Comparison of Two Ground-based Mass Estimation Methods on Real Data

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***Abstract*—This paper focuses on the estimation of the aircraft mass in ground-based applications. Mass is a key parameter for climb prediction. It is currently not available to ground-based trajectory predictors because it is considered a competitive parameter by many airlines. There is hope that the aircraft mass might become widely available someday, but in the meantime it is possible to estimate an *equivalent mass* from the data already available, assuming the thrust to be known (maximum or reduced climb thrust for example).**

In this paper, using Mode-C radar data, we compare the performances of two mass estimation methods proposed in recent publications. In a previous paper ([1]), these two methods were compared using simulated data. Both methods estimate the aircraft mass by fitting the modeled energy rate (i.e. the power of the forces acting on the aircraft) with the energy rate observed at several points of the past trajectory. The first method, proposed by Schultz *et al.* ([2]), dynamically adjusts the weight parameter so as to fit the energy rate, using an adaptive sensitivity parameter to weight each observation. The second method, introduced in one of our previous publications ([1]), estimates the mass by minimizing the quadratic error on the observed energy rate, taking advantage of the polynomial expression of the modeled power when using the BADA model.

The actual mass is unavailable in our radar data. However, we can use the estimated mass to compute a trajectory prediction. This prediction is then compared to the actual trajectory giving us some insight on the accuracy of the estimated mass. We have compared the obtained predictions with the ones obtained using the BADA reference mass. The root mean square error on the predicted altitude is reduced by 45 % using the least squares method. With the adaptive method this error is divided by two.

Keywords: aircraft trajectory prediction, mass estimation, BADA, energy rate, specific power

INTRODUCTION

With the emergence of new operational concepts ([3], [4]) centered on trajectory-based operations, predicting aircraft trajectories with great accuracy has become a key issue for most ground-based applications in Air Traffic Management and Control (ATM/ATC). Some of the most recent algorithms applied to ATM/ATC problems require to test a large number of alternative trajectories. As an example, in [5] an iterative quasi-Newton method is used to find trajectories for departing aircraft, minimizing the noise annoyance. Another example is [6] where Monte Carlo simulations are used to estimate the risk of conflict between trajectories, in a stochastic environment. Some of the automated tools currently being developped for ATC/ATM can detect and solve conflicts between trajectories, using Genetic Algorithms, or Differential Evolution or Particle Swarm Optimization ([9])

To be efficient, all these methods require a fast and accurate trajectory prediction, and the capability to test a large number of "what-if" trajectories. Such requirements forbid the sole use of on-board trajectory prediction, which is certainly the most accurate, but is not sufficient for these most promising applications. Even with the existing (or future) datalink capabilities that could transmit the on-board prediction to ground systems, there remains a need for a fast and accurate ground-based prediction.

Most trajectory predictors rely on a point-mass model to describe the aircraft dynamics. The aircraft is simply modeled as a point with a mass, and the second Newton's law is applied to relate the forces acting on the aircraft to the inertial acceleration of its center of mass. Such a model is formulated as a set of differential algebraic equations that must be integrated over a time interval in order to predict the successive aircraft positions, knowing the aircraft initial state (mass, current thrust setting, position, velocity, bank angle, etc.), atmospheric conditions (wind, temperature), and aircraft intent (thrust profile, speed profile, route).

Unfortunately, the data that is currently available to ground-based systems for trajectory prediction purposes is of fairly poor quality. The speed intent and aircraft mass, being considered competitive parameters by many airline operators, are not transmitted to ground systems. The actual thrust setting of the engines (nominal, reduced, or other, depending on the throttle's position) is unknown. There are uncertainties or noise in the Weather and Radar data. Some studies ([10], [11], [12]) detail the potential benefits that would be provided by additional or more accurate input data. In other works, the aircraft intent is formalized through the definition of an Aircraft Intent Description Language ([13], [14]) that could be used in air-ground data links to transmit some useful data to ground-based applications. There is hope that, in the future, all the necessary data required to predict aircraft trajectories will be available. In the meantime, we propose to learn some of the unknown parameters of the point-mass model – typically the aircraft mass – from the data that is already available.

Focusing on the aircraft climb, we are interested in this paper in estimating the aircraft mass, which is one of the key parameters for climb performance, using the past trajectory points. This approach, where some unknown parameters are adjusted by fitting the model to the observed past trajectory, is not new. The past publications following this path ([15], [16], [17], [18], [2], [19], [20], [21]) propose several methods, with different choices for the adjusted parameter (mass, or thrust, for example), the modeled variable that is fitted on past observations (rate of climb, energy rate), and the algorithm that is applied (stochastic method, adaptive mechanism, least squares, etc.).

Among the publications dealing with mass estimation, let us cite [15], where Warren and Ebrahimi propose an *equivalent weight* as a workaround to use a point-mass model without knowing the actual aircraft mass. Nominal thrust and drag profiles are assumed. The equivalent mass is found by minimizing the gap between the computed and observed vertical rates. A second study ([16]) raises doubts about the reliability of the vertical rate for this purpose, and suggests to use the energy rate instead. The proposed method is tested on simulated trajectories only. In more recent works, Schultz, Thipphavong, and Erzberger ([2]) introduce an adaptive mechanism where the modeled mass is adjusted by fitting the modeled energy rate with the observed energy rate.

