Airspace Network Characterization for Effect of Intermediate Waypoints on Collision Risk Assessment

Fergus Symon, Sameer Alam, Md. Murad Hossain
School of Engineering & IT,
University of New South Wales,
Australian Defense Force Academy Campus,
Canberra, Australia

Abstract—This paper investigates how estimated collision risk in upper airspace varies with changes in underlying airspace network complexity. Direct Route model (which assumes great circle route between entry and exit waypoints) and Intermediate Waypoint model (which uses airway-waypoint routes between entry and exit waypoints) were employed. One month of traffic data (over 200,000 flights) from 12 countries in the middle-east region was analyzed for collision risk estimates and the airspace network was characterized for several complex network indicators. Results show that Intermediate waypoint leads to a significant increase in collision risk estimates. Results also show correlation between estimated collision risk and specific network complexity measures. From an operational perspective this means that in airspaces with a highly structured airspace, collision risk may be underestimated when using the widely accepted direct route model.

Keywords—Collision Risk; Airways Network; Complex network

I. INTRODUCTION

One of the key challenges faced by the Air Navigation Service Providers (ANSPs) is how to accommodate continued growth in air traffic while meeting the safety targets. ANSPs are exploring new paradigms (e.g. SESAR [1] and NextGen [2]) and procedures (for e.g. Reduced Vertical Separation Minima [3]) for efficient and safe management of airspace.

Although mid-air collision is a rare event, their impact is significant due to large number of fatalities involved. International Civil Aviation Organization (ICAO) standards separating aircrafts in time and space have well served the purpose until the surge in air traffic during the last decade. ANSPs are now compelled to relax these standards and adopt new procedures to accommodate increasing traffic [4]. There is also a compelling need for safety risk assessment of these new procedures [5].

One of the vital indicators for estimating air traffic safety is the Airspace Collision Risk Assessment [6]. Most of the collision risk models are based on the Reich Model [7], which was developed in early 60’s to estimate the collision risk for flights over the North Atlantic and to specify appropriate separation rules for the flight trajectories. However, there is no one universal model for collision risk assessment due to different communication, navigation and surveillance capabilities of ANSPs in different regions of the world.

EUROCONTROL uses a sophisticated collision risk model developed by Mathematical Drafting Group which uses precision 4D radar data/ADS-B data to account for flights vectoring frequently in European airspace [8]. Whereas the African region (ARMA) and Middle-East region (MIDRMA) use the ICAO Collision Risk Model [9] based on entry and exit flight plan data due to presence of large volumes of procedural airspace and limited Communication, Navigation & Surveillance (CNS) capabilities.

In particular when airway network structure is more complex, this may lead to significant variations in collision risk estimates between different Flight Information Regions of the world. One of the motivations of this paper is how to improve the collision risk estimates given the limited amount of flight data available in regions with limited CNS capabilities. Another research question that we attempt to address is how the collision risk estimates varies with network measures for airspace network complexity. This also raises the question which airspace network measures are most significant in identifying the need for collecting more traffic data.

The characterization of airspace network features given the changes and impact on collision risk estimates may provide a motivation on better data collection practices and may identify the airspace network features where such data collection may influence the collision risk estimates significantly.

The airspace in consideration for this paper is Reduced Vertical Separation Minimum (RVSM) Airspace. Within RVSM airspace, air traffic control (ATC) separates aircraft by a minimum of 1,000 feet vertically between flight level (FL) 290 and FL 410 inclusive [3].

This paper is organized as follows. Section II outlines airspace network characteristics. Section III outlines collision risk analysis for procedural airspace. Section IV explains the steps conducted in evaluating the airspace area considered. Section V presents the network characteristics and collision risk results obtained. Section VI draws conclusions.

II. AIRSPACE NETWORK CHARACTERISTICS

Network analysis to characterize complex systems has become widespread during the last few decades where complex network frameworks have been applied in a growing range of disciplines including air traffic [12].
A. Airspace Network

In any Air Traffic Management system, airspace network features such as number of airways, crossing angle, number of crossings etc. have significant impact on collision risk [13].

![Figure 1: Airspace network modelled as a graph (network), comprising of waypoint as vertices or nodes linked by airways (links) connecting them.](image)

As illustrated in Figure 1, an airspace network can be modelled as a graph (network), comprising the waypoint as vertices or nodes linked by airways connecting them. Interestingly, many real networks, including airspace networks, share a certain number of topological properties; for example, most are small worlds [14], that is, the average topological distances between nodes increase very slowly (logarithmically or even more slowly) with increases in the number of nodes. Additionally, ‘hubs’ (nodes with very large degrees \(k\)) compared with the mean of the degree distribution \(\langle k \rangle\) are often encountered. More precisely, in many cases, the degree distributions exhibit heavy tails which are often well approximated for a significant range of values of \(k\) by a power-law behavior [15].

B. Network Characteristics

Different networks have different topological features which characterize its connectivity, interaction and the dynamical processes executed by the network [16].

The analysis, discrimination and synthesis of Airspace networks therefore rely on the use to measurements capable of expressing the most relevant topological features, which enable us to characterize the airspace properties. Several indices are used in this paper to measure the topological configuration of the airspace network.

- **Average Degree**
  \[ \langle k \rangle = \frac{1}{n} \sum_{i=1}^{n} k_i \]  
  The average degree of a network refers to the average number of neighbors a node has in the network. A major crossing point will have higher average degree.

- **Clustering Coefficient**

\[ C_i = \frac{1}{k_i(k_i-1)} \sum_{j} k_j a_{ij} a_{jk} a_{ik} \]  
This captures the local cohesiveness of the node and also represents the network transitivity. This value ranges between 0 and 1.

- **Closeness Centrality**
  \[ C_c(i) = \frac{n-1}{\sum_{j \neq i} d_{ij}} \]  
  The centrality measures the relative importance of a node within a network. Closeness centrality measures the extent to which nodes are closer to all other nodes along the shortest path and reflects their accessibility in a given network. This value ranges between 0 - 1.

- **Betweenness Centrality**
  \[ C_b(i) = \frac{\sigma_{ij}(i)}{\sigma_{kj}} \]  
  where \(\sigma_{kj}\) is the total number of shortest paths from node \(k\) to node \(j\), and \(\sigma_{ij}(i)\) is the number of those paths that pass through node \(i\). Betweenness centrality is a useful measure of the load placed on a given node in the network as well as its importance to the network than just connectivity. It measures the extent to which a particular node lies between other nodes in a network.

- **Characteristic Path Length**
  \[ L = \frac{1}{n(n-1)} \sum_{i \neq j} d_{ij} \]  
  The smaller the \(L\), the more compact the network is. Thus \(L\) could be used as an indicator of the highly dense airspace network leading to large number of crossings.

In order to better understand the relation between these network characteristics and collision risk, in the sequel of the paper both will be evaluated for a selected airspace area. Prior to doing so, we first outline collision risk analysis.

III. COLLISION RISK ANALYSIS

A. Collision Risk

Collision Risk is defined by ICAO [9] as “the expected number of mid-air aircraft accidents in a prescribed volume of airspace for a specific number of flight hours due to loss of planned separation”. The collision risk assessment methodology consists of two elements: first, risk estimation, which concerns the development and use of methods and techniques with which the actual level of risk of an activity can be estimated; and second, risk evaluation, which concerns the level of risk considered to be the maximum tolerable value for a safe system. The level of risk that is deemed acceptable was termed the target level of safety (TLS) [10]. The risk evaluation process consists of comparing the estimated risk against a TLS to provide a quantitative basis for judging the safety of air traffic operations in a given volume of airspace.
B. Collision Risk in Vertical Dimension

A mid-air collision between two aircraft nominally separated by 1000ft could only occur if either or both aircraft were to deviate vertically from their assigned flight level such that the vertical separation between the aircraft is lost. There are two main reasons why an aircraft may not be at its assigned flight level – normal height deviations and large height deviations.

