Comparison of Two Ground-based Mass Estimation Methods on Real Data

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**Objective**

**Context**

Two mass estimation methods:
- Adaptive method [Schultz et al., 2012]
- Least square method [Alligier et al., 2013]

A comparison was done on synthetic data [Alligier et al., 2013]

In this work: comparison on real data
- Mode-C radar data + weather data
- Actual mass not known
- Comparison of the trajectory prediction accuracies
Objective

altitude

time
Objective

altitude

time
Objective

altitude

time
Objective

Estimation of the Aircraft Mass
An *energy-rate* oriented approach

Newton’s laws

\[ \frac{1}{2} \frac{dv^2}{dt} + g \frac{dz}{dt} = \frac{\text{power}(\text{mass})}{\text{mass}} \]

\( \frac{dv^2}{dt} \) is the energy-rate, \( g \frac{dz}{dt} \) is the power.

\( f \) is given by a physical model of the forces.

Using past positions given by radar

- We compute the observed *energy-rate* from radar data.
- We search a *mass* such that:

\[ \text{observed energy-rate} = f(\text{mass}) \]
1. Computing the \( \frac{\text{power}}{\text{mass}} \) provided by BADA

2. The adaptive method [Schultz et al., 2012]

3. The least square method [Alligier et al., 2012]

4. Experimental setup

5. Results
1. Computing the power provided by BADA

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A Point Mass Model

\[
m \cdot \frac{dV_{TAS}}{dt} = Thr - D - m.g \cdot \sin(\gamma)
\]
A simplified model (longitudinal+vertical)

\[
\begin{align*}
V_{\text{TAS}} \cdot \frac{dV_{\text{TAS}}}{dt} + g \cdot \frac{dz}{dt} &= \frac{(Thr - D) \cdot V_{\text{TAS}}}{m}
\end{align*}
\]

- \(z\): altitude
- \(Thr\) (Thrust): thrust of the engines
- \(D\) (Drag): drag of the aircraft
- \(m\): mass
- \(V_{\text{TAS}}\) (True Air Speed): velocity in the air
- \(\frac{dV_{\text{TAS}}}{dt}\): longitudinal acceleration
- \(\frac{dz}{dt} = V_{\text{TAS}} \cdot \sin(\gamma)\): rate of climb
A physical model

\[ V_{TAS} \cdot \frac{dV_{TAS}}{dt} + g \cdot \frac{dz}{dt} = \frac{(Thr - D) \cdot V_{TAS}}{m} \]

- \( V_{TAS} \cdot \frac{dV_{TAS}}{dt} \): Energy rate
- \( g \cdot \frac{dz}{dt} \): Power
- \( \frac{(Thr - D) \cdot V_{TAS}}{m} \): Mass
A physical model

\[ V_{TAS} \frac{dV_{TAS}}{dt} + g \frac{dz}{dt} = \frac{(Thr - D) \cdot V_{TAS}}{m} \]

**BADA model**

- Max climb thrust:
  \[ Thr = f(T, V_{TAS}, z) \]

- Drag:
  \[ D = f(T, V_{TAS}, z, m) \]
A physical model

\[ V_{\text{TAS}} \cdot \frac{dV_{\text{TAS}}}{dt} + g \cdot \frac{dz}{dt} = \frac{(Thr - D) \cdot V_{\text{TAS}}}{m} = f(T, V_{\text{TAS}}, z, m) \]

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Equation at a given point

\[ V_{\text{TAS}} \cdot \frac{dV_{\text{TAS}}}{dt} + g \cdot \frac{dz}{dt} = \frac{(Thr - D) \cdot V_{\text{TAS}}}{m} = f(T, V_{\text{TAS}}, z, m) \]

Using radar and weather data, we know:

