Optimization of ATM Scenarios Considering Overall and Single Costs

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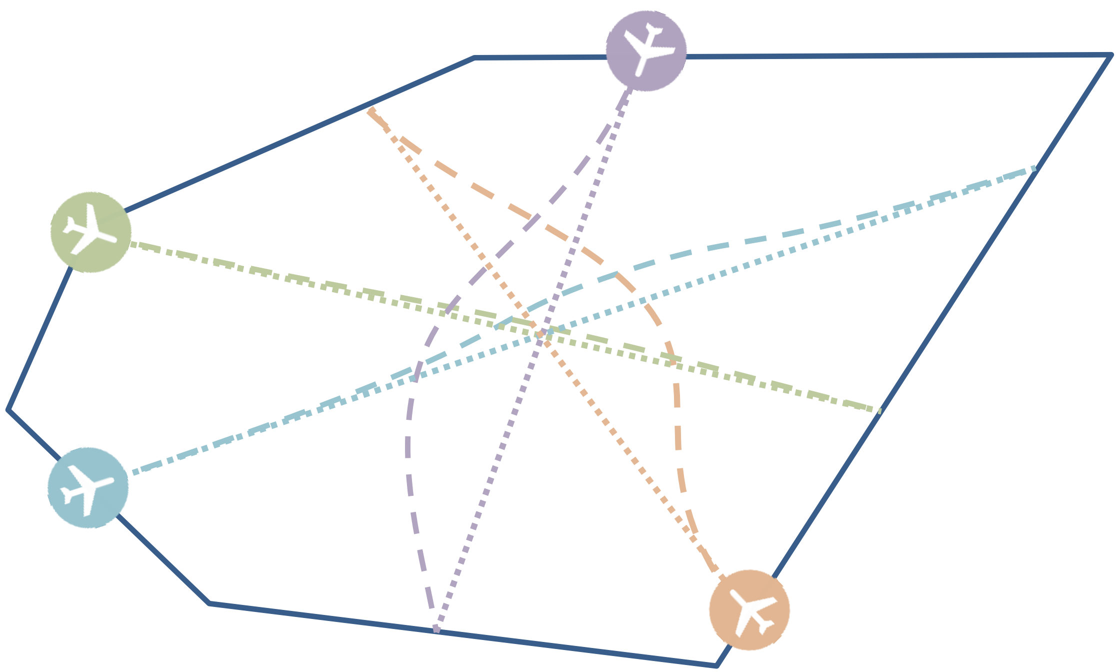
Problem description

- Multiple aircraft crossing ATM sector
- Minimum individual costs
- Maintain separation
- Maximum fairness



Problem formulation

Determine the optimal control histories (aircraft index i)

$$\mathbf{u}_{i,opt}(t) \in \mathbb{R}^{m_i}, \qquad i = 1, \dots, N$$

the corresponding optimal state histories

$$\mathbf{x}_{i,opt}(t) \in \mathbb{R}^{n_i}, \qquad i = 1, \dots, N$$

and any additional parameters $\mathbf{p}_i \in \mathbb{R}^{k_i}$

that minimize the Bolza cost functional

$$J = \Phi\left(\mathbf{x}_i(t_f), t_f\right) + \int_{t_0}^{t_f} \mathcal{L}(\mathbf{x}_i(t), \mathbf{u}_i(t), \mathbf{p}_i, t) dt$$

subject to the state dynamics

$$\dot{\mathbf{x}}_i(t) = \boldsymbol{f}_i(\mathbf{x}_i(t), \mathbf{u}_i(t), \mathbf{p}_i, t_i), \qquad i = 1, \dots, N$$

the initial and final boundary conditions

$$\boldsymbol{\Psi}_0(\mathbf{x}_i(t_0), t_0) = \mathbf{0}, \qquad \boldsymbol{\Psi_0} \in \mathbb{R}^p$$

$$\boldsymbol{\Psi}_f\left(\mathbf{x}_i(t_f), t_f\right) = \mathbf{0}, \qquad \boldsymbol{\Psi_f} \in \mathbb{R}^q$$

and the equality and inequality path constraints

$$\boldsymbol{C}_{eq}(\mathbf{x}_i(t), \mathbf{u}_i(t), \mathbf{p}_i, t) = \mathbf{0}, \qquad \boldsymbol{C}_{eq} \in \mathbb{R}^r, \qquad i = 1, \dots, N$$

$$\boldsymbol{C}_{in}(\mathbf{x}_i(t), \mathbf{u}_i(t), \mathbf{p}_i, t) \leq \mathbf{0}, \qquad \boldsymbol{C}_{in} \in \mathbb{R}^s, \qquad i = 1, \dots, N$$

Optimization Process

Optimization using direct methods: **Discretize then Optimize!**

Infinite optimal control problem	Discretization of problem	Finite parameter optimization problem	Optimization	Solution
Optimization parameters: Continuous functions $\mathbf{u}_\mathrm{i}(t), \mathbf{x}_\mathrm{i}(t), \mathbf{p}_i, t_f$ Analytical optimality conditions cannot be evaluated.	Reduce infinite problem by discretization e.g. Trapezoidal Collocation	Optimization parameters: Values at discrete points $\mathbf{u}_{i,k}, \mathbf{x}_{i,k}, \mathbf{p}_i, t_f$ Optimization problem can be solved numerically.	Optimization algorithm for parameter optimization problems e.g.: Sequential quadratic programming (WORHP, SNOPT, IPOPT, fmincon)	Resulting values for states and controls at discrete points can be interpolated to the full state and control histories.

Simulation model

- Kinematic models are sufficient, dynamics far from envelope
- 2D models are used

State dynamics

$$\dot{x}_i = V_{K,i} \cdot \cos \chi_{K,i}$$
$$\dot{y}_i = V_{K,i} \cdot \sin \chi_{K,i}$$

State and control vectors

$$\mathbf{x}_i = [x_i, y_i]^{\mathrm{T}}$$
$$\mathbf{u}_i = \left[V_{K,i}, \chi_{K,i}\right]^T$$

Separation path constraints (applied pair wise)

$$d_{\min}^2 = (5NM)^2 \leq (x_i - x_j)^2 + (y_i - y_j)^2$$

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Different flight times in one phase

- Aircraft need different times to cross sector
- Multi-Phase-Approach: Sequence needs to be known a priori
- Solution: Fading of aircraft dynamics

$$\dot{\mathbf{x}}_i = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \end{pmatrix} = \begin{pmatrix} V_{K,i} \cdot \cos \chi_{K,i} \\ V_{K,i} \cdot \sin \chi_{K,i} \end{pmatrix} \cdot \delta_\mathrm{x} \cdot \delta_\mathrm{y}$$

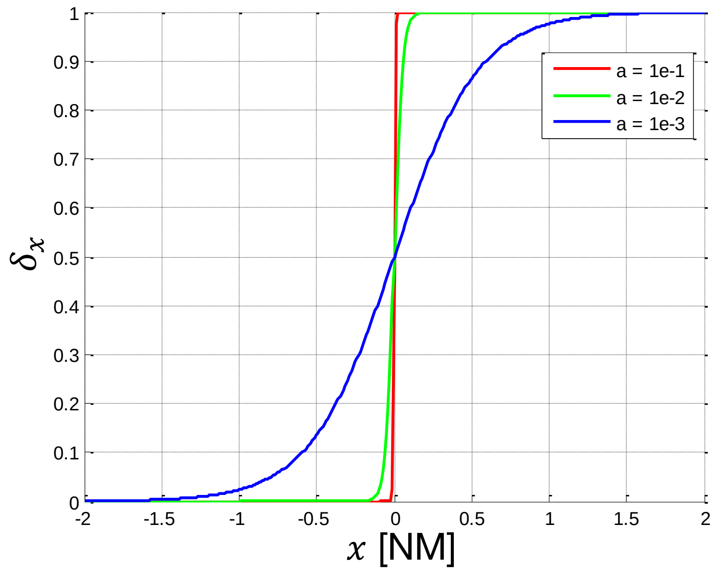
With

$$\delta_\mathrm{x} = \pm \frac{1}{2} \tanh\left(a \cdot |x - x_f|\right) + \frac{1}{2}$$

$$\delta_y = \pm \frac{1}{2} \tanh\left(a \cdot |y - y_f|\right) + \frac{1}{2}$$

Steepness parameter

$$a = 1{,}0799 \cdot 10^{-2} \cdot 1/m$$



Cost functions and fairness I

- Flight time is used as approximation for cost
- Fading of model:

$$\dot{\mathbf{X}}_\mathrm{i} = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{t}_i \end{pmatrix} = \begin{pmatrix} V_{K,i} \cdot \cos \chi_{K,i} \\ V_{K,i} \cdot \sin \chi_{K,i} \\ 1 \end{pmatrix} \cdot \delta_\mathrm{x} \cdot \delta_\mathrm{y}$$

- For comparison: Relative cost increase

$$c_i = \frac{t_{i,final} - t_{i,final,min}}{t_{i,final,min}} \cdot 100\%$$

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Cost functions and fairness II

- Fairness means neat distribution of relative cost increases
- Leads to multi criteria optimization problem
- Different formulations:
  - All cost increases should be minimized

$$\mathbf{J} = \begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix}$$

  - The statistical values of the cost increases should be minimized

$$\mathbf{J} = \begin{pmatrix} c_{sum} \\ c_{var} \end{pmatrix} = \begin{pmatrix} \sum c_i \\ \frac{1}{N} \sum (c_i - c_m)^2 \end{pmatrix}$$

Multi criteria optimization methods I

Scalarization techniques

- Weighted sum (fairness cannot be considered)

$$J_{Sum} = \sum_{i=1}^{N} w_i \cdot c_i$$

- p-Norm (all $c_i \geq 0$)

$$J_p = \left(\sum_{i=1}^{N} c_i^p \right)^{1/p}$$

- Distance to target cost (find target by min of overall cost, k_T tuning parameter)

$$J_T = \sum_{i=1}^{N} (c_i - c_T)^2$$

$$c_T = \min(c_i) \cdot k_T$$

Multi criteria optimization methods II

- Min-Max optimization (two step method, k_c allows tuning)

1. $$\min J_{max} = \min \|c\|_\infty = \min \lim_{p\to\infty} \left(\sum_{i=1}^{N} c_i^p\right)^{1/p} = \min \max_i (c_i)$$

2. $$\min J_{Sum} = \min \sum_{i=1}^{N} c_i \,, \qquad s.th. \quad c_i \leq k_c \cdot c_{\max} \quad , k_c \geq 1, \quad i = 1, \ldots, N$$

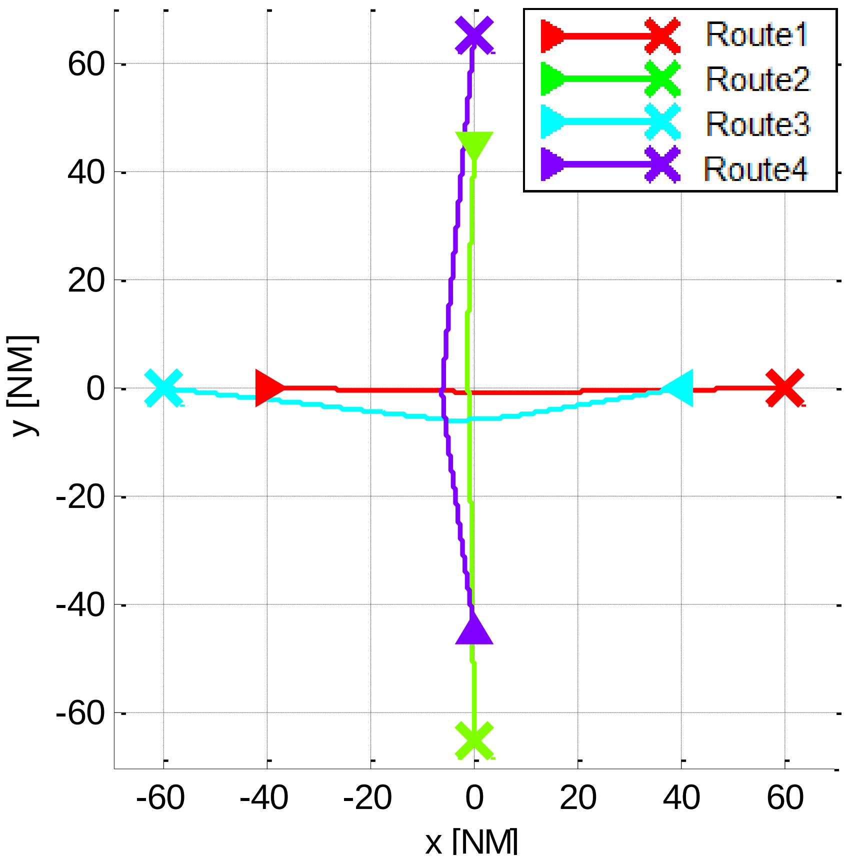
- Mean-Variance minimization (two step method, k_c allows tuning)

1. $$\min J_{sum} = \min \sum_{i=1}^{N} c_i$$

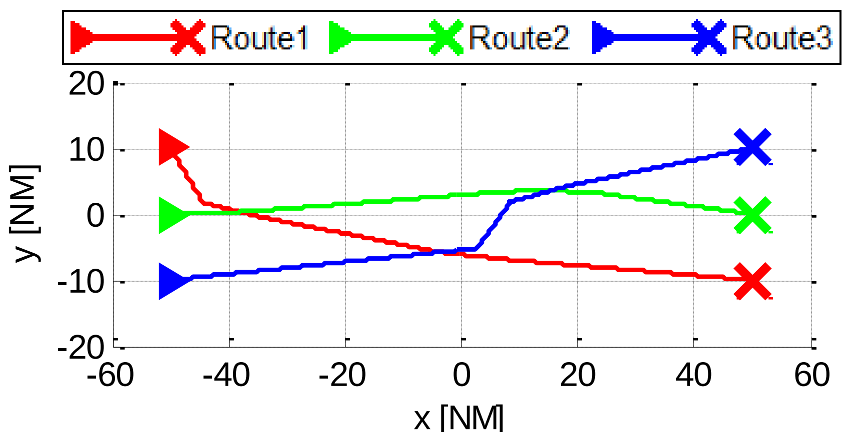
2. $$\min J_{var} = \min \frac{1}{N} \sum_{i=1}^{N} (c_i - \bar{c})^2 \qquad s.th. \quad c_{sum} \leq c_{sum,min} \cdot k_c \qquad k_c \geq 1$$

Example Scenarios

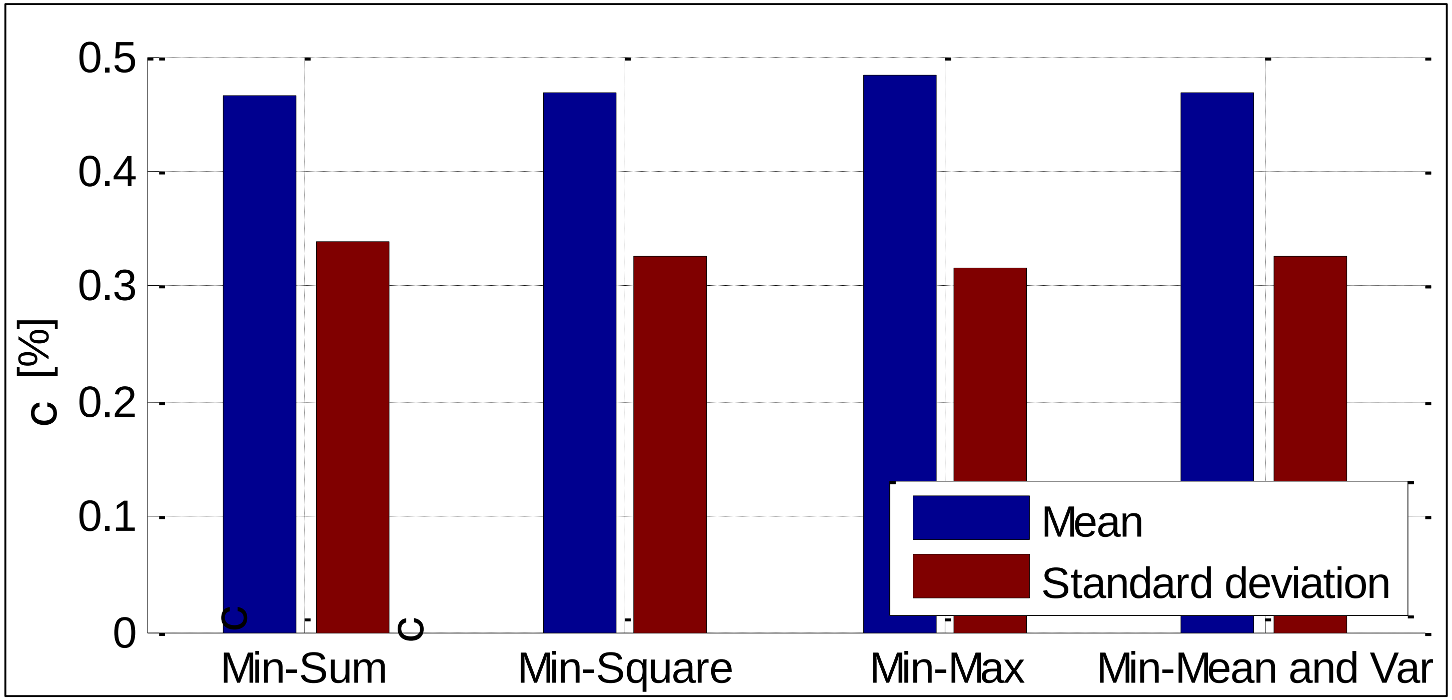
Example 1



Example 2



Results for Example 1



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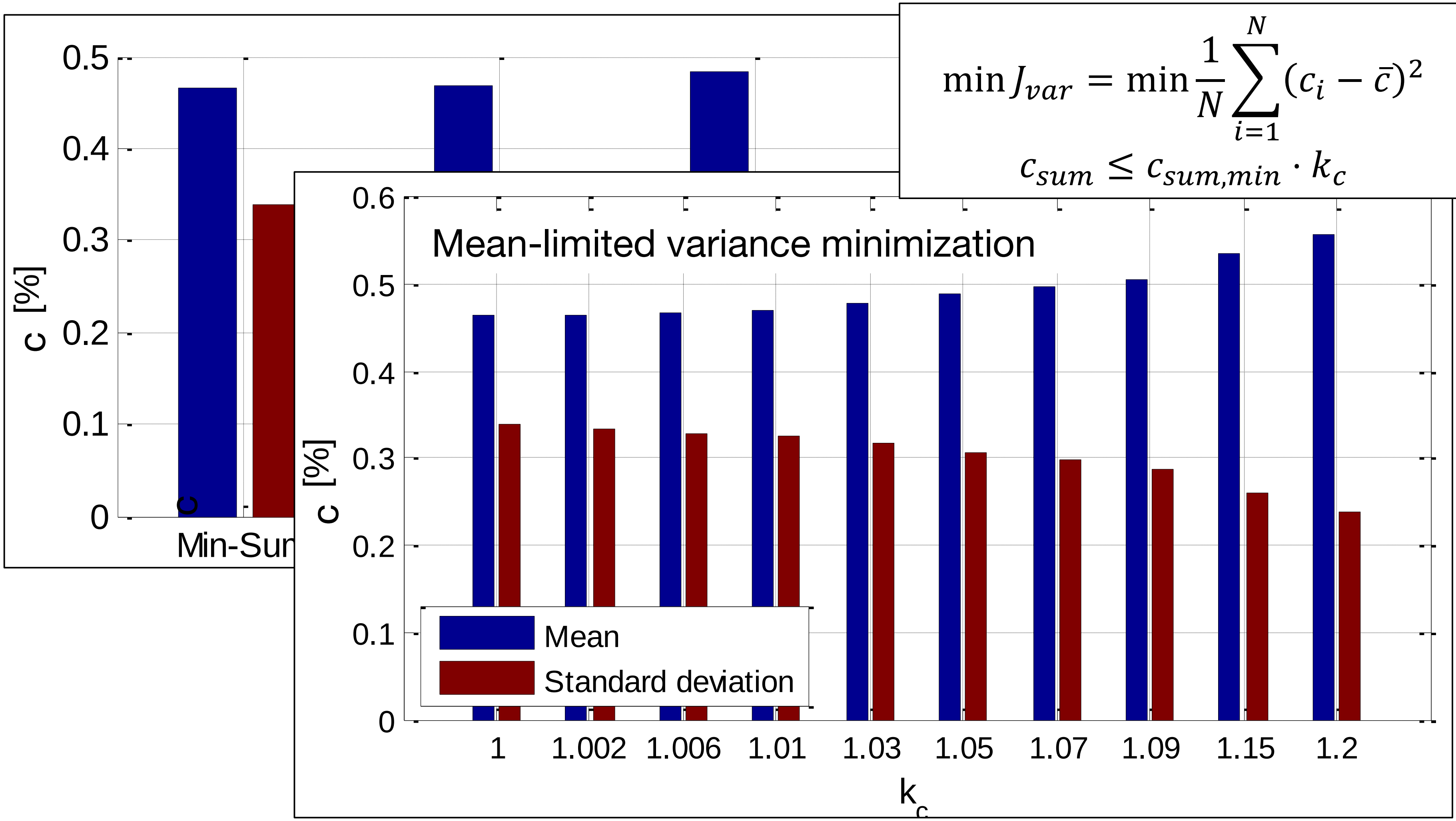
Results for Example 1

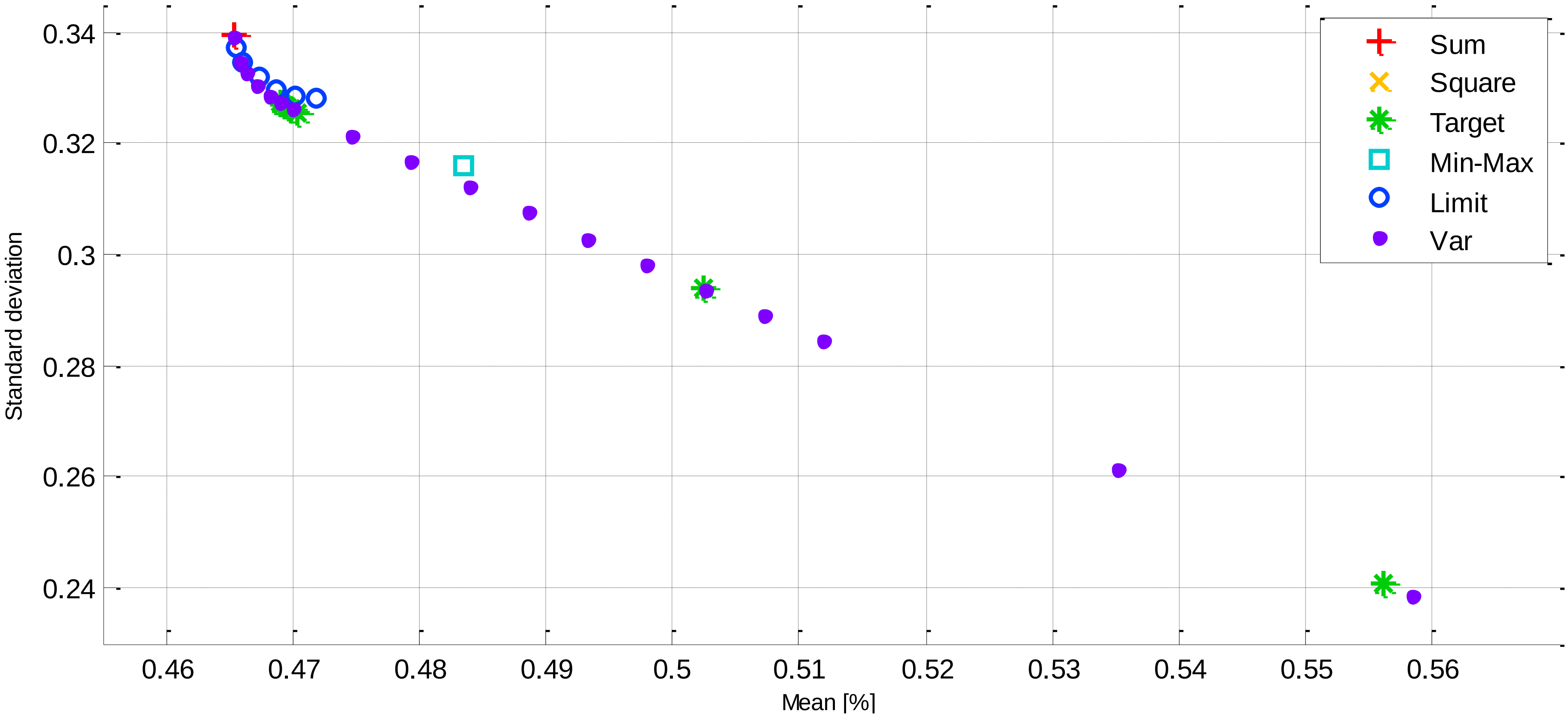
$$\min J_{var} = \min \frac{1}{N}\sum_{i=1}^{N}(c_i - \bar{c})^2$$

$$c_{sum} \leq c_{sum,min} \cdot k_c$$

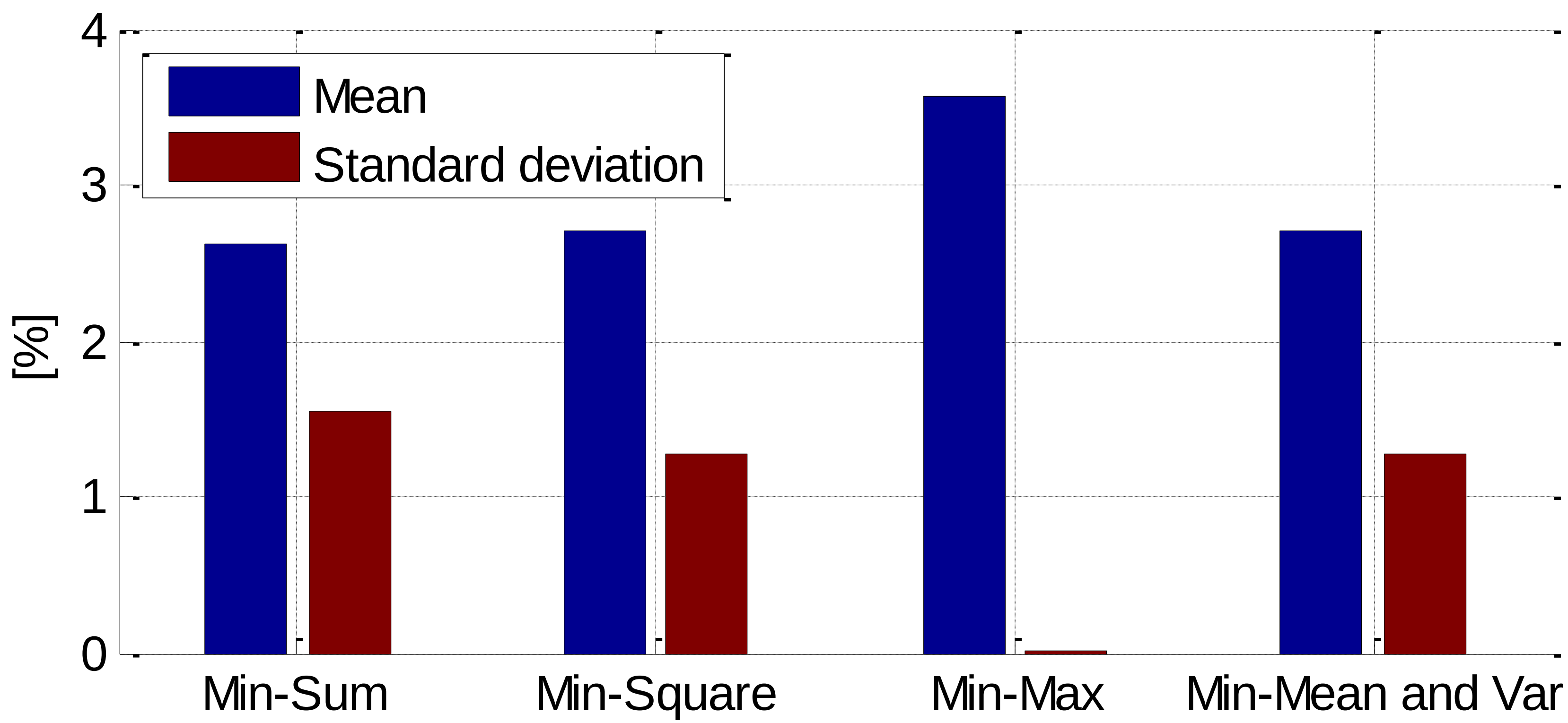


Results for Example 1

- Mean / Variance Pareto Front

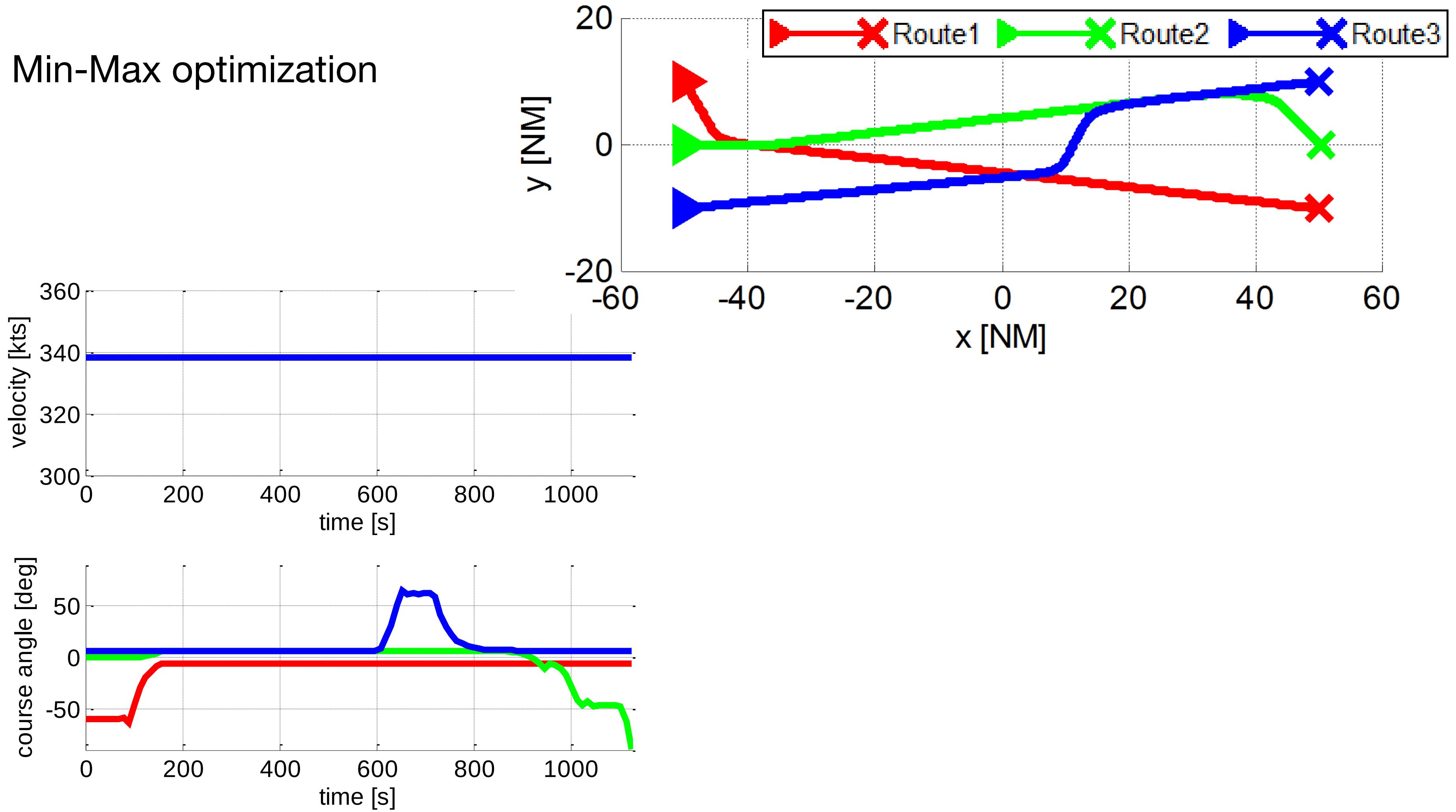


Results for Example 2



Results for Example 2

Min-Max optimization



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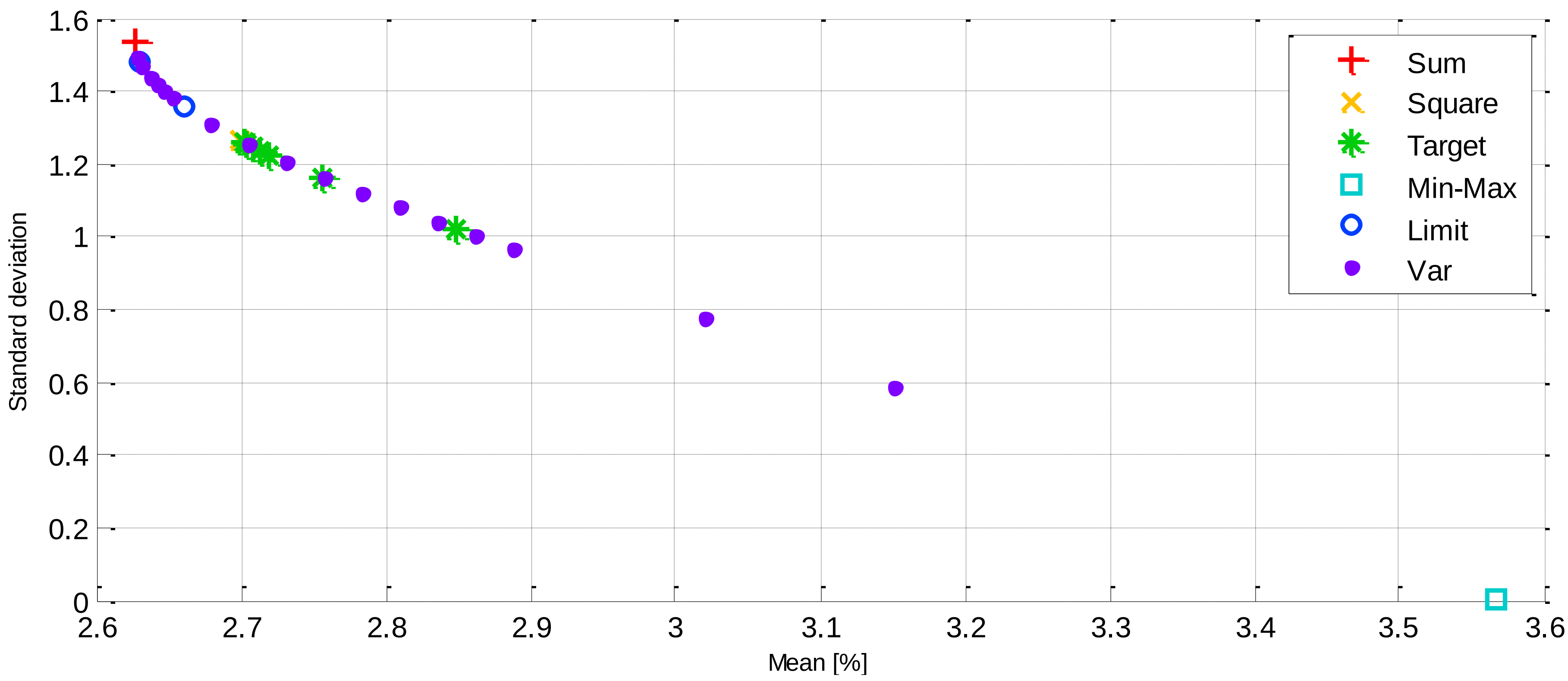
Results for Example 2



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Conclusions and outlook

Conclusions

- Modelling of ATM scenarios as multi criteria optimal control problems
- Optimal control problem solved using direct collocation
- Use of fading dynamics to model different sector crossing times
- Implementation of different methods to solve multi criteria optimization problems
- Results show:
  - Methods are comparable, but depend on scenario
  - “Overall cost” for the increase of fairness strongly depends on scenario
  - Approximation of Pareto front combining the methods

Outlook

- Multi criteria optimization methods can be improved (e.g. Tchebychev scalarization)
- More elaborate model / wind may be added
- Controller workload, number of maneuvers etc. may be considered

Thank you very much!

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