This adaptive method provides good results on simulated traffic and this method has also been successfully applied on actual radar data ([22], [23]).

In [19], [20], we use a Quasi-Newton algorithm (BFGS) combined to a mass estimation method to learn the thrust profile minimizing the error between the modeled and observed energy rate. The thrust law, once learned on historical data, is used to predict the future trajectory of any new aircraft, together with the mass estimated on the past trajectory points. This method has been tested on two months of real data, showing good results. Concerning the mass estimation method, we showed that, when using the BADA[1] model of the forces (or a similar model), the aircraft mass can be estimated at any past point of the trajectory by solving a polynomial equation, knowing the thrust setting at this point. When using several points, and assuming a constant mass over the whole trajectory segment, the mass can be estimated by minimizing the quadratic error on the energy rate.

In the current paper, we propose to compare the least squares method and the adaptive method using Mode-C radar data. A similar study ([1]) was done on synthetic data. This study has shown that both methods perform well on noisy data with a slight advantage to the least squares method. In this paper, we compare these two methods using actual radar data. However, the actual mass is not available, making the comparison of the methods more tricky. Thus, we used two different ways to evaluate the performance. The first way is to use the estimated mass to predict the trajectory and compare the accuracy obtained with the two estimated mass. The second way is to estimate a mass on the future points of the trajectory. This mass is compared to the mass estimated on the past points.

The rest of this paper is organized as follows: Section I describes the forces' model and the equations governing the aircraft dynamics. Section II describes the two mass estimation methods. The data and experimental setup are detailed in section III, and the results are shown and discussed in section IV, before the conclusion.

[1]BADA: the Eurocontrol Base of Aircraft DAta

I. Models and Equations

A. Aircraft Dynamics with the Effect of Wind

Ground-based trajectory predictors used for air traffic management and control purposes usually rely on a simplified point-mass model to predict aircraft trajectories. In such a model, all forces acting on the aircraft body are exerted at the center of mass, making several simplifying approximations. The inertial moments and angular accelerations of the aircraft around its center of gravity are not included in the model. The aircraft is modeled as a point of mass m, subject to the second Newton's law that gives us the inertial acceleration $\overrightarrow{a_i} = \frac{d\overrightarrow{V_i}}{dt} = \dot{\overrightarrow{V_i}}$ of the center of mass (the dot above a vector denotes the time derivative of this vector):

$$m\dot{\overrightarrow{V_i}} = \overrightarrow{Thr} + \overrightarrow{D} + \overrightarrow{L} + m\overrightarrow{g} \tag{1}$$

In equation (1), mass is considered a stationary variable[2] for what concerns its impact on the aircraft dynamics. At a larger scale, though, the fuel burn and the consequent loss of mass must be taken into account when integrating the equations to predict the future trajectory. Concerning the forces, it is assumed that the thrust $\overrightarrow{Thr}$ exerted by the aircraft engines is aligned to the airspeed vector $\overrightarrow{V_a}$, and in the same direction. The drag $\overrightarrow{D}$ exerted by the relative wind on the flying airframe is also aligned to $\overrightarrow{V_a}$, by definition, and in the opposite direction. The lift force $\overrightarrow{L}$ caused by the motion of the airframe through the air is perpendicular to these vectors and in the plane of symmetry of the aircraft. The flight is assumed to be symmetric and there is no aerodynamic sideforce. The effects of Earth rotation on the aircraft dynamics are neglected (flat Earth approximation).

The effect of wind $\overrightarrow{W}$ on the aircraft velocity and acceleration cannot be neglected, however. It can be written as follows:

$$\overrightarrow{V_i} = \overrightarrow{V_a} + \overrightarrow{W} \tag{2a}$$

$$\overrightarrow{a_i} = \dot{\overrightarrow{V_a}} + \dot{\overrightarrow{W}} \tag{2b}$$

We can project equation (1) onto the airspeed vector $\overrightarrow{V_a}$ axis. This gives us the following equation, where "." denotes the dot product of two vectors:

$$m\overrightarrow{V_a}.\frac{d\overrightarrow{V_i}}{dt} = \left(\overrightarrow{Thr} + \overrightarrow{D} + \overrightarrow{L} + m\overrightarrow{g}\right).\overrightarrow{V_a} \tag{3}$$

Combining equations (2) and (3), and introducing h the geodetic height of the aircraft, and $\dot{h} = \frac{dh}{dt}$ the inertial vertical velocity (counted positive upward), equation (3) can be reformulated as a law governing the total energy rate, denoting W_{Up} the upward component of the wind:

$$\underbrace{\left(\frac{Thr - D}{m}\right)V_a}_{\text{specific power}} = \underbrace{V_a\dot{V_a} + g\dot{h}}_{\text{specific energy rate}} + \underbrace{(\dot{\overrightarrow{W}}.\overrightarrow{V_a} - gW_{Up})}_{\text{wind effect}} \tag{4}$$