- \( T \), \( z \), \( \frac{dz}{dt} \), \( V_{\text{TAS}} \), \( \frac{dV_{\text{TAS}}}{dt} \)
Equation at a given point

\[ V_{TAS} \cdot \frac{dV_{TAS}}{dt} + g \cdot \frac{dz}{dt} = \frac{(Thr - D) \cdot V_{TAS}}{m} = \frac{f(T, V_{TAS}, z, m)}{m} \]

Using radar and weather data, we know:

- \( T \), \( z \), \( \frac{dz}{dt} \), \( V_{TAS} \), \( \frac{dV_{TAS}}{dt} \)

The resulting equation:

\[ E = f(T, V_{TAS}, z, m) = \frac{P(m)}{m} \]

\( P \), a known function
Computing the \( \frac{\text{power}}{\text{mass}} \) provided by BADA

2. The adaptive method [Schultz et al., 2012]

3. The least square method [Alligier et al., 2012]

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The adaptive method [Schultz et al., 2012]

Principle

- Assuming an initial guess $m_0$
- At each point $i$, the mass $m_i$ is estimated using $m_{i-1}$
The adaptive method [Schultz et al., 2012]

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E_i = \frac{P_i (m_i)}{m_i}
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- Assuming an initial guess $m_0$
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\[
E_i = \frac{P_i}{m_i} \begin{pmatrix} m_i \\ m_i \end{pmatrix} \iff m_i = \frac{P_i}{E_i} \begin{pmatrix} m_i \\ E_i \end{pmatrix}
\]
The adaptive method [Schultz et al., 2012]

Principle

- Assuming an initial guess $m_0$
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E_i = \frac{P_i (m_i)}{m_i} \iff m_i = \frac{P_i (m_i)}{E_i} \approx \frac{P_i (m_{i-1})}{E_i}
\]
The adaptive method [Schultz et al., 2012]

**Principle**
- Assuming an initial guess $m_0$
- At each point $i$, the mass $m_i$ is estimated using $m_{i-1}$

At each new point $i$:

$$m_i = \frac{P_i(m_{i-1})}{E_i}$$
The previous update formula can be rewritten:

\[
m_i = m_{i-1} \left[ 1 + \frac{m_{i-1}}{P_i(m_{i-1})} \left( E_i - \frac{P_i(m_{i-1})}{m_{i-1}} \right) \right]^{-1}
\]

error on the energy rate when using \(m_{i-1}\)
Introduction of the sensitivity parameter $\beta$

[Schultz et al., 2012]

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Introducing a sensitivity parameter $\beta_i$:

\[
m_i = m_{i-1} \left[ 1 + \frac{\beta_i m_{i-1}}{P_i(m_{i-1})} \left( E_i - \frac{P_i(m_{i-1})}{m_{i-1}} \right) \right]^{-1}
\]
Logic of the sensitivity parameter $\beta$
[Schultz et al., 2012]

- $\beta$ low $\Rightarrow$ point $i$ has nearly no impact, $m_i \simeq m_{i-1}$
- $\beta$ is dynamically adjusted according to $\Delta \dot{E}_i, \ldots, \Delta \dot{E}_{i-p}$

**Update rule [Schultz et al., 2012]**

If $i > 0$ and $\Delta \dot{E}_i > 0.0001$ and

$$\left| \frac{\Delta \dot{E}_i - \Delta \dot{E}_{avg}}{\Delta \dot{E}_{avg}} \right| < 3$$

then

$$\beta_i = \max(0.205, \beta_{i-1} + 0.05)$$

else

$$\beta_i = 0.005$$
Logic of the sensitivity parameter $\beta$

[Schultz et al., 2012]

$$m_i = m_{i-1} \left[1 + \beta_i \frac{m_{i-1}}{P_i(m_{i-1})} \left(E_i - \frac{P_i(m_{i-1})}{m_{i-1}}\right)\right]^{-1}$$

This mechanism increases robustness

- If $\Delta \dot{E}_i$ repeatedly high in the same order of magnitude, $\beta$ will increase, strengthening adaptation
- Isolated low or high $\Delta \dot{E}_i$ has a lower impact on adaptation
Logic of the sensitivity parameter $\beta$

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The variation is limited

- The estimated mass is kept within 80% and 120% of the reference mass
- The variation is limited to 2% of the reference mass
1. Computing the $\frac{\text{power}}{\text{mass}}$ provided by BADA

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The least square method [Alligier et al., 2012]

**Principle**

- All the points are considered at once
- Minimizes the sum of square error on the energy rate

For all point $i$, $i \in J_{1,n}$, we have:

$$P_i(m_i) = m_i - E_i$$

Masses $m_1,...,m_n$ are not independent variables

⇒ Equations cannot be satisfied altogether (in general)

Then, we search $(m_1,...,m_n)$ minimizing:

$$E(m_1,...,m_n) = \sum_{i=1}^{n} \left( P_i(m_i) - E_i \right)^2$$
The least square method [Alligier et al., 2012]

Principle

- All the points are considered at once
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For all point $i$, $i \in [1; n]$, we have:

$$\frac{P_i (m_i)}{m_i} = E_i$$
The least square method [Alligier et al., 2012]

**Principle**

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For all point \( i, i \in [1; n] \), we have:

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Masses \( m_1, \ldots, m_n \) are not independent variables
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\[
\mathcal{E}(m_1, \ldots, m_n) = \sum_{i=1}^{n} \left( \frac{P_i(m_i)}{m_i} - E_i \right)^2
\]
Relationship between the $m_i$

**fuel consumption**

BADA model of the fuel consumption:

$$\frac{dm}{dt} = -f(T, V_{TAS}, z)$$
Relationship between the $m_i$

BADA model of the fuel consumption:

$$\frac{dm}{dt} = -f(T, V_{TAS}, z)$$

$$m_i = m_n + \int_{t_i}^{t_f} f(T(t), V_{TAS}(t), z(t)) dt$$

$$m_i \approx m_n + \sum_{k=i}^{n-1} f(t_k + 1) + f(t_k) \frac{2(t_{k+1} - t_k)}{t_{k+1} - t_k}$$
Relationship between the $m_i$

fuel consumption

BADA model of the fuel consumption:

\[
\frac{dm}{dt} = -f(T, V_{TAS}, z)
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\[
m_i = m_n + \int_{t_i}^{t_n} f(T(t), V_{TAS}(t), z(t)) \, dt
\]

\[
\Rightarrow m_i \approx m_n + \sum_{k=i}^{n-1} \frac{f(t_{k+1}) + f(t_k)}{2} (t_{k+1} - t_k)
\]

\[
\Rightarrow m_i = m_n + \delta_i
\]
Minimizing this error

The error function can be rewritten:

\[ E(m_1, \ldots, m_n) = E(m_n) = \sum_{i=1}^{n} \left( \frac{P_i(m_n + \delta_i)}{m_n + \delta_i} - E_i \right)^2 \]
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Minimizing this error can be done by solving:

\[ \mathcal{E}'(m) = 0 \]
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With the BADA model, \( P_i \) polynomial of the second degree.
Minimizing this error

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\[ E'(m) = 0 \]

With the BADA model, \( P_i \) polynomial of the second degree

\( \Rightarrow \) Solving \( E'(m) = 0 \) leads to find roots of a polynomial of degree at most \( 3(n-1) + 4 \)
Minimizing this error

The original error function:

\[ E(m_n) = \sum_{i=1}^{n} \left( \frac{P_i(m_n + \delta_i)}{m_n + \delta_i} - E_i \right)^2 \]

⇒ Numerical issues solving \( E'(m) = 0 \)
Minimizing this error

The original error function:

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⇒ Numerical issues solving \( E'(m) = 0 \)

An approximated error function:

\[ E_{approx}(m_n) = \sum_{i=1}^{n} \left( \frac{P_i(m_n + \delta_i)}{m_n + \delta_{avg}} - E_i \right)^2 \]

with: \( \delta_{avg} = \frac{1}{n} \sum_{i=1}^{n} \delta_i \)
Minimizing this error

