Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft

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General Picture

It is known that nowadays we face several challenges related with Air Traffic as,

- Constant growing of Air Traffic
- Free-Flight
- Environment impact
- Departures and Arrivals Scheduling
- Possible UAV inclusion
- ...

Beside of that several issues related with Air Traffic Management (ATM), it is always necessary to ensure SAFETY.
Subject And Motivation

1. In this sense, our intention is to approach the Collision Avoidance Problem;
2. And in this huge particularly subject we will address it on a Generalized point of view;
3. Our main goal is try to find a Real Time solution for the problem;
4. Additionally, we also want a solution computed with few Computationally Resources.
Hypothesis

The main idea behind the collision avoidance problem is to avoid the distance between aircraft became dangerous for both. In that way, considering the case of two aircraft represented by points $P_0$ and $P_1$ with velocities $V_0$ and $V_1$ respectively, the distance between them is represented by the vector $\vec{r}$. Then for collision avoidance problem the main requirement to be fulfill is,

$$\|\vec{r}\|^2 \geq \Delta \quad (1)$$

where $\Delta$ is a value, which independently of aircraft size they never collide (on a general sense).
Hypothesis

The equation (1) *per se* does not solve the problem, indeed with only that equation the problem can not be formulated. Hence, it was considered aircraft with different priorities and assumed relative motions.

Hence, from now on, we consider the following Hypothesis;

- **The system is based on a Priority System;**
- **Aircraft keep their velocity vectors during the computation.**
Two Aircraft Formulation

Definition (Object to Object System)

Consider the referential $OXYZ$ where a point of mass with position $P_0$ and velocity $V_0$ coincide with origin of that referential. Suppose also there is other point of mass with position $P_1$, relatively to referential $OXYZ$, and velocity $V_1$. The system formed by that two points of mass in the referred configuration, separated by the distance $||\vec{r}||$, with respective velocities, is called Object to Object System.
Two Aircraft Formulation

Definition (Line of Sight)

The Line formed by the two objects on an Object to Object System, with distance $\|\vec{r}\|$, is called *Line of Sight*. 
Two Aircraft Formulation

Definition (Safety Sphere)

The sphere built on Object to Object System where $P_1$ is its center and the safety distance radius $\|\vec{r}_{CAD}\|$, is called Safety Sphere.
Two Aircraft Formulation

Definition (Collision Cone)

At Object to Object System the infinite cone with apex coincident with point of mass $P_0$ and the straight lines tangent to a sphere with radius $||\vec{r}_{CAD}||$, where $||\vec{r}|| > ||\vec{r}_{CAD}||$, and center at the point of mass $P_1$ is called the Collision Cone.
Two Aircraft Formulation

Definition (Collision)

It is considered that two points of mass of an Object to Object System are in collision if

\[ \| \vec{r} \| < \| \vec{r}_{CAD} \|, \text{ where } \| \vec{r}_{CAD} \| \text{ is the safety distance.} \]
Two Aircraft Formulation

Lemma

Let’s be $\vec{V}_{01}$ the relative velocity vector between the points of mass of an Object to Object System with constant velocities during a time interval $\Delta t$. For a time $t_0$ where $||\vec{r}(t_0)|| > ||\vec{r}_{CAD}||$, if the angle $\delta$ formed by vector $\vec{V}_{01}$ and the line of sight $\vec{r}$ is greater than the half aperture angle $\delta_{CAD}$ of a collision cone, then $\exists \Delta t > 0 : \forall t \in [t_0, t_0 + \Delta t], ||\vec{r}(t)|| > ||\vec{r}_{CAD}||$. 
Two Aircraft Formulation

Theorem (Set of Solutions)

