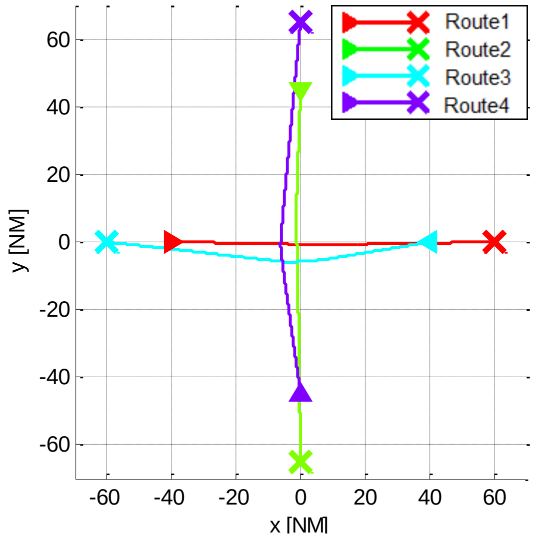

Figure 1 Cost minimal solution for scenario 1

2) Scenario 2

Three aircraft are involved in the second scenario. They all start at an x-position of $x_0 = -50\ NM$. Their y-positions are spread regularly in 10 NM steps with the middle aircraft being positioned on the x-axis. They are all starting at the same time and need to mirror their y-positions until the end of the sector that is located at $x_{final} = 50\ NM$ (see Figure 2).

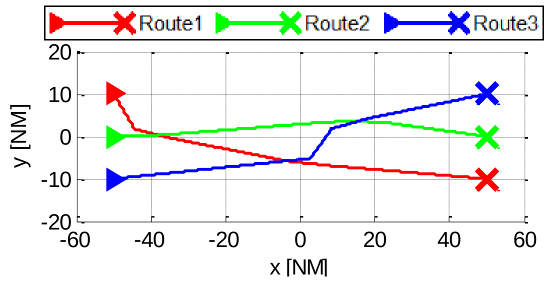

Figure 2 Cost minimal solution for scenario 2

Once again, the solution minimizing the overall costs is shown in Figure 2. In this scenario the relative increase of costs for aircraft 1 is $c_1 = 3.724$ % the relative increase for aircraft 2 is $c_2 = 0.452$ % and the increase for aircraft 3 also is $c_3 = 3.724$ %. Consequently, the mean in this scenario is $c_m = 2.627$ % while the variance is $c_{var} = 2.362$ % and the standard deviation is $c_{std} = 1.537$ % which is quite high.

B. Multi criteria optimization results (fair and cost efficient)

As the pure cost minimal results have been shown, now, the results of the different approaches for a fair optimization of the scenarios – always also minimizing the overall costs – are presented. For means of comparison, the results for mean and variance are shown here, although they are not the real criterion for Pareto optimal points in the first problem formulation (20).

1) Scenario 1

When using the p-norm approach with $p = 2$ the mean of the costs in the first scenario becomes $c_m = 0.468$ % while the variance is $c_{var} = 0.327$ %. As can be seen from Figure 3, the results are very similar to the ones created by a minimization of the sum. The same holds for the optimization of the mean and the variance and a weighting of $w_m = w_{var} = 1$. The results created by the minimization of the maximum costs, which in this case is equal to the limited minimum sum optimization if $k_c = 1$, tend to be better in fairness and a little worse in the overall costs. The results for the minimum target optimization as well as the results for the minimization of the variance strongly depend on the chosen parameters. The results for some selected parameters are shown in Figure 4 and Figure 5.

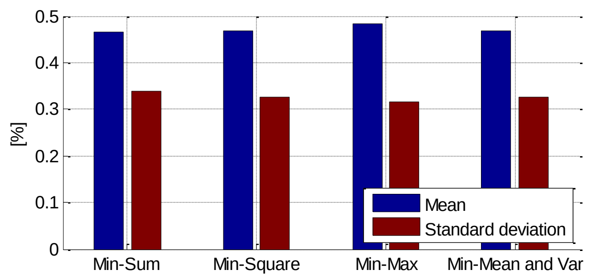

Figure 3 Results in mean and standard deviation for scenario 1.