The original error function:

\[ E(m_n) = \sum_{i=1}^{n} \left( \frac{P_i(m_n + \delta_i)}{m_n + \delta_i} - E_i \right)^2 \]

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with: \( \delta_{avg} = \frac{1}{n} \sum_{i=1}^{n} \delta_i \)

⇒ Solving \( E'_{approx}(m) = 0 \) leads to find roots of a polynomial of degree 4
1. Computing the $\frac{\text{power}}{\text{mass}}$ provided by BADA

2. The adaptive method [Schultz et al., 2012]

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4. Experimental setup

5. Results
Real data

Radar and weather data

- Radar Mode-C from Paris Air Traffic Control Center
- Weather data from Météo France
- 4939 trajectories of A320
- Trajectories of 12.5 minutes long (ie. 51 points)
- Each 15 seconds, we observe: $T, V_{\text{TAS}}, z, \frac{dz}{dt}, \frac{dV_{\text{TAS}}}{dt}$

Estimate a mass with observed variables
Problem: The actual mass is not known in our data
Estimate the mass on past points and predict the future trajectory.
1. Computing the $\frac{\text{power}}{\text{mass}}$ provided by BADA

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5. Results
Trajectory prediction accuracy

Computing trajectory prediction

BADA 3.9 requires:

- initial mass: \( m = \hat{m}(t = 0) \)
- speed profile: \( V_{TAS}(t) = V_{TAS}^{(obs)}(t) \)

<table>
<thead>
<tr>
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<th>mean</th>
<th>stddev</th>
<th>mean abs</th>
<th>rmse</th>
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<tr>
<td>( m_{ref} )</td>
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Table: Statistics on \( (H_p^{(pred)}(\hat{m}_{past}) - H_p^{(obs)}) \) at \( t = 600 \) s, in feet.
An example: trajectory with the largest error

Trajectory with the largest prediction error:

![Graph with time (t) on the x-axis and specific energy rate (W/kg) on the y-axis, showing a trajectory with large prediction errors. The figure includes a legend indicating the specific energy rate (Ei) and the estimated mass (\(\hat{m}_{\text{past}}^{\text{LS}}\)) with a dashed line.](image-url)
Relation between past error and future error

Quantile regression to estimate the 95% quantile

Quantile regression to estimate the 95% quantile
Relation between past error and future error

Quantile regression to estimate the 95 % quantile

$|H_p^{(pred)}(\hat{m}_{past}^{LS}) - H_p^{(obs)}| [ft]$ vs $\varepsilon^{[past]}(\hat{m}_{past}^{LS}) [W/kg]$
Relation between past error and future error

Quantile regression to estimate the 95 % quantile
Conclusion

Accuracy improvements

- When compared to $m_{ref}$, the RMSE on the altitude is reduced by:
  - 45% for the Least Square method
  - 50% for the adaptive method
- Adaptive method performs better
- Error on past points gives hints on the prediction error

Beyond accuracy

- Adaptive method:
  - Simpler to implement
  - Can be used with a black box model of the power
- Least Square:
  - Do not have a $\beta$ sensitivity parameter to be tuned
Future work and preliminary results

Machine learning techniques

- Learn the mass $\hat{m}_{\text{future}}^{LS}$
- Gradient Boosting Machine (GBM) with 10-folds cross-validation
- With all the available variables: arrival airport, aircraft operator, distance to go, etc.

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Table: Statistics on $\left( H_p^{(\text{pred})} (\hat{m}_{\text{past}}) - H_p^{(\text{obs})} \right)$ at $t = 600$ s, in feet.
Thank you, any questions?
In *International Conference on Research in Air Transportation (ICRAT), Berkeley, California, 22/05/12-25/05/12*, page (on line), http://www.icrat.org. ICRAT.

In *10th USA/Europe Air Traffic Management Research and Development Seminar*.