Consider an Object to Object System, where the point of mass $P_1$ has a sphere of collision centered in it of radius $\|\vec{r}_{CAD}\|$ and $\|\vec{r}\| > \|\vec{r}_{CAD}\|$. If the point of mass $P_1$ has constant velocity, then the set of velocity vector variations $\Delta\vec{V}$ of point of mass $P_0$ that produces a non-interception condition with $P_1$ is given by,

$$\Gamma = \{\Delta\vec{V} \in \mathbb{R}^3 \mid \vec{V}_{01}^* = \vec{V}_{01} + \Delta\vec{V}; \arccos(\hat{\vec{V}}_{01}^* \cdot \hat{r}) > \delta_{CAD}\}$$

where $\vec{V}_{01}$ is the relative velocity vector and $\delta_{CAD}$ is the half aperture angle of collision cone and $\hat{\vec{V}}_{01}^*$, $\hat{r}$ are the normalized vectors.
Generalised Formulation

Definition (Object to Multi Object System)

Considering the referential $OXYZ$ where a point of mass with position $P_0$ and velocity $V_0$ coincides with origin of that referential. Suppose also there are others point of mass with position $P_i$, relatively to referential $OXYZ$, and velocity $V_i$, where $i = 1, ..., N - 1$ with $N$ as the number of points of mass. The system formed by that $N$ points of mass in the referred configuration, separated by the distance $\|\vec{r}_i\|$, with respective velocities, is called Object to Multi Object System.
Generalised Formulation
Generalised Formulation
Generalised Formulation

Theorem (Generalized Set Of Solution)

At Object to Multi-Object System, the point of mass $P_i$ has a sphere of collision centered in it of radius $\|\vec{r}_{CAD_i}\|$ and $\|\vec{r}_i\| > \|\vec{r}_{CAD_i}\|$. If the points of mass $P_i$ have constant velocities, then the set of variation velocity vector $\Delta \vec{V}$ of the point of mass $P_0$ which produces a non-interception with $P_i$ is given by,

$$\Gamma = \left\{ \Delta \vec{V} \in \mathbb{R}^3 \mid \vec{V}^*_{0i} = \vec{V}_{0i} + \Delta \vec{V}; \arccos(\hat{\vec{V}}^*_{0i} \cdot \hat{\vec{r}}_i) > \delta_{CAD_i} \right\}$$

where $i = 1, \ldots, N - 1$ with $N$ as the number of points of mass and $\hat{\vec{V}}^*_{01}, \hat{\vec{r}}$ are normalized vectors.
Problem Solution
Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Predictive Collision Avoidance

However, the solutions that were found do not work anymore, we need some kind of predictability to transform our solution on an invariant solution. Hence, we start to construct the navigation model in order to find a 4D waipoint,

\[ \dot{\lambda}(t) = \frac{V(t) \cos \gamma(t) \cos \psi(t)}{(R + h(t)) \cos \varphi(t)} \]  \hspace{1cm} (2a)

\[ \dot{\varphi}(t) = \frac{V(t) \cos \gamma(t) \sin \psi(t)}{R + h(t)} \]  \hspace{1cm} (2b)

\[ \dot{h}(t) = V(t) \sin \gamma(t) \]  \hspace{1cm} (2c)

\[ t \in [t_k, t_{k+1}] \]  \hspace{1cm} (3)
Predictive Collision Avoidance

\[ V(t) = V \]
\[ \gamma(t) = \gamma \]  \hspace{1cm} (4)
\[ \varphi(t) = \varphi \]

For notation proposes, it is also assumed that,

\[ t_0 = t_k \]  \hspace{1cm} (5)

and

\[ V(t_0) = V_0 \]
\[ \gamma(t_0) = \gamma_0 \]  \hspace{1cm} (6)
\[ \varphi(t_0) = \varphi_0 \]
Predictive Collision Avoidance

So, the dynamic equation system take the next form,

\[
\dot{\lambda}(t) = \frac{V \cos \gamma \cos \psi}{(R + h(t)) \cos \varphi(t)} \tag{7a}
\]

\[
\dot{\varphi}(t) = \frac{V \cos \gamma \sin \psi}{R + h(t)} \tag{7b}
\]

\[
\dot{h}(t) = V \sin \gamma \tag{7c}
\]