Normal height deviations arise due to typical Assigned Altitude Deviation (AAD) and Altimetry System Errors (ASE) whereas large height deviations occur due to operational issues such as a level burst or TCAS alert. The focus of this paper is on normal height deviations which are also termed as technical vertical risk (TVR) as they happen due to purely technical reasons.

Technical vertical risk is computed using historic flight data and takes into account, among several factors, the accuracy of navigation, the airway structure, the aircraft population and the total flying time within the region. The assessment of the technical vertical risk requires the risk estimate to be less than the technical TLS of $2.5 \times 10^{-9}$ fatal accidents, per flight hour (noting that a mid-air collision counts at two fatal accidents). Appendix A provides the collision risk equations to be used for this risk calculation.

C. ICAO’s Form 4 Data

One of the key elements in TVR estimation is flight data collection. ICAO has stipulated the use of Form 4 Air Traffic Flow data [11] for collecting RVSM traffic data from ANSPs. The ICAO Form 4 data provides sufficient detail, but often to quite low resolution for Collision Risk models to give an estimate of TVR. ICAO Form 4 records following flight data:

- Flight date
- Aircraft call sign
- Aircraft type
- Departure aerodrome
- Arrival aerodrome
- Entry Waypoint
- Entry level
- Entry time
- Exit Waypoint
- Exit level
- Exit time

The ICAO Form 4 data is then processed to compute:

- Total flight time for each region
- Average ground speed for each region
- Number of flight crossings in each region
- Flight time proportions for each aircraft which is used to calculate:
- Average aircraft dimensions
- Altimetry system error (ASE) probability

D. Limitations of the approach

The common approach in conducting these computations is to assume that there is a great circle route between entry and exit waypoints for estimating the crossing frequency. However, in any given airspace/sector a flight may go through several intermediate waypoints before it reaches the exit point. As a result the actual flight path may not be a straight line between entry and exit waypoints but segment of chords which join the intermediate waypoints.

Assumption of great circle route between entry and exit waypoints results in a simplified airspace network structure and therefore an incorrect number of crossings computed as well as an incorrect crossing frequency which in turn affects the collision risk estimates.

IV. METHODOLOGY

The proposed assessment methodology comprises of two stages.

In stage 1, technical vertical risk is computed using the direct route model. In this stage, an airspace network is generated using the entry and exit point data extracted from the Form 4 data. Network analysis is performed and network characteristics are identified.

In stage 2, technical vertical risk is again computed, using the Intermediate Waypoint Model. Again, the airspace network is generated incorporating the intermediate waypoints between entry and exit points using ICAO Form 4 data.

As illustrated in Figure 2, ICAO Form 4 data is the basis for the flight data input to the two models. Both the models use the same collision risk model and databases for aircraft positional error distribution and kinematic factors (speed and dimension).

In this paper we use one month traffic data from 12 countries in the Middle-East (MIDRMA) region. Two risk assessment exercises were carried out; one with direct route model (great circle route between airspace entry and exit point) and with the flight route model with intermediate waypoints between entry and exit points. In both the exercises the airspace network features (with and without intermediate waypoints) were characterized for each country and analyzed for their effect on collision risk estimates.

A. Intermediate Waypoints Model

The introduction of an intermediate waypoint model has several effects. The most immediate is the increased likelihood
for an aircraft crossing to occur. This is because it is not uncommon for two flight paths, if represented by the naive flight path (Figure 3, left) not to have any possible intersection. However, often the airway structure is such that these two flights will meet along a common path and then split and deviate, as seen in Figure 3 (right).

Another introduced effect of an intermediate waypoint model is an increased average ground speed. This is because of the additional distance covered by each flight in the ICAO Form 4 data. While this has no effect on the crossing frequency, while calculating the expected number of fatal accidents $N_{az}$, the increased ground speed $V$ will drive a minor decrease in the expected number of fatal accidents (often balancing the increase in crossing frequency).

B. Crossing Frequency Model

The passing frequencies are the frequency with which two aircraft at adjacent flight levels pass each other. They can be in the same direction ($n_z$(same)), opposite directions ($n_z$(opp)) or pass each other on crossing tracks ($n_z(\theta)$).

The same and opposite direction crossing frequencies can be calculated by taking the number of plane passing’s, dividing by the total hours of flight and multiplying by the probability of lateral overlap as shown in equation (6) and (7)

$$n_z(\text{same}) = \frac{\text{number of crossings} \times P_y(0)}{\text{total flight time in FIR/UIR}}$$

$$n_z(\text{opp}) = \frac{\text{number of crossings} \times P_y(0)}{\text{total flight time in FIR/UIR}}$$

The crossing traffic frequency is calculated in a similar manner as the same and opposite directions (with a value calculated for each crossing angle). However, it is not multiplied by the probability of lateral overlap and a larger crossing diameter is taken when the crossings are counted. This is because if the crossings were counted on the average aircraft diameter ($\lambda_{xy}$), this would result in a very small number of crossings. Therefore, a larger, proximity, distance ($\delta_x$) is taken in order to better estimate the frequency. The number of crossings is then scaled down by a factor of the aircraft diameter on the proximity distance as shown in (8).

$$n_z(\theta) = \frac{\text{number of crossings} \times \frac{\lambda_{xy}}{\delta_x}}{\text{total flight time in FIR/UIR}}$$

C. Counting Crossings

The two pieces of required information for calculating the number of crossings for a FIR/UIR is the complete ICAO Form 4 data for the time period and a list of waypoints and their coordinates corresponding to the names used in the ICAO Form 4 data.

The first step in the process is to read in the list of waypoints and their coordinates. These are stored for use in the later calculations.

The second step in the process is to read in the ICAO Form 4 data, filtering out any data that is either incomplete or suspected to be incorrect. From the first pass of the data, the number of flights ($N$), total flying time ($T$, in hours) and average ground speed ($\bar{V}$, in knots) can be calculated. Additionally taken is a list of entry-exit point pairs flown within the FIR/UIR.

The third step is to determine the crossing pairs within the data. This is done by taking the list of entry-exit points from the ICAO Form 4 data scan and computing whether the great circle arc formed by that flight path intersect with any of the other entry-exit great circle arcs.

Finally, each flight in the ICAO Form 4 data is processed to count the number of flights that they intersect with. This is done by picking a flight and checking it against all of the other flights. For each flight that they are adjacent to, their entry and exit points are compared, if they are both the same then the flights are checked whether they intersect in either the same or opposite directions. If the flight has a different entry and exit pair then it is checked whether the two entry-exit pairs intersect and if they do then it is checked whether the two flights intersect in a crossing path.

V. EXPERIMENTS

A. Region and Traffic Data

ICAO’s Middle East Regional Monitoring Agency (MIDRMA) is the administrator of the RVSM airspace in the Middle-East region.

As shown in Figure 4, MIDRMA region consists of following countries Bahrain, Egypt, Iran, Iraq, Jordan, Kuwait, Lebanon, Oman, Qatar, Saudi Arabia, Syria, UAE and Yemen.
MIDRMA provided Air Traffic data, waypoint data and aeronautical information data for all the 12 member states. Data was collected for the month of Oct 2011 for all the member countries using ICAO Form 4. In total there were 203,764 flights flying in the RVSM airspace (FL 290 to FL 420 inclusive) in the region. Table I summarizes the number of flight in each region for the Month of Oct 2012 and the total number of flight hours flown.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Flight Data (Oct 2011) RVSM Airspace</th>
<th>Number of Flights</th>
<th>Flight Duration (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bahrain</td>
<td>39206</td>
<td>23624</td>
<td></td>
</tr>
<tr>
<td>Egypt</td>
<td>26322</td>
<td>18160</td>
<td></td>
</tr>
<tr>
<td>Iran</td>
<td>17030</td>
<td>20165</td>
<td></td>
</tr>
<tr>
<td>Iraq</td>
<td>2810</td>
<td>2795</td>
<td></td>
</tr>
<tr>
<td>Jordan</td>
<td>6277</td>
<td>1513</td>
<td></td>
</tr>
<tr>
<td>Kuwait</td>
<td>12122</td>
<td>3395</td>
<td></td>
</tr>
<tr>
<td>Lebanon</td>
<td>1151</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>Oman</td>
<td>30000</td>
<td>18846</td>
<td></td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>7716</td>
<td>2049</td>
<td></td>
</tr>
<tr>
<td>Syria</td>
<td>7716</td>
<td>5398</td>
<td></td>
</tr>
<tr>
<td>UAE</td>
<td>20725</td>
<td>3445</td>
<td></td>
</tr>
<tr>
<td>Yemen</td>
<td>5025</td>
<td>23624</td>
<td></td>
</tr>
</tbody>
</table>

**B. Intermediate Waypoints**

In order to collect information about intermediate waypoint for given entry and exit points in an FIR/UIR, MIDRMA issued a circular to all member states to develop a database for all the entry and exit points and the most commonly flown route on them in their respective regions. The data collection and verification exercise underwent for over two months period. All twelve member states collected and reported data on intermediate waypoints for all the entry exit points in their respective FIR/UIRs.

**C. Experimental Parameters & Supplementary data**

Table II shows the various collision risk model parameters used in the experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length ($\lambda_x$)</td>
<td>173.51 ft</td>
</tr>
<tr>
<td>Wingspan ($\lambda_y$)</td>
<td>163.35 ft</td>
</tr>
<tr>
<td>Diameter ($\lambda_{xy}$)</td>
<td>159.91 ft</td>
</tr>
<tr>
<td>Height ($\lambda_z$)</td>
<td>45.451 ft</td>
</tr>
</tbody>
</table>

Aircraft dimension parameters (representing the average dimension of aircraft that fly in the region) are calculated and weighted as per the flight time proportion of each aircraft group.

Based on member state provided navigation data, the proportion of flights flying with satellite navigation in MIDRMA region was set to 75%. Airspeed parameters were used as recommended by ICAO [3]. Aircraft performance was modeled using Eurocontrol’s Base of Aircraft Data (BADA). For computing Probability of Vertical Overlap the Eurocontrol’s ASE parameter database and AAD flight sample lists were used.

The collision risk computation process is illustrated in Figure 5. First the countries for which the collision risk is to be done are selected. Various supplementary data files such as Waypoint/Airport names and coordinates, BADA database, ASE and AAD parameter, aircraft dimension files are then read and processed. After that the flight data for the selected countries is read and processed to compute flight time proportion and crossing frequencies. Probability of lateral overlap and probability of vertical overlap are then computed. These intermediate results are then inputted into equation (9) and technical vertical risk is computed.
VI. RESULTS

A. Collision Risk and Passing frequency
We first present the results of crossing frequency per flight hour and the technical vertical risk with direct route model and intermediate waypoint model. As can be seen from Table III, and Figure 6, with intermediate waypoint model, Egypt, Iraq Lebanon and Oman had shown a significant increase in crossing frequency as well as technical vertical risk. Bahrain, Iran and Saudi Arabia did not have any significant change in their crossing frequency and technical vertical risk. Most surprisingly UAE has shown a decrease in its crossing frequency as well as technical vertical risk.

<table>
<thead>
<tr>
<th>Nation</th>
<th>Direct Route</th>
<th>Inter. Waypoint</th>
<th>Direct Route</th>
<th>Inter. Waypoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bahrain</td>
<td>0.020211</td>
<td>0.02019</td>
<td>3.63E-11</td>
<td>3.62E-11</td>
</tr>
<tr>
<td>Egypt</td>
<td>0.019650</td>
<td>0.02843</td>
<td>3.53E-11</td>
<td>5.10E-11</td>
</tr>
<tr>
<td>Iran</td>
<td>0.023403</td>
<td>0.02348</td>
<td>4.20E-11</td>
<td>4.21E-11</td>
</tr>
<tr>
<td>Iraq</td>
<td>0.009957</td>
<td>0.05476</td>
<td>1.79E-11</td>
<td>9.82E-11</td>
</tr>
<tr>
<td>Jordan</td>
<td>0.012597</td>
<td>0.01288</td>
<td>2.26E-11</td>
<td>2.31E-11</td>
</tr>
<tr>
<td>Kuwait</td>
<td>0.000297</td>
<td>0.00117</td>
<td>5.34E-13</td>
<td>2.10E-12</td>
</tr>
<tr>
<td>Lebanon</td>
<td>0.003515</td>
<td>0.00872</td>
<td>6.31E-12</td>
<td>1.56E-11</td>
</tr>
<tr>
<td>Oman</td>
<td>0.027840</td>
<td>0.04504</td>
<td>5.00E-11</td>
<td>8.07E-11</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>0.020981</td>
<td>0.02096</td>
<td>3.77E-11</td>
<td>3.76E-11</td>
</tr>
<tr>
<td>Syria</td>
<td>0.028031</td>
<td>0.02904</td>
<td>5.03E-11</td>
<td>5.21E-11</td>
</tr>
<tr>
<td>UAE</td>
<td>0.009877</td>
<td>0.00640</td>
<td>1.77E-11</td>
<td>1.15E-11</td>
</tr>
<tr>
<td>Yemen</td>
<td>0.006218</td>
<td>0.00720</td>
<td>1.12E-11</td>
<td>1.29E-11</td>
</tr>
</tbody>
</table>

The highest variability can be seen in Iraq and Oman where the collision risk increase significantly with the intermediate waypoint model.

Similarly, the increase in the collision risk estimate of Iraq, for intermediate waypoint model, can be attributed to significant increase in crossings (5 fold increase) due to crossing traffic from Iran and Saudi Arabia.

B. Network Characteristics Analysis
We then analyzed airspace network with Direct Route and Intermediate Waypoint model.

Average Degree (Figure 8): For intermediate waypoint model the average degree for almost all the countries (except Lebanon) increased significantly. This mainly occurred due to
presences of major crossings (especially in large airspaces) which were not captured in direct route model affecting the collision risk computation. Lebanon is a small FIR with a semi-circular design. The airspace structure is simple with all airways from the boundary of FIR merging at the Beirut VOR.

Clustering Coefficient (Figure 9): The nodes in Bahrain, Lebanon and Syrian airspace network appear to have higher tendency to form clusters in intermediate waypoint model. In Egypt, Iran, Saudi Arabia lower clustering coefficient indicates low collision risk.

Closeness Centrality (Figure 10): For Iran and Saudi Arabia, given their large airspaces, the closeness centrality is very low. For Lebanon, this measure is high (in both the models) due to very small airspace and very few airways merging at VOR. In Bahrain and Syria it is reduced for intermediate waypoint model due to their structured airspace leading to minimal or no change in collision risk estimates.

Betweenness Centrality (Figure 11): As expected this measure has gone down for all the FIRs in MIDRMA except Lebanon. The Intermediate waypoint model reduces the possibility of a particular node lying between other nodes, as oppose to the direct route model, in a network. This indicates that the more unstructured a network is will lead to a higher collision risk.

Characteristic Path Length (Figure 12): The airspace network of Iran and Saudi Arabia appears to be denser (lower characteristic path length) with the intermediate waypoint network. This is possibly due to the presence of large areas of procedural airspace. A denser network results in higher collision risk estimates.

VII. CONCLUSIONS

Collision Risk was estimated using one month data from 12 countries in the Middle East region. Two models were employed, one uses direct routes and other uses intermediate waypoints for aircraft flying at RVSM altitude. Complex network measures were employed to analyze the resulting two networks to gain an insight into how the collision risk estimates varies with network measures for airspace network complexity. Results indicate that the Intermediate waypoints leads to a significant increase in collision risk estimates specifically for airspace networks with higher average degree and higher closeness centrality measures. Results also indicate that Collision Risk decreases in networks with lower betweenness centrality. It was also found that a denser network results in higher collision risk estimates. From an operational point of view, results indicate that countries which have highly structured airspace are actually overestimating the collision risk with direct route model.
In future, we will extend this work by developing new clustering algorithms to identify presence of clusters in the airspace network and assessing their impact on collision risk.

ACKNOWLEDGMENT

The first two authors would like to thank Mr. Fareed Alawai and Mr. Fathi Thawadi of ICAO-MIDRMA for providing the necessary traffic data and aeronautical information for the MIDRMA.

APPENDIX A. VERTICAL COLLISION RISK EQUATIONS

Technical vertical risk represents the risk of a collision between aircraft on adjacent flight levels due to normal or typical height deviations of RVSM approved aircraft. The technical vertical collision risk is assessed against a technical TLS of $2.5 \times 10^{-9}$ fatal accidents per flight hour using a suitable collision risk model.

Following [1], the vertical collision risk model for aircraft on adjacent flight levels of the same route, flying in either the same or the opposite direction satisfies:

$$N_{az} = 2P_x(S_z)P_y(0)n_z\text{(equiv)}\left[1 + \frac{\bar{V}}{2\bar{V}} + \frac{\lambda_{xy} \bar{z}}{\lambda_x 2\bar{V}}\right]$$  \hspace{1cm} (9)

where

$$n_z\text{(equiv)} = n_z\text{(opp)} + n_z\text{(same)} - \frac{\bar{V}}{2\bar{V}} + \frac{\lambda_{xy} \bar{z}}{\lambda_x 2\bar{V}}$$

$$+ \frac{1}{P_x(0)}\left[1 + \frac{\bar{y}}{2\bar{V}} + \frac{\lambda_{xy} \bar{z}}{\lambda_x 2\bar{V}}\right]\sum_{i=1}^{n} n_z(\theta_i)$$

$$+ \frac{\pi \lambda_{xy}}{V_{rel}(\theta_i) 2\lambda_x}$$  \hspace{1cm} (10)

with the various symbols in (1)-(2) explained below.

The left-hand side variable $N_{az}$ represents the expected number of aircraft accidents due to normal technical height deviations of RVSM approved aircraft for the given traffic geometry. The longitudinal overlap frequency parameters $n_z\text{(same)}$ and $n_z\text{(opp)}$ together with the kinematics factors in brackets (as functions of the relative speeds and aircraft dimensions) represent a major part of the different levels of exposure to the risk of the loss of vertical separation for the two traffic geometries covered by the collision risk model of equation (9). (The subscript $z$ in $n_z\text{(same)}$ and $n_z\text{(opp)}$ refers to aircraft on adjacent flight levels.

There are two aircraft dimensions used by the technical vertical risk, being the average diameter ($\lambda_{xy}$) and the average height ($\lambda_x$). The probability of vertical overlap ($P_x(S_z)$) is the probability that two aircraft will overlap vertically, separated by 1000ft ($S_z$). This indicates the probability that they will overlap while correctly flying at adjacent flight levels. The probability of lateral overlap ($P_y(0)$) is the probability of two aircraft being in lateral overlap, if they are both correctly flying at adjacent flight levels. This is calculated by taking the proportion of time that an aircraft in the region are flying using satellite navigation (GNSS) versus radio navigation (VOR/DME).

There are five relative speed parameters that appear in the technical vertical risk:

- \(\Delta V\) is the relative along-track airspeed between two aircraft flying at adjacent flight levels and flying in the same direction.
- \(\bar{V}\) is the average ground speed of the aircraft.
- \(\bar{y}\) is the average relative cross-track speed between two aircraft flying at adjacent flight levels.
- \(\bar{z}\) is the average relative cross-track vertical speed between two aircraft that have lost $S_z$ feet of vertical separation.
- \(V_{rel}(\theta)\) is the average horizontal relative speed between aircraft flying at adjacent flight levels and intersecting at an angle given by the equation (11):

$$V_{rel}(\theta) = \bar{V}\sqrt{2(1 - \cos \theta)}$$  \hspace{1cm} (11)

REFERENCES