Expressing the power of the forces acting along the true airspeed axis, and the total energy (kinetic and potential) of

[2]We assume in fact that $\frac{d}{dt}(mV_i) = m\dot{V_i}$, and neglect the impact of $\dot{m}$ on the acceleration.

the aircraft gives us an interesting insight to equation (4). We can see how the aircraft dynamics are governed by the specific power (i.e. power per unit of mass) and energy rate:

$$\textit{Power} = (\textit{Thr} - D)\, V_a \tag{5a}$$

$$\textit{Energy} = \frac{1}{2} m V_a^2 + mgh \tag{5b}$$

$$\frac{\textit{Power}}{m} = \frac{d}{dt}\left(\frac{\textit{Energy}}{m}\right) + (\dot{\overrightarrow{W}}.\overrightarrow{V_a} - gW_{Up}) \tag{5c}$$

For historical and technical reasons, the geodetic altitude h and the inertial vertical velocity $\dot{h}$ are not much used in air traffic control operations. Instead, a *pressure altitude* H_p (also called *geopotential pressure altitude* in [24]) is computed on board the aircraft and transmitted to ground systems by Mode-C or Mode-S transponders. The relationship between the pressure altitude and the geodetic altitude is the following, with T denoting the air temperature, and ΔT is the difference with the temperature that would occur using the International Standard Atmosphere (ISA) model:

$$g\dot{h} = g_0 \left(\frac{T}{T - \Delta T}\right) \frac{dH_p}{dt} \tag{6}$$

Neglecting the vertical component of the wind W_{UP} and using the relationship between $\dot{h}$ and $\dot{H}_p$ stated in equation (6), equation (4) can be re-written as follows, introducing g_0 the gravitational acceleration *at mean sea level*, and a corrective factor related to the temperature:

$$\underbrace{\frac{\textit{Thr} - D}{m} V_a}_{\text{specific power}} = \underbrace{V_a \frac{dV_a}{dt} + g_0 \left(\frac{T}{T - \Delta T}\right) \frac{dH_p}{dt}}_{\text{specific energy rate}} + \underbrace{\frac{d\overrightarrow{W}}{dt}.\overrightarrow{V_a}}_{\text{wind effect}} \tag{7}$$

Considering an aircraft trajectory picked up from historical data, the energy rate and wind effect (right-hand part of equation (7)) can be computed at any point of the observed trajectory. The specific power (left-hand part) is a function of the mass m and the thrust and drag forces (Thr and D).

In the rest of this paper, we focus on estimating the mass for climbing aircraft, using equation (7). In the two methods presented in section II, the mass is adjusted so that equation (7) is satisfied. This requires a model of the thrust and drag forces.

B. Modeling the Forces

Using equation (7) to actually compute a trajectory requires a model of the aerodynamic drag D of the airframe flying through the air. We also need a computational model of the engines' thrust *Thr*. In our experiments, we used version 3.9 of the Eurocontrol Base of Aircraft Data (see [25]) to compute these forces.

The BADA model provides different parametric models of the thrust force *Thr* for jet, turboprop, and piston engines (see section 3.7 of [25]). These models are tuned by regression using manufacturers' data. They allow us to compute the standard maximum climb thrust $\textit{Thr}_{\text{max climb}}$ as a function of H_p, ΔT, and V_a:

$$\textit{Thr}_{\text{max climb}} = f_1(H_p, V_a, \Delta T) \tag{8}$$

Given S the wing surface and Φ the bank angle, the equation for the drag D is the following:

$$C_L = \frac{2mg_0}{\rho V_a S \cos\Phi} \tag{9a}$$

$$C_D = a_D + b_D C_L^2 \tag{9b}$$

$$D = \frac{C_D \rho V_a^2 S}{2} \tag{9c}$$

The coefficients a_D and b_D are values depending on the phase of flight (landing gear up or down, flaps extended, etc.).

With the atmosphere model and the equations of [24], the air density ρ and temperature T can be expressed as a function of the temperature differential ΔT. So the drag is as a function of the aircraft mass m, the true air speed V_a, the geopotential pressure altitude H_p and the temperature differential ΔT. Moreover, one can notice that the drag D is a polynomial of the second degree with respect to the mass that has the following form:

$$D = f_2(H_p, V_a, \Delta T) + m^2 \times f_3(H_p, V_a, \Delta T, \Phi) \tag{10}$$

C. Fuel consumption

A fuel consumption model is also required when computing a full trajectory. In climbing phase, the fuel consumption is modeled by equation (11), where the mass variation $\frac{dm}{dt}$ is described as a function of H_p, V_a and ΔT.

$$\frac{dm}{dt} = -f_4(V_a, H_p, \Delta T) \tag{11}$$

II. Mass Estimation

The two mass estimation methods compared here rely on the idea of adjusting the mass m in order to equalize the specific power and the specific energy rate.