Now, we need to solve it. The altitude has direct resolution which can be substituted on the other equations. With that we are able also into solve the latitude equation with a close-form solution. The problem is the longitude equation which have no close-form solution. For solve it we need to apply some numerical approach.
Predictive Collision Avoidance

Starting with altitude we have,

\[ h(t) = h_0 + V \sin \gamma (t - t_0) \quad (8) \]

Replacing the equation (8) on differential equation \( \dot{\varphi}(t) \) a closest solution still possible, as demonstrated,

\[ \varphi(t) = \varphi_0 + \frac{\sin \psi}{\tan \gamma} \ln \left[ 1 + \frac{V \sin \gamma}{R + h_0} (t - t_0) \right] \quad (9) \]

but if analyzed the previous equation, a singularity arise when the path angle \( \gamma \) comes to \( k\pi \), \( \forall k \in \mathbb{N} \). Passing to the limit the equation (9) the singularity can be fixed,
Predictive Collision Avoidance

\[ \lim_{\gamma \to 0} \varphi(t) = \varphi_0 + \frac{V \sin \psi}{R + h_0} (t - t_0) \]  

(10)

then the latitude equation \( \varphi(t) \) can be represented as a piecewise function in order to path angle \( \gamma \) as,

\[
\varphi(t) = \begin{cases} 
\varphi_0 + \frac{\sin \psi}{\tan \gamma} \ln \left[ 1 + \frac{V \sin \gamma}{R + h_0} (t - t_0) \right] , & \gamma \neq k\pi \\
\varphi_0 + \frac{V \sin \psi}{R + h_0} (t - t_0) , & \gamma = k\pi 
\end{cases}
\]

(11)

with \( \forall k \in \mathbb{N} \)
Simulation - Architecture

Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Simulation - Used Tools

- **CPU** - i7-920
- **Operating System** - Fedora 20 x86-64
- **Programming Languages** - mainly C, python for some additional tasks
- **Non-standart Libraries** - clapack, plplot for real time data plotting;
- **Compiler** - gcc version 4.8
- **IDE** - Eclipse
Simulation - Structure

Flight Plan

• Load Flight Plan
• Check Type
• Read Memory
• Process Navigation
• Write Memory

RTNSim

Aircraft Type Definition

Shared Memory

• Read Memory
• Check Priority
• Check Type
• Process Evasion
• Write Memory

ACAS

Real Time Plot

Data

Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Simulation - 10 Aircraft

Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Simulation - Altitude

![Graph showing altitude changes over time for multiple aircraft.](image-url)

- $A_1$
- $A_2$
- $A_3$
- $A_4$
- $A_5$
- $A_6$
- $A_7$
- $A_8$
- $A_9$
- $A_{10}$

Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Simulation - Ground Projection

Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
### Simulation - Minimum Distance [m]

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Simulation - 20 Aircraft
## Simulation - Minimum Distance [m]

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Automatic Collision Avoidance System based on Geometric Approach applied to Multiple Aircraft
Discussion and Conclusions

• We were able to identify the Set of Solutions of generalized collision avoidance problem, based on a concept of collision cone and a priority system.

• However, the solutions only work for a instantaneous time, then a predictive method was added, where was possible to delivery the solutions to the aircraft by mean of a 4D waypoint.

• The implemented algorithms, until now, point us for possible real implementation of those at least on UAV scenarios, of course, the approach should be mature.
Future Work

- The Priority System should be exhaustive studied.
- Due to the generalized approach, the method could adapted to ground, meteo and restricted zones collision avoidance scenarios.
- The implemented software, also should be submitted to an exhaustive verification and validation.
- The decentralized architecture should be take into account for future implementations
- A distributed implementation maybe will reduce significantly the time consumption.
Questions?