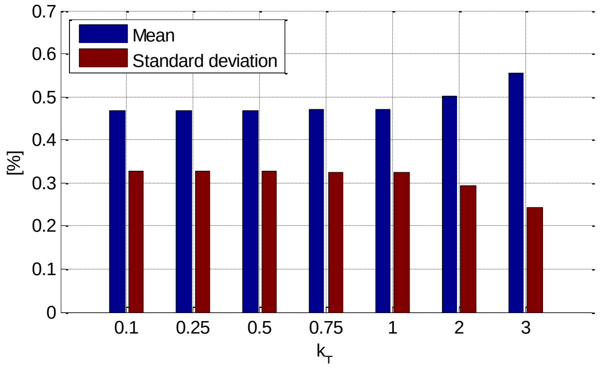

Figure 4 Optimal mean and standard deviation for different parameters k_T in scenario 1 using the minimum target optimization.

When plotting the mean cost increases over the cost increase standard deviations for the different optimization methods, an approximation of the Pareto front of the problem may be drawn. Figure 6 shows the results for the different methods in different colors. It can clearly be seen that not one optimal solution in mean and standard deviation exists but that by improving the one the other gets worse. This behavior is as expected in a multi criteria optimization problem. In this example a preferable point may be identified at the very left of the diagram as there the improvement in the standard deviation would be far greater than the increase of the mean.

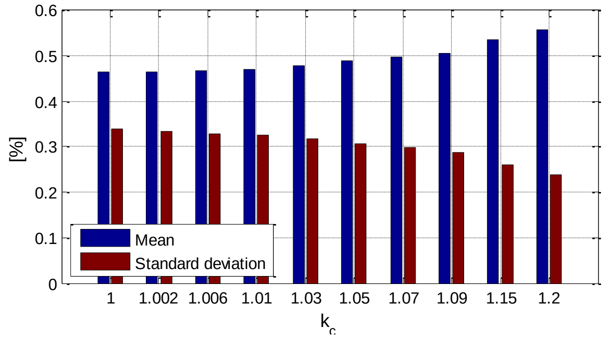

Figure 5 Optimal mean and standard deviation for different parameters k_c in scenario 1 using the minimum variance optimization.

2) Scenario 2

The results for scenario 2 are quite different from those of scenario 1 as the problem is far more interconnected and the cost increases are far higher. The minimum mean in the costs can again be achieved by minimizing the overall costs (of course) but this time this increase is $c_m = 2.63$ % with a standard deviation of $c_{std} = 1.54$ %. Figure 7 shows the mean and the standard deviation for the minimum square optimization (p-Norm with $p = 2$), the minimization of the maximum and the optimization of mean and variance both weighted by one $w_m = w_{var} = 1$. It is striking that the Min-Max optimization is able to reduce the variance to zero with a comparably small increase in the mean of the costs. Once again, here these results are equal to the ones of the limited minimum sum optimization if $k_c = 1$.

The trajectories and controls that result in that case can be seen in 8 and are quite different from the trajectories that were calculated for minimum overall costs in Figure 2.

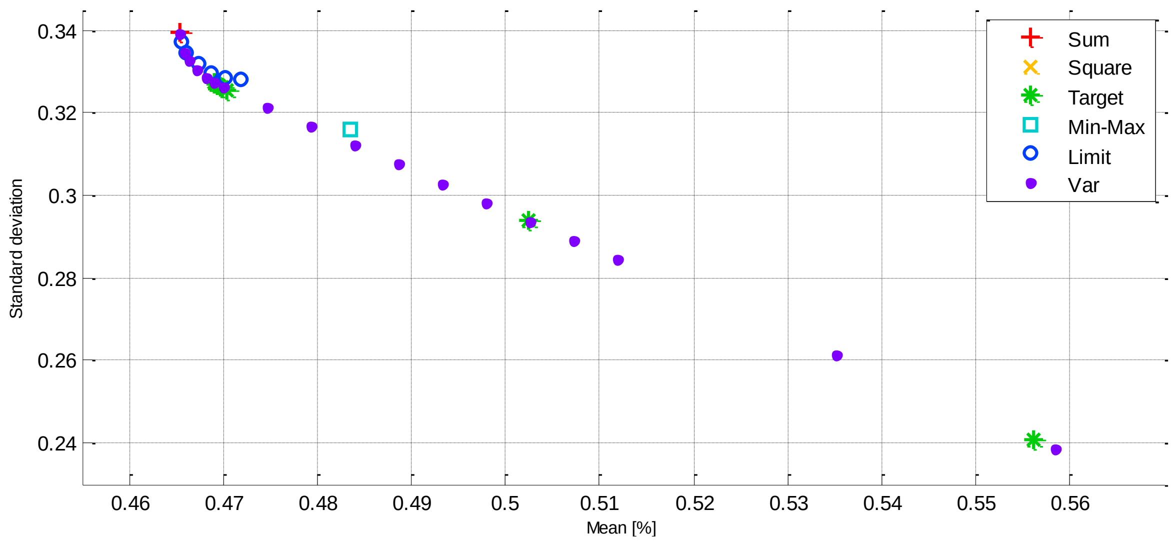

Figure 6 Approximation of the Pareto front in mean and standard deviation for scneario 1.

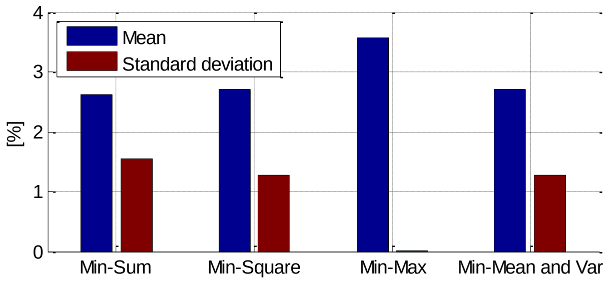

Figure 7 Results in mean and standard deviation for scneario 2.

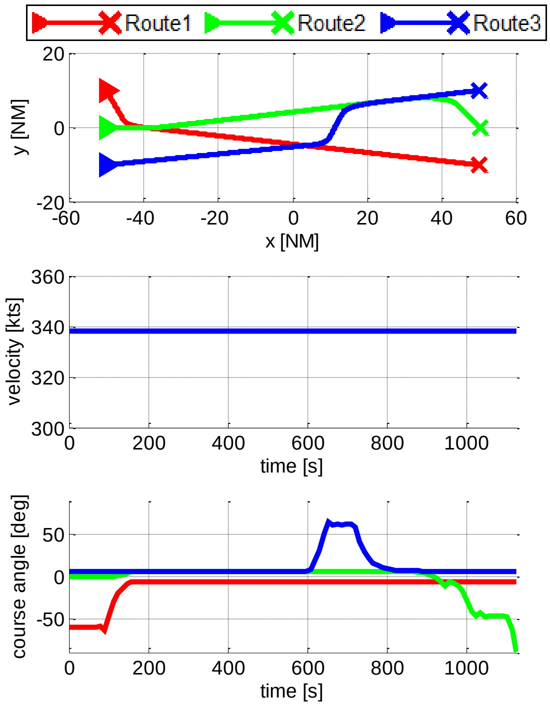

Figure 8 Optimal trajectories for the Min-Max optimization for scenario 2 and the corresponding control inputs (velocity remains at maximum value all the time for all aircraft).

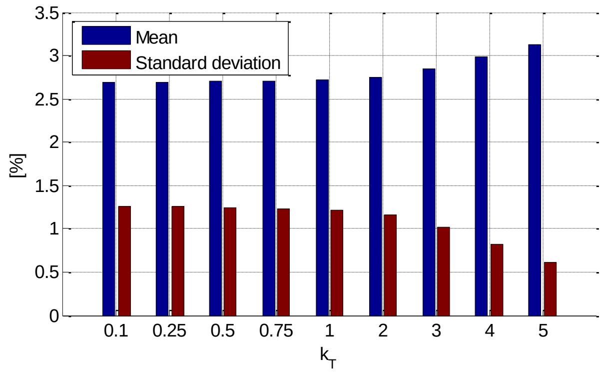

Figure 9 Mean and standard deviations for different scaling parameters k_T in the minimum target optimization for scenario 2.

When using the minimum target optimization for the second scenario with different parameters (Figure 9) the results are quite similar to the results calculated in the first example. Once again, the cost increases with increasing scaling factors k_T while the standard deviation simultaneously decreases.

The results for the minimization of the variance with different parameters k_c look very similar to the results in scenario 1 and can be seen in Figure 10. Anyway, the numeric values are far bigger, as the scenario is generally more “expensive” to solve. Thus, in this scenario an increase in fairness can be achieved with a comparably low relative increase in the mean.

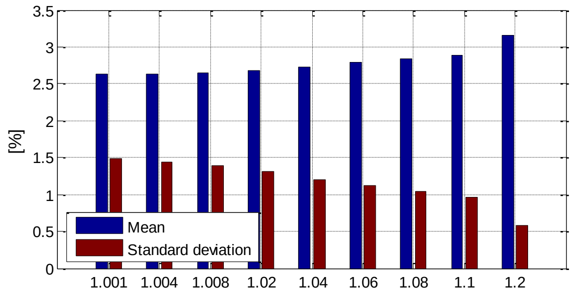

Figure 10 Optimal mean and standard deviation for different parameters k_c in scenario 2 using the minimum variance optimization.

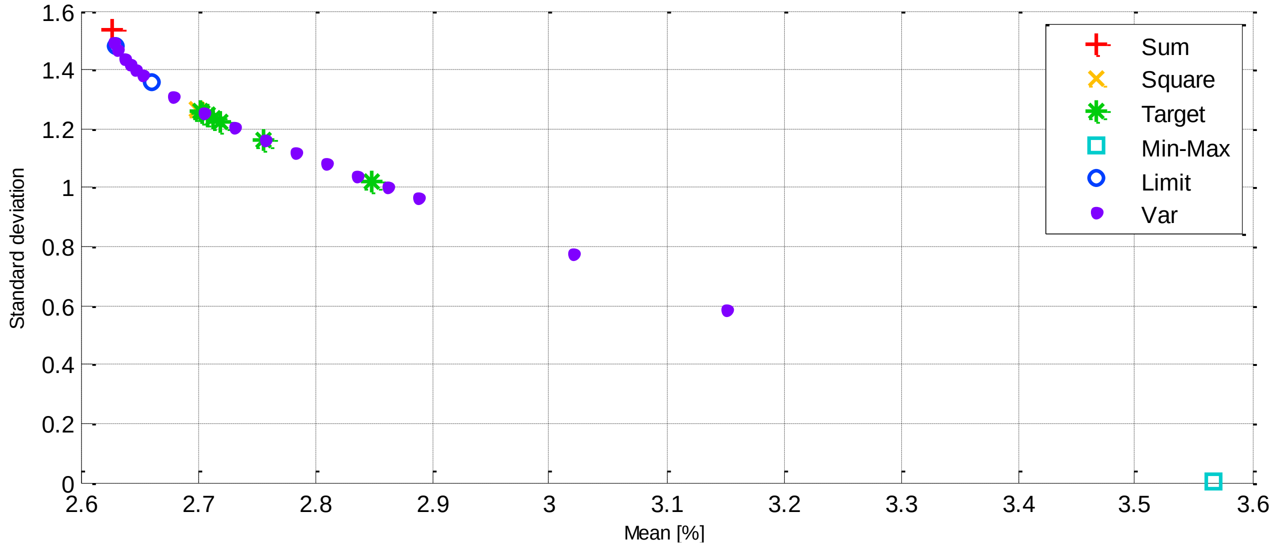

Figure 11 Approximation of the Pareto front in mean and standard deviation for scneario 2.

When comparing all methods for scenario 2 and plotting the mean of the costs against their standard deviation the result is quite similar to the result created in scenario 1. These results can be seen in Figure 11.This time, all optimal points are located on one clean line which is very likely to be the real Pareto front of the problem as formulated in (23). All methods taken together form a clear impression of the achievable performance in that scenario.

Moreover, here no significant point by means of huge decreases in one dimension bringing relatively small increases in the other can be found. So, the human decision maker is challenged and no preferable point can be detected.

V. Conclusions and Outlook

In the paper at hand methods for optimizing ATM scenarios with respect to overall costs and cost distribution were presented. The goal of the method is the calculation of trajectories in conflicting scenarios that are on the one side as cost-efficient as possible, but on the other side do not burden the cost to one or little of the participants in the scenario. The problem was formulated as a multi criteria optimal control problem. This problem was first transcribed into a parameter optimization problem by direct collocation methods. Afterwards, the resulting multi criteria parameter optimization problem can be tackled by different scalarization techniques to obtain a regular parameter optimization problem. In the paper at hand several methods have been used and tested in two examples.

The results of the calculations show that no method is superior to any other – neither in the results nor in computational time. Instead, all methods are able to calculate some points that are somewhere close to the Pareto front of the overall problem. In scenario 1, the limit optimization is a little inferior to the other methods. Using a combination of the methods presented, a good approximation for the Pareto front may be found – depending on the scenario considered. In most of the scenarios no preferable solution exists in which either the mean or the variance could be lowered without significantly increasing the other. So, finally, the decision for one point inside the solution still remains with the decision maker, i.e. the human controller.

The presented study was a very first study on the topic as no other publication is known to the authors where multi criteria optimization is combined with optimal control to calculate optimized solutions for ATM conflicts. As such, there of course is a lot of potential for future research. First of all, further investigations on Pareto optimality of the calculated points can be done. Furthermore, the cost functions used here are only based on time which is a first approximation of the real costs and can be extended to much more realistic models. Besides the aircraft cost functions, also controller workload and the number of maneuvers might be considered as both are crucial to safety. Moreover, uncertainties in the scenarios like wind may be added. And finally, more sophisticated methods to solve multi criteria optimization problems, like parameter sweeps over a Tchebychev scalarization, could be applied and used within a study considering real Pareto optimality.

Acknowledgment

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