In order to be more specific, let us introduce P and Q, defined as follows, considering equations (5) and (7) :

$$P = \textit{Power} - m \times \underbrace{\left[\frac{d}{dt}\left(\frac{\textit{Energy}}{m}\right) + (\dot{\overrightarrow{W}}.\overrightarrow{V_a})\right]}_{Q} \tag{12a}$$

$$Q = V_a \frac{dV_a}{dt} + g_0 \left(\frac{T}{T - \Delta T}\right) \frac{dH_p}{dt} + \frac{d\overrightarrow{W}}{dt}.\overrightarrow{V_a} \tag{12b}$$

The quantity Q is the sum of the energy rate and wind effect. It can be computed at any point of the past trajectory using the recorded radar track, Weather data, and equations (2). Considering the forces model given by equations (8) and (9c) in section I-B, only the mass m is missing to compute the power. Thus, at each point i of the trajectory, the power is a function $Power(m_i)$ of the mass m_i at point i. The total energy model equation (7) becomes:

$$\frac{P_i(m_i)}{m_i} = 0 \;\Leftrightarrow\; \textit{Power}_i(m_i) = m_i Q_i \tag{13}$$

A. The Adaptive Method

The idea of the adaptive method introduced by Schultz *et al.* in [2] is to dynamically adjust the weight mg so that the modeled energy rate (i.e. the power of the forces acting on the aircraft) fits the observed energy rate. The weight is adjusted for each new trajectory point and the weight update depends on a sensitivity parameter which is dynamically adapted, comparing the energy rate error of the new observation to the average value over the five last points. Small values of the sensitivity parameter compensate for the volatility of radar track data, giving less importance to the outliers (i.e. the points that differ too much from the average), whereas high values allow the algorithm to better follow the energy rate variations.

Let us now describe more formally the two parts of this adaptive algorithm: the weight adjustment and the sensitivity parameter adaptation. Due to our choice of notations and to the form of our equation (7), and also because we adjust the mass m instead of the weight mg, our description of the adaptive method is slightly different from the one given by Schultz *et al.*. Otherwise, the mechanism is exactly the same.

In the dynamic weight adjustment, the power at point i is modeled using the previous mass m_{i-1}. The current mass m_i is then obtained by applying equation (13), using Q_i the energy rate and wind effect observed at point i:

$$m_i = \frac{Power_i(m_{i-1})}{Q_i}. \tag{14}$$

This equation (14) can be rewritten as follows:

$$m_i = m_{i-1}\left(1 - \frac{P_i(m_{i-1})}{Power_i(m_{i-1})}\right)^{-1}. \tag{15}$$

For the reasons explained at the begining of this section, a sensitivity parameter β_i is introduced in the update term of equation (15). Finally, the mass is updated using the following equation:

$$m_i = m_{i-1}\left[1 + \beta_i\left(\frac{-P_i(m_{i-1})}{Power_i(m_{i-1})}\right)\right]^{-1}. \tag{16}$$

The sensitivity parameter β_i is adapted by comparing the observed variations of the energy rate, given by $P_i(m_{i-1})$ in equation (16), to the average variation over the five previous points. The adaptation rule given in [2] is the following, where $\Delta\dot{E}_i = \frac{P_i(m_{i-1})}{m_{i-1} g V_a}$ (with our notations):

$$\begin{aligned}
&\text{if } i > 0 \text{ and } \Delta\dot{E}_i > 0.0001 \\
&\qquad \text{and } \left|\frac{\Delta\dot{E}_i - \Delta\dot{E}_{avg}}{\Delta\dot{E}_{avg}}\right| < 3 \\
&\quad \text{then} \\
&\qquad \beta_i = max(0.205, \beta_{i-1} + 0.05) \\
&\quad \text{else} \\
&\qquad \beta_i = 0.005
\end{aligned} \tag{17a}$$

In equation (17), $\Delta\dot{E}_{avg}$ is the average value of $\Delta\dot{E}_i$ over the last five previous points. Note that there might be less than five points when the algorithm "warms up", at the beginning of the trajectory.

With this mechanism, if $\Delta\dot{E}_i$ is repeatedly high in the same order of magnitude, β_i will increase, strengthening the adaptation. Otherwise, β_i has a low value. As a consequence, an isolated high $\Delta\dot{E}_i$ does not have a great impact on the adaptation. This improves the robustness of this mass estimation process.

The algorithm starts with an initial mass m_0 (typically the reference mass given by the BADA model). The mass variation at each iteration is bounded: in our experiments, it is limited to 2%[3] of the reference mass. During the whole process, the estimated mass is bounded within 80% and 120% of the reference mass.

B. Least Squares Method

In the adaptive method presented in section II-A, the mass is iteratively updated with each new trajectory point. The algorithm starts with an initial mass m_0 and ends up with a final mass m_n after n iterations.

In the least squares method, the mass is directly estimated by minimizing the sum of the squared errors over n points. The total error $\mathcal{E}$ being minimized is the following:

$$\mathcal{E}(m_1, \ldots, m_n) = \sum_{i=1}^{n}\left(\frac{Power_i(m_i)}{m_i} - Q_i\right)^2 \tag{18a}$$

$$= \sum_{i=1}^{n}\left(\frac{P_i(m_i)}{m_i}\right)^2 \tag{18b}$$

The mass variation is ruled by equation (11) (see section I-C). With this equation, the mass m_i at point i and time t_i can be written as a simple function of the final mass m_n, knowing the values of the state variables (temperature, altitude, velocity, etc) observed at point i. The mass at point i is the following, with f_4 modeling the fuel burn:

$$m_i = \simeq m_n + \delta_i \tag{19}$$

The quantity $\delta_i = \sum_{k=i}^{n-1} \frac{f_4(t_{k+1}) + f_4(t_k)}{2}(t_{k+1} - t_k)$ can be computed from the available data for every point i of the observed past trajectory. Therefore, the sum of squares error $\mathcal{E}$ can be rewritten as follows:

$$\tilde{P}_i(m_n) = P_i(m_n + \delta_i) \tag{20a}$$

$$\mathcal{E}(m_n) = \sum_{i=1}^{n}\left(\frac{\tilde{P}_i(m_n)}{(m_n + \delta_i)}\right)^2 \tag{20b}$$

The aircraft mass is estimated by minimizing $\mathcal{E}(m_n)$ given by equation (20b). This minimization can be done efficiently when using the model of forces provided by BADA. With this model, the power ($Power_i(m_i)$) can be expressed as a second-degree polynomial of the mass m_i, using the functions f_1, f_2, and f_3.

Consequently, $\tilde{P}_i(m_n) = Power_i(m_n+\delta_i) - (m_n+\delta_i)Q_i$ is a second-degree polynomial of the final mass m_n. The overall error $\mathcal{E}$ is a sum of rational terms (i.e. ratios of polynomial functions). The minimum m^* of this function satisfies the

[3] This value differs from the one given in [2], but it gives better results in our experiments.

equation $\mathcal{E}'(m^*) = \frac{d\mathcal{E}}{dm}(m^*) = 0$. When introducing a common denominator in $\mathcal{E}'$, the equality $\mathcal{E}'(m^*) = 0$ becomes a polynomial equation of degree at most $3(n-1)+4$. Solving such a high degree polynomial might be a difficult task due to numerical issues [26]. Therefore, instead of minimizing $\mathcal{E}$ we minimize an approximation $\mathcal{E}_{approx}$ as defined by equation (21) below:

$$F_{avg}(m_n) = \frac{1}{n}\sum_{i=1}^{n}(m_n + \delta_i) \tag{21a}$$

$$\mathcal{E}_{approx}(m_n) = \sum_{i=1}^{n}\left(\frac{\tilde{P}_i(m_n)}{F_{avg}(m_n)}\right)^2 \tag{21b}$$

With this approximation, the optimal mass m^* must satisfy the fourth-degree polynomial equation (22) below, in order to cancel out $\mathcal{E}'_{approx}$.

$$\sum_{i=1}^{n}\tilde{P}_i(m^*)\left[\tilde{P}_i{}'(m^*)F_{avg}(m^*) - \tilde{P}_i(m^*)F_{avg}{}'(m^*)\right] = 0 \tag{22}$$

One can solve analytically this fourth-degree polynomial equation using Ferrari's method. However, even for a third-degree polynomial, analytical methods might not be numerically stable [27]. In our experiments, we used the numerical method[4] provided by the GNU Scientific Library. This numerical method appears to be as fast as the analytical method in our experiments. Among the four potential solutions given by this numerical method, we select the solution[5] in $]0;+\infty[$ minimizing $\mathcal{E}_{approx}$. The obtained value is the estimated aircraft mass m^* at point n.

III. Data and Experimental Setup

A. Data Pre-processing

Recorded radar tracks from Paris Air Traffic Control Center are used in this study. This raw data is made of one position report every 1 to 3 seconds, over two months (July 2006, and January 2007). In addition, the wind and temperature data from Météo France are available at various isobar altitudes over the same two months.

The raw Mode C altitude[6] has a precision of 100 feet. Raw trajectories are smoothed using splines. Basic trajectory data is made of the following fields: aircraft position (X,Y in a projection plan, or latitude and longitude in WGS84), ground velocity vector $V_g = (V_x, V_y)$, smoothed altitude (H_p, in feet above isobar 1,013.25 hPa), rate of climb or descent $\frac{dH_p}{dt}$. The wind $W = (W_x, W_y)$ and temperature T at every trajectory point are interpolated from the weather datagrid.

Using the position, velocity and wind data, we compute the true air speed V_a. The successive velocity vectors allow us to compute the trajectory curvature at each point. The aircraft bank angle is then derived from true airspeed and the curvature of the air trajectory.

B. Filtering and Sampling Climb Segments

To compare the performances of the two methods, we focus on a single aircraft type (Airbus A320). Our dataset comprises all flights of this type departing from Paris-Orly (LFPO) or Paris-Charles de Gaulle Airport (LFPG). Needless to say, this approach can be replicated to other aircraft types and airports.

The trajectories are filtered so as to keep only the climb segments. An additional 80 seconds is clipped from the beginning and end of each segment, so as to remove climb/cruise or cruise/climb transitions.

The trajectories are then sampled every 15 seconds. Each climb segment contains 51 points. The altitude at the 11^{th} point is always 18,000ft. The first 11 points (past trajectory) are used to estimate the mass. The remaining points (future trajectory) are used to compute the error between the predicted and actual trajectory.

IV. Results

A. Trajectory Prediction using the Estimated Mass

In order to compute a trajectory prediction using the BADA model, you have to specify a mass and a speed profile. Both are usually unknown. In our experiment, we want to compare the impact of the mass estimation methods. Thus, we assume the speed profile to be known; the prediction is computed using the observed speed $V_a{}^{(obs)}$ on the future points of the trajectory. The trajectory is computed using the speed profile $V_a = V_a{}^{(obs)}(t)$ and the estimated mass. With this setup, the predicted speed is equal to the observed speed whereas the predicted altitude can be very different from the observed altitude depending on the mass used.

Some statistics on the differences between the predicted altitude and the observed altitude are presented on the table III. In addition to the two estimated methods, we have also computed the trajectory using the reference mass m_{ref} given by the BADA model. The performance obtained with this mass is the baseline performance. Using the least squares method, the root mean square error on the altitude at a 10 minutes look-ahead time is reduced by 45% when compared to the baseline. The adaptive method reduces this root mean square error by 50%. The predicted altitude underestimates the observed one, especially when using the least square method.

method	mean	stdev	mean abs	rmse	max abs
m_{ref}	-83	1479	1168	1482	5495
Adaptive	-304	685	582	750	5535
Least Squares	-533	653	631	843	6033

Table I: These statistics, in feet, are computed on the differences between the predicted altitude and the observed altitude $\left(H_p^{(pred)}(\hat{m}_{11}) - H_p^{(obs)}\right)$ at the time $t = 600$ s.

However, the maximum error on the predicted altitude is higher using estimation methods than using the reference mass. There are different sources of errors: measurements error, errors on the model of forces, and error on the thrust setting.

[4] This method of the GNU scientific library uses a balanced-QR reduction of the companion matrix.

[5] Actually, under reasonable hypotheses on the observed variables, one can prove that there is exactly one solution in $]0;+\infty[$ that cancels out $\mathcal{E}'_{approx}$.

[6] This altitude is directly derived from the air pressure measured by the aircraft. It is the height in feet above isobar 1012.25 hPa.

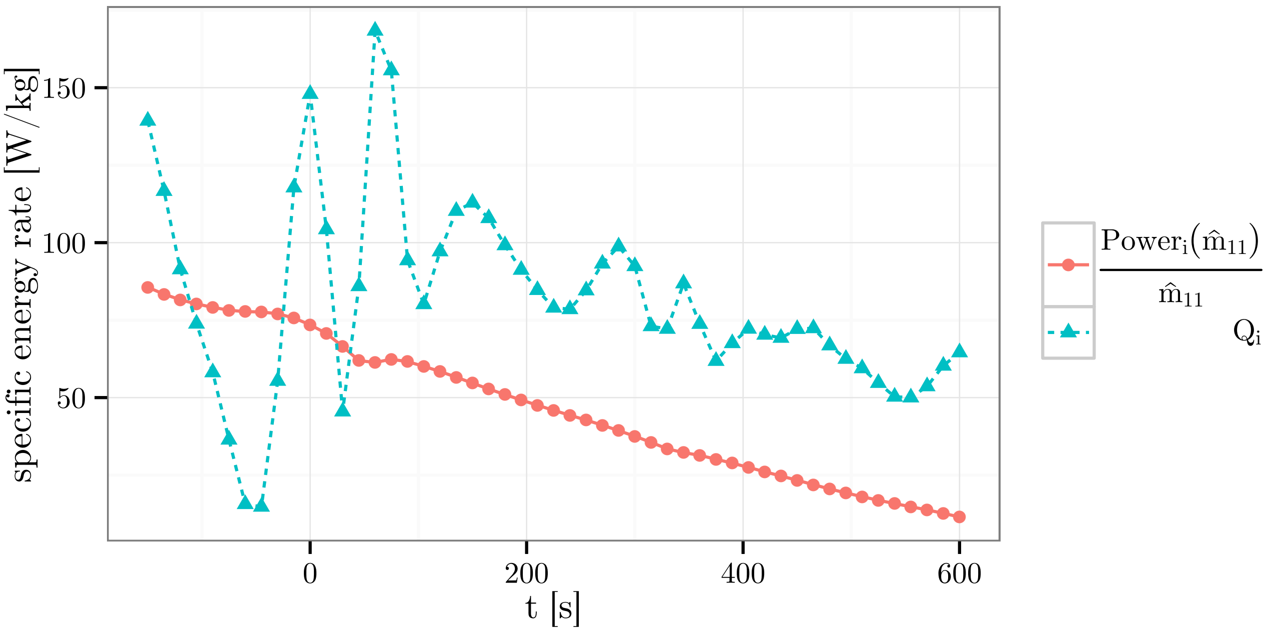

Figure 1: This figure portrays the computed specific energy rate $\frac{Power_i(\hat{m}_{11})}{\hat{m}_{11}}$ and the observed specific energy rate Q_i. In this figure, $\hat{m}_{11}$ was estimated with the least squares method. The largest error on the predicted altitude at the time $t = 600s$ is obtained with this trajectory.

We have assumed a *max climb* thrust setting on past and future points. This hypothesis might not be true. For instance, the observed specific energy rate can exhibits large variation not in compliance with a *max climb* hypothesis. This case is illustrated by the figure 1.

B. Distribution of the Estimated Mass

The figure 2 is the distribution of the estimated masses obtained with the two methods. The masses estimated by the adaptive method are comprised between 51,200 kg and 76,800 kg. This is due to the mechanism bounding the mass during the adaptive process. On the contrary, the mass estimated by the least square method is not bounded. With this method, some estimated masses are larger than the maximum mass in the BADA model. Such high estimated mass is not realistic. If we consider that the BADA model of forces is exact, this suggests that the hypothesis of a *max,climb* thrust is not true for all the past points of these trajectories.

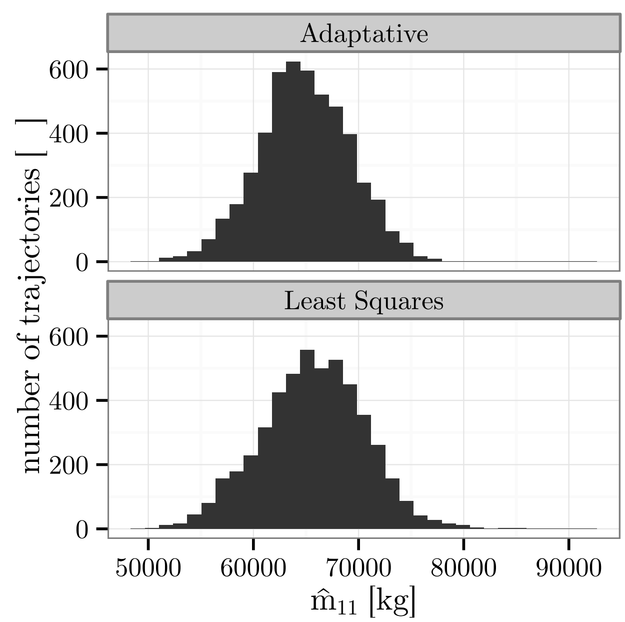

Figure 2: Distribution of the estimated masses obtained with the two methods.

C. Estimation of the "Actual" Mass Knowing the Future Trajectory

On our radar data set, the actual mass is unavailable. Thus, we cannot compare the estimated mass $\hat{m}_{11}$ with the actual mass. However, we can estimate a mass $\hat{m}_{51,future}$ using the 41 future points of the trajectory. From the mass $\hat{m}_{51,future}$, we can compute the mass $\hat{m}_{11,future}$ at the 11th point using the fuel consumption model with the equations (19). With the table II, we can observe that the predictions obtained with the least squares method are more accurate than the ones obtained with the adaptive method. The adaptive method underestimates the mass needed to make a good prediction. This is probably due to the fact that in the adaptive method, the last points have more impact on the estimation process than the first ones. With the least squares method, the mass is adjusted so that the sum over all the points of the square difference between

method	mean	stdev	mean abs	rmse	max abs
Adaptive	358	569	543	672	3303
Least Squares	-63.1	121	105	137	817

Table II: These statistics, in feet, are computed on the differences between the predicted altitude and the observed altitude $\left(H_p^{(pred)}(\hat{m}_{11,future}) - H_p^{(obs)}\right)$ at the time $t = 600$ s.

specific power and observed energy rate is minimized. Let us note $\hat{m}_{11,future}^{LS}$ the mass $\hat{m}_{11,future}$ estimated using the least squares method.

Using the statistics presented in the table III, we can observe that the mass $\hat{m}_{11}$ estimated on the past points overestimates the mass $\hat{m}_{11,future}^{LS}$ by at least 1.27 %. This means that the specific power increases between the past points and the future points. This is in accordance with previous publications ([19], [20]) on the same radar data. In these publications, a thrust setting profile is learned from historical data. The learned thrust setting found is increasing with respect to the altitude. With the table III, we can note that the root mean square error on the mass is divided by two when compared to the baseline.

method	mean	stdev	mean abs	rmse	max abs
m_{ref}	0.366	7.56	5.93	7.57	31.7
Adaptive	1.27	3.28	2.71	3.52	21.7
Least Squares	2.46	3.26	2.94	4.08	34.9

Table III: These statistics, in percentage, are computed on the relative differences between the estimated mass on the past point and on the future points $\frac{\hat{m}_{11}-\hat{m}_{11,future}^{LS}}{\hat{m}_{11,future}^{LS}}$.

D. Assessment of the Prediction Accuracy

In the least squares method, the mass $\hat{m}_{11}$ is estimated using the past points by fitting the computed specific power to the observed specific energy rate. We might gain some information by looking on how good this fit is. In order to measure this goodness of fit, with the equation (23) we introduce $e^{[i:j]}(m)$ the root mean square error on the points i to j of the difference between the observed specific energy rate and the computed specific power using the mass m.

$$e^{[i:j]}(m) = \sqrt{\frac{1}{j-i+1}\mathcal{E}^{[i:j]}{}_{approx}(m)} \tag{23}$$

This value is computed on each trajectory using its past points. The figure 3 presents the absolute error on the altitude at $t = 600$ s with respect to $e^{[1:11]}\left(\hat{m}_{11}^{LS}\right)$. Using these plotted points we have applied a local linear quantile regression giving us an estimation of the 95 % quantile. Due to the lack of points above 40 W/kg, this estimation might not be reliable. Below 40 W/kg, we can note that the 95 % quantile increases with respect to $e^{[1:11]}\left(\hat{m}_{11}^{LS}\right)$. With $e^{[1:11]}\left(\hat{m}_{11}^{LS}\right) = 2$ W/kg, 95 % of the error are inferior to 1,123 ft. If $e^{[1:11]}\left(\hat{m}_{11}^{LS}\right) = 32$ W/kg, 95 % of the error are inferior to 2,200 ft. Thus, using only the past points of the trajectory, the error $e^{[1:11]}\left(\hat{m}_{11}^{LS}\right)$ is a good indicator on how accurate will be the prediction.

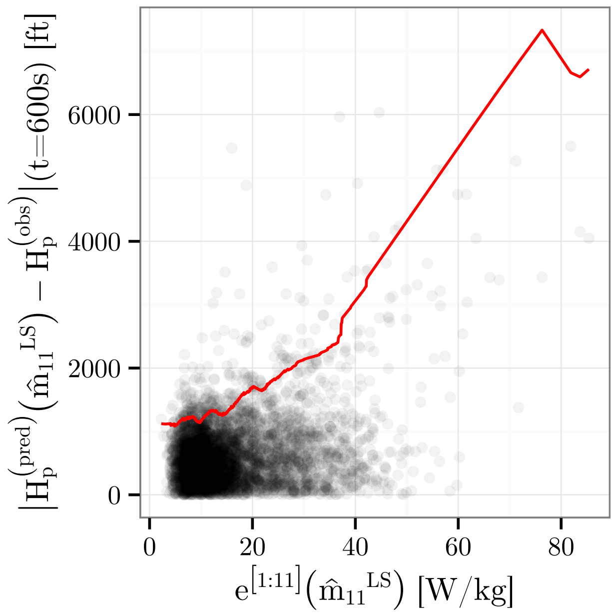

Figure 3: Relation between $e^{[1:11]}\left(\hat{m}_{11}^{LS}\right)$ and the absolute error on the predicted altitude $\left|H_p^{(pred)}(\hat{m}_{11}^{LS}) - H_p^{(obs)}\right|(t = 600\text{ s})$. The red line is the estimated 95 % quantile. It is obtained by local linear quantile regression [28].

Conclusion

To conclude, let us summarize our approach and findings, before giving a few perspectives on future works. In this study, we compare two mass estimation methods (adaptive and least squares), using real Mode-C radar data. The adaptive method, recently introduced by Schultz *et al.* in [2], dynamically adjusts the weight to fit the modeled energy rate to the observation. The least squares method was proposed in [1]. This method minimizes the sum of squared errors on the energy rate, using several points of the past trajectory. It takes advantage of the fact that the specific power is a polynomial function of the mass when modeling the thrust and drag forces with the BADA model. Although it is model-dependent, we believe that the least squares method could be extended to some other point-mass models.

The two mass estimation methods are tested on a set of actual radar trajectories. For that purpose, the estimated mass is used to compute a trajectory prediction. The accuracies of the predicted trajectories obtained are compared between the two methods. The root mean square error on the altitude is reduced by at least 45 % when compared to the root mean square error obtained using the BADA reference mass. The adaptive method is slightly more accurate than the least squares method. In order to quantify the error on the estimated mass, an "actual" mass is estimated using future points. The root mean square error of the relative difference between this actual mass and the estimated mass is below 4 %. The mass estimated by the least square method is obtained by minimizing the sum of square error between the computed specific power and the observed specific energy rate. On this study, we have seen that this minimized error is a good indicator on how accurate the prediction is.

From an operational point of view, the resulting improvement in the climb prediction accuracy would certainly benefit air traffic controllers, especially in the vertical separation task as shown in [2].

In future works, it could be interesting to compare the two methods, adaptive and least squares using a thrust setting profile learned from historical data. Such a profile has been learned in previous works [19], [20]. With this thrust setting profile, the mass will be no longer overestimated. The least squares method is too sensitive to outliers. In order to mitigate the impact of outliers, we may investigate robust estimators instead of the least squares estimator. These different masses might be also used in a Machine Learning approach. They could be used to learn the estimated "actual" mass for instance.

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