Analysis of the Impact of Intent Uncertainty on the Accuracy of Predicted Trajectories for Arrival Management Automation

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Abstract—Accurate trajectory predictions are required for high efficient Air Traffic Management (ATM) procedures. However, this process is strongly affected by the knowledge of the actual performance of the aircraft, the real weather that will affect the flight and the knowledge about how the aircraft will be operated throughout the trajectory. In addition, all automation tools present some limitations which constrain the capability of reducing the errors between predicted and actual trajectories. This paper presents a process for reducing the impact of such limitations in the case of continuous descent approaches by using ADS-B tracks. This method aims at finding the aircraft intent which best fits the recorded flight data based on the restrictions of a selected trajectory prediction infrastructure. Based on this optimal aircraft intent, a sensitivity analysis of the impact of the intent uncertainties in the described descent procedures is also presented.

Keywords: aircraft trajectory prediction; uncertainty characterization; arrival management automation

I. INTRODUCTION

The efficiency of both current and future ATM systems relies on the capability of accurately predicting the evolution of traffic with time. This capability constitutes a key enabler of the future Trajectory Based Operations (TBO) paradigm which will evolve the ATM processes from the current tactical, airspace centered management towards a highly automated, strategic and trajectory-focused environment.

Nowadays, there are three main ATM modernization initiatives (Single European Sky (SES) in Europe [1], NextGen in United States [2] and the Australian ATM Strategic Plan (AATMSP) in Australia [3]) which establish specific targets in terms of capacity, efficiency, safety and environmental impact. The accomplishment of these targets needs to be validated in order to evaluate how well the end users are exploiting their own capabilities, and those provided by the ATM infrastructure. However, in all circumstances a high fidelity representation of the actual traffic is the cornerstone upon which the metrics will be defined.

On board, the Flight Management System (FMS) has at its disposal accurate information about the aircraft performance for the specific aircraft which it is installed on, the actual weather conditions and the actual aircraft state. Access to such information leads to very accurate trajectory predictions which are used to command and control the aircraft in order to obtain the maximum adherence to the reference trajectory. However, the main drawback of the FMS is its no accessibility to precise weather forecasts due to the size of such information packages and the limitations of the data link bandwidth in flight.

On ground, the Decision Support Tools (DST) lack from real time information about aircraft performance or actual aircraft state, but have access to the most up-to-date weather forecasts and make use of the actual state of the whole traffic within a specific area, in order to ensure that separation minima among individual trajectories is preserved in all circumstances [4].

Thus, predictability is considered a primary capability which will facilitate the ATM processes and procedures, especially in high dense environments such as the Terminal Maneuvering Areas (TMA), where arrival, departures and cruise overflights operate in the same area at the same time. In such complex environment, the prediction of descending trajectories requires special attention due to their influence on the surrounding traffic and their sensitivity to small variations of the condition under which the predictions have been calculated. In particular, the sensitivity analysis of Continuous Descents Approaches (CDA) turns out to be of high interest because their importance in the TBO [5]. CDA procedures may be characterized by idle thrust descent which aims at reducing the fuel consumption, and therefore, emissions and noise.
impact, by optimizing the flight segment between the Top of Descent (TOD) until at least the metering fix point. The prediction of such trajectories is very sensitive to most of the factors involved in the prediction process [6]: wind forecast, idle thrust modeling (aircraft performance modeling in general), TOD location and aircraft intent description.

However, the problem of performing a realistic sensitivity analysis begins with the need of obtaining the best representation of the actual trajectory considering the limitations of the ground-based trajectory predictors (TP). Commonly, these tools are not capable of representing the whole variety of aircraft behaviors, and therefore, it is required to calculate the best fit to the actual trajectory considering the computational constraints of the considered TP. Once the best possible trajectory representation is available, a sensitivity analysis can be systematically performed in order to evaluate the influences of different factors on the predictions.

The remainder of the paper is structured as follows. Section II describes the methodology applied for obtaining the best possible prediction based on the use of recorded flight data and the formulation of the sensitivity analysis which will make use of such best prediction. Section III presents the results of the analysis performed to the selected study modeling the intent uncertainty by means of probabilistic distributions. Section IV evaluates the impact of the intent uncertainties on the selected state variables. Finally, Section V includes the main conclusions and remarks of the study and outlines future research lines.

II. METHODOLOGY

A. Modeling of CDA procedures

The process of describing the CDA procedures depends basically on the functionality of the TP which will compute the predicted descents. The most common approach followed by the majority of ground-based TP is to reduce the prediction problem to a 3 degrees of freedom (3DoF) problem where the aircraft is considered as a mass point subject to external forces, under symmetric and coordinated flight conditions, whose mass variation is only caused by the fuel consumption. The aircraft maneuverability is also restricted to small angles of attack and subsonic Mach numbers. These simplifications have proved to be sufficiently accurate for ATM purposes, and therefore, simplify the description of the trajectory to a system of differential equations which requires from 2 vertical and 1 lateral constraints for being mathematically solvable.

In general terms, there are different alternatives to describe CDA procedures which basically are characterized by low engine thrust settings and, where possible, a low drag configuration [7]. A typical description can be represented by a descent at idle engine regime with a Mach/CAS\(^1\) transition from the TOD until a designated metering fix point (e.g., 10,000 ft of pressure altitude). Although, there are other alternatives for describing those procedures (constant path angle while idle thrust), it is assumed that the limitations of the considered TP do not allow the description of the CDAs by other different formulations.

Making use of the Aircraft Intent Description Language (AIDL) [8][9], a CDA procedure can be described as follows:

- **Lateral Profile.** Description of the route to be followed within the TMA which respects all Air Traffic Control (ATC) restrictions. This route is given by a sequence of geodesics and circular arcs.

- **Vertical Profiles.** The first profile describes how the speed is to be controlled to execute the planned Mach/CAS transition, while the second establishes the instant from which the aircraft changes the cruise phase (constant pressure altitude and Mach speed) to the idle descent.

- **Configuration Profiles.** During the flight segment under study the aircraft will operate in clean configuration which implies that the landing gear is maintained retracted, the high lift devices are not extended and the speed brakes not used throughout the whole trajectory.

The descent is assumed to finish at 10,000 ft of pressure altitude.

Such description of the CDA procedure basically provides two parameters for adjusting the predictions: the position of the TOD, and the CAS value at which the speed control change from Mach to CAS.

B. Best prediction fitting to recorded flight data

The limitations of the considered ground-based TP constrain the ability to accurately predict the actual trajectories. However, based on the parameters available in the description of the aircraft intent, it is possible to compute the best prediction taking advantage of recorded real flight data.

Considering a set of real flight data originated, for instance, from the aircraft and shared through the Automated Dependent Surveillance – Broadcast (ADS-B) system, it is possible to define an optimization problem which returns the best predicted trajectory restricted to the limited description of the aircraft intent defined in previous Section II.A. The set of real flight data usually comprises the following information:

- **Time stamp (t).** This is the time at which the broadcast of information is executed.
- **Longitude (\(\lambda\)) and latitude (\(\phi\)).** Horizontal projection of the aircraft position respect to the ground at the instant defined by the time stamp.
- **Pressure altitude (Hp).** Altitude of the aircraft respect to the atmosphere barometric conditions at the instant defined by the time stamp.
- **Ground speed (Vg).** Module of the aircraft speed respect to the ground at the instant defined by the time stamp.
- **Heading (\(\chi\)).** Direction of the aircraft speed respect to ground at the instant defined by the time stamp.

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1. Calibrated Airspeed

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ADS-B reports also provide the corresponding ICAO aircraft type designator, which can be used to select the appropriate aircraft performance model.

Once real flight data are available, and appropriate aircraft performance and weather models are selected, the optimization problem which provides the best fit to the real flight data can be defined by the minimization of the vertical error between the real and computed trajectories subject to the a set of constraints which prevent trajectories from going out of the flight envelope,

\[
\min \text{RMS}_{Hp}(m_0, \text{CAS}, \text{TOD}) \text{such that}
\]

\[
\begin{align*}
\text{OEW} & \leq m_0 \leq \text{MTOW} \\
\text{CAS}_0 & \leq \text{CAS} \leq \text{CAS}_{op} \\
\text{TOD} - 20 \, s & \leq \text{TOD} \leq \text{TOD} + 20 \, s
\end{align*}
\]

where the \( \text{RMS}_{Hp} \) represents the root mean square altitude error calculated over the whole trajectory.

\[
\text{ERR}_{Hp} = H\hat{p}(\lambda, \phi)|_{\text{real}} - H\hat{p}(\lambda, \phi)|_{\text{computed}}
\]

\[
\text{RMS}_{Hp} = \sqrt{\frac{\sum_i^N \text{ERR}_{Hp}^2}{n}}
\]

From the selected optimization parameters, the initial aircraft mass \( m_0 \) and the CAS are initially unknown and need to be determined by the optimization process, while the instant of the TOD can be easily obtained from the ADS-B data. However, letting small fluctuations of this point around the real position (±20s) leads to a lower \( \text{RMS}_{Hp} \), and therefore, to a more accurate fitting (to the cost of an increased error of the TOD position).

The initial mass \( m_0 \) is a basic parameter which impacts significantly the computed trajectory. This influence depends on the type of aircraft intent which describes the trajectory. This information is not accessible by ground-based automation tools because airlines consider it as sensitive proprietary information which is intrinsically linked to the commercial strategy of the company. Nevertheless, the optimization process provides the value of the initial aircraft mass which attains the best trajectory fit without using no additional information than the restriction that \( m_0 \) must lie within the standard operational limits for the given aircraft type defined by the Operating Empty Weight (OEW) and the Maximum Take Off Weight (MTOW).

Finally, the constraint on the CAS ensures that no speeds lower than the initial value (\( \text{CAS}_0 \)) or higher than the operational \( \text{CAS}_{op} \) at the considered altitude are returned as output of the optimization process. As the descent starts, the CAS increases while the altitude decreases. Hence, Mach/CAS transitions at values lower than \( \text{CAS}_0 \) do not have sense. \( \text{CAS}_0 \) can be obtained from the ADS-B data following the next process:

- Calculation of the initial True Airspeed (\( \text{TAS}_0 \)).

Equation (4) shows the relationship of the ground speed vector (\( \vec{V}_g \)) and the wind vector (\( \vec{W}_w \)) with True Airspeed vector (\( \vec{V}_{TAS} \)).

\[
\vec{V}_{\theta_0} = \vec{V}_{TAS_0} + \vec{W}_w
\]

The vector \( \vec{W}_w \) represents the three wind speed components as functions of the time and geolocation provided by the selected weather model. The vector \( \vec{V}_{\theta_0} \) can be directly obtained from the surveillance data package by means of the ground speed module (\( V_g \)) and the heading (\( \chi \)).

- Considering the actual atmospheric conditions, the expression which relates TAS and CAS is defined by (5).

\[
\text{CAS} = \sqrt{\frac{2 \kappa}{\kappa - 1} \left( \frac{p_0}{\rho_0} \left( \frac{1 + \frac{\kappa - 1}{2 \kappa} \frac{\rho}{\rho_0} \frac{T}{T_0} - 1 \right) \right)^\frac{\kappa - 1}{\kappa} - 1}
\]

where \( \kappa \) is the air adiabatic index; \( p_0 \) and \( \rho_0 \) are the standard pressure and density at mean sea level (MSL); \( p \) and \( \rho \) are the static pressure and air density at the considered altitude.

As part of the initial conditions required for computing the predicted continuous descending trajectory under the aforementioned hypothesis, the initial Mach number (\( M_0 \)) needs also to be calculated, since this value will be maintained constant until the crossover altitude defined by the transitioning CAS. Following Equation (6) relates \( M_0 \) with \( \text{TAS}_0 \):

\[
M_0 = \frac{\text{TAS}_0}{a_0} = \frac{\text{TAS}_0}{\sqrt{\kappa R T_0}}
\]

where \( a \) represents the speed of sound at the referred atmosphere condition; \( R \) is the specific air constant; and \( T \) the atmospheric static temperature.

C. Sensitivity of CDA predictions to aircraft intent errors

Considering that it is possible to obtain a high fidelity prediction of a real CDA trajectory based on the methodology exposed above, the problem of analyzing the influence of the errors introduced by the inputs required for computing a prediction can easily decoupled.

The process of computing a prediction basically requires from the following inputs:

- an Aircraft performance model (APM), which provides the basic performance characteristic required for integrating the aircraft motion problem.
- a Weather model (WM), which provides the atmospheric conditions (temperature and pressure) and the wind field.
- a set of initial conditions (IC).
• a description of the aircraft intent (AI).

Although all inputs add errors to the output trajectory prediction, the results presented in this paper are mainly focused on evaluating the sensitivity of the predictions to aircraft intent errors. The errors introduced by the remaining inputs are considered of an order of magnitude lower than those introduced by the variations of the aircraft intent. Nevertheless, to minimize their effect, the best approach is to use a very accurate APM (such as BADA\textsuperscript{2} models [10]) and a WM generated by a posteriori re-analysis of the weather forecasts incorporating the actual measured weather.

The error in the initial conditions is almost negligible because the first point of the ADS-B data package is used by the prediction infrastructure as initial aircraft state. The only parameter which needs to be studied in detail is the initial aircraft mass $m_0$ because there is no information about that in the surveillance data. As explained in the previous section, the mass $m_0$ is obtained as output of the optimization process. Thus, the influence of variations of such initial condition needs to be evaluated to understand how the prediction capabilities of the considered TP are affected.

The aircraft intent uncertainties can range from small variations in the instants at which the aircraft changes its behavior to totally different strategies to command and control the planned trajectory. Assuming the limitations of the selected TP in use, and the fact that the most suitable AI is available thanks to a previous fitting process, the problem of studying the sensitivity of the predictions is reduced to evaluate the following sources of uncertainty:

- Location of the TOD, defined by the time at which the descending path begins.
- CAS value at which the transition Mach/CAS is performed.

This approach does not consider uncertainties in the lateral path, because basically the predicted trajectory can match with high fidelity the real flown path flown.

Due to the considered description of the AI, there are state variables which are considered as invariants. An invariant is a variable which is not affected by the uncertainty introduced by the inputs. In this case, the invariants of the trajectory are:

- Initial pressure altitude ($H_{p0}$). Cruise altitude defined by the recorded flight data.
- Initial Mach speed ($M_{0}$). Cruise Mach defined by the recorded flight data.
- Final pressure altitude ($H_{f}$). The trajectory in all circumstances is defined by a CDA from the cruise altitude down to 10,000ft.
- CDA description. Mach/CAS profile with the throttle set to idle.

The input uncertainties are therefore supported by other variables which depend strictly on the description of the AI. In this case, the variables affected by the AI uncertainties are:

- Flown distance. Any variation of the AI will produce deviation in the total distance flown until reaching the final target altitude.
- Flight duration. The effect of the modifications of the AI will lead to variations on the total spent time.
- Fuel consumption. This is the most sensitive variable. Any variation of any of the inputs produces an impact on the total amount of fuel consumed during the flight.

The sensitivity analysis exposed in the following section will be focused on the study of the first two state variables because they are the most interesting ones from the ATM perspective. The variation of the fuel consumption, although might be useful for environmental impact assessment for instance, is not considered because is meaningless for controlling the arrival traffic nearby an airport.

III. METHODOLOGY VALIDATION BASED ON REAL FLIGHT DATA

The use of available ADS-B tracks enable the capability of adjusting the features of limited TPs to try to reduce as much as possible the prediction errors. To that aim, an optimization process can be defined to obtain the best fit to the recorded flight data. This process requires accurate representations of the aircraft performance and the weather faced by the aircraft along the trajectory.

BADA provides two different APM to reproduce real aircraft performance, the BADA3 family and the BADA4 family. Both families are accurate representations of the nominal performance of the majority of current commercial aircraft, but use different model specifications and performance datasets.

To accurately reconstruct the aircraft trajectory, a high fidelity weather model which provides with the meteorological conditions that the aircraft encountered during the flight, chiefly the temperature, pressure and wind (both direction and magnitude) is paramount. To that aim, two meteorological forecasts from the following sources were used:

- United States National Oceanic and Atmospheric Administration (NOAA), in particular, the output of the GFS (Global Forecast System) Grid 4 Domain models, which are freely accessible through the NOAA website [11].
- European Centre for Medium-Range Weather Forecasts (ECMWF), which makes use of sophisticated numerical models and weather observations to generate high accurate weather models [12].

A. Best prediction fitting to recorded flight data

For the validation of the proposed methodology is has been selected a real ADS-B track which represent a CDA from cruise altitude to a metering fix at which the aircraft (B747-400) reaches the target altitude of 10,000ft.

\textsuperscript{2} Base of Aircraft DAta
Based on the recorded flight data, the inputs to the optimization process are:

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.847</td>
<td>498.8</td>
<td>285.3</td>
<td>225</td>
<td>22.279</td>
</tr>
<tr>
<td>Wₓ₀ [m/s]</td>
<td>Wᵧ₀ [m/s]</td>
<td>W₂ [m/s]</td>
<td>a₀ [m/s]</td>
<td>Hp₀ [ft]</td>
</tr>
<tr>
<td>-6.03</td>
<td>36.96</td>
<td>0</td>
<td>301</td>
<td>36,000</td>
</tr>
</tbody>
</table>

In order to obtain the best possible results, 6 alternatives (3 APMs and 2 WMs) were evaluated.

The BADA 3 family provides a unique model for the B747-400 aircraft type, which is named as B744, while the BADA4 family provides two models, the B744GE and the B744ERGE. The former model represents a B747-400 equipped with General Electric CF6-80C2B1F engines with a MTOW of 396,894kg, while the latter models the performance of an extended range B747-400 equipped with General Electric CF6-80C2B5F engines with a MTOW of 412,770kg.

The appropriate WMs form NOAA and ECMWF, defined over the geographical area and time period under consideration were used with the three possible APMs. Table II summarizes the results obtained as output of the optimization process defined in Section II.B.

<table>
<thead>
<tr>
<th>APM</th>
<th>WM</th>
<th>m₀ [kg]</th>
<th>CAS [kn]</th>
<th>TOD [sec]</th>
<th>RMS₁₀₀ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B744</td>
<td>GFS</td>
<td>269,370</td>
<td>301.8</td>
<td>950</td>
<td>51.8</td>
</tr>
<tr>
<td>B744GE</td>
<td>GFS</td>
<td>210,013</td>
<td>288.5</td>
<td>950</td>
<td>26.5</td>
</tr>
<tr>
<td>B744ERGE</td>
<td>GFS</td>
<td>210,006</td>
<td>288.3</td>
<td>949.5</td>
<td>26.5</td>
</tr>
<tr>
<td>B744</td>
<td>ECMWF</td>
<td>269,370</td>
<td>301.8</td>
<td>950</td>
<td>51.8</td>
</tr>
<tr>
<td>B744GE</td>
<td>ECMWF</td>
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<td>949.5</td>
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</tr>
<tr>
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<td>ECMWF</td>
<td>210,006</td>
<td>288.3</td>
<td>949.5</td>
<td>26.5</td>
</tr>
</tbody>
</table>

The use of BADA4 models provides better fitting results than the BADA3 model, which suggests that, in this particular case, it is more appropriate to use the B744GE or the B744ERGE. These two models present the same results because they represent basically the same aircraft but equipped with slightly different power plants and with extra fuel tanks in the case of the extended range model.

The influence of assuming one of the two proposed WMs in the final results is almost null. Both models provide enough fidelity of the actual weather conditions, and therefore, both are good alternatives for executing the planned analysis.

Fig. 1 shows a 3D representation of the recorded trajectory (blue) and the prediction (red) obtained as output of the optimization process when the used APM and WM were the B744GE and ECMWF.

In addition to the exposed RMS₁₀₀ metric, it is also relevant to evaluate the deviations in flight duration at the end of the trajectory. Form ATC point of view, the crossing time over a metering fixed point is a very intuitive control parameter which facilitate the traffic management. Thus, knowing the errors introduced by the assumptions and limitations of the considered TP may help to a better understanding of the real capabilities on ground which in turn will improve the efficiency of the ATC directives.

Table III includes the time deviations at the end of the trajectory for the six proposed cases.

<table>
<thead>
<tr>
<th>APM</th>
<th>WM</th>
<th>Δt [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B744</td>
<td>GFS</td>
<td>+4</td>
</tr>
<tr>
<td>B744GE</td>
<td>GFS</td>
<td>-10</td>
</tr>
<tr>
<td>B744ERGE</td>
<td>GFS</td>
<td>-10</td>
</tr>
<tr>
<td>B744</td>
<td>ECMWF</td>
<td>+5</td>
</tr>
<tr>
<td>B744GE</td>
<td>ECMWF</td>
<td>-3</td>
</tr>
<tr>
<td>B744ERGE</td>
<td>ECMWF</td>
<td>-3</td>
</tr>
</tbody>
</table>

In this case, the differences between the results obtained by using the GFS or the ECMWF weather model are noticeable. The predictions are better with the ECMWF model due to its higher resolution. ECMWF provides values with a resolution of 0.125° in longitude and latitude while the resolution of GFS is 0.5°.
The variability of the initial mass \( m_0 \) is defined by the following interval and probability density function,

- \( m_0 \in [m_0^\text{ref} - 5,000kg; m_0^\text{ref} + 5,000kg] \)
- uniform distribution \((\bar{X} = 209,972kg; \sigma = 2,896kg)\) defined within the above described interval. Fig. 3 shows the stochastic distribution used in the sensitivity analysis.

![Stochastic distribution of initial masses](image)

**Figure 3.** Stochastic distribution of initial masses

Based on this distribution and using the nominal values for the remaining inputs, the distributions of the studied outputs (flown distance and duration referred to the reference trajectory) are depicted in following Fig. 4.

![Distributions of \( \Delta d \) and \( \Delta t \) at the end of the trajectory](image)

**Figure 4.** Distributions of \( \Delta d \) and \( \Delta t \) at the end of the trajectory

The behaviors of the flown distance and elapsed time variables replicate perfectly the uniform distribution used for modeling the initial mass variability. For higher/lower masses the distance and time until reaching the target altitude at the end of the trajectory increases/decreases for a fixed TOD.

2) **Sensitivity to transition CAS errors**

Once the descending phase starts, the CAS increases. Hence, the Mach/CAS transition has to be performed at a CAS value higher or equal to the initial CAS\(_0\). However, due to structural limitation it is not possible to reach very high CAS values. An upper limit of 330kn has been defined in order to ensure that no predictions breech any of the operational flight limitations.

The variability of the transition CAS is defined by the following interval and probability density function,

- \( \text{CAS} \in [\text{CAS}_0; 330 \text{ kn}] \)
- triangular distribution \((\bar{X} = 298.2kn; \sigma = 7.8kn)\) defined within the above described interval whose central value is the reference CAS calculated in III.A. Fig. 5 shows the stochastic distribution used in the sensitivity analysis.

![Stochastic distribution of transition CAS](image)

**Figure 5.** Stochastic distribution of transition CAS

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Fig. 2 compares the vertical profiles of the recorded (blue) and predicted trajectories (red) as a function of \( H_p \) with the time.

![Recorded vs. Predicted vertical profiles with time](image)

**Figure 2.** Recorded vs. Predicted vertical profiles with time

### B. Sensitivity of CDA predictions to aircraft intent errors

The study of the sensitivity of the trajectory predictions to AI errors requires from a definition of the AI which best represents the real flight data. Based on the methodology and the results presented in previous section, it is possible to obtain such AI which minimizes the vertical errors along a complete CDA trajectory. This AI leads to a reference trajectory which is used for analyzing the prediction sensitivity to uncertainties of the key parameters: the initial aircraft mass \( m_0 \), the transition CAS and the location of the TOD.

Initially, the impact of each individual parameter is studied separately to identify their influences on the predicted trajectories. Finally, the uncertainties of the three selected parameters will be combined to evaluate the global impact on the variables under analysis.

The sensitivity analysis is performed modeling the inputs by probabilistic distributions which aim at representing the realistic variations of the parameters within a predefined interval. The stochastic generated values are used afterwards in a Monte Carlo simulation process.

1) **Sensitivity to initial mass errors**

The initial mass \( m_0 \) is considered the best approximation to the actual aircraft mass at the beginning of the trajectory. There are no references available for validating its suitability rather than the fact that the predicted trajectory is really close to the actual trajectory.

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Based on this distribution and using the nominal values for the remaining inputs, the distributions of the studied outputs are depicted in following Fig. 6.

**Figure 5.** Stochastic distribution of transition CAS

**Figure 7.** Stochastic distribution of TOD location

Based on this distribution and using the nominal values for the remaining inputs, the distributions of the studied outputs are depicted in following Fig. 8.

**Figure 6.** Distributions of Δd and Δt at the end of the trajectory

**Figure 8.** Distributions of Δd and Δt at the end of the trajectory

The behaviors of the flown distance and elapsed time variables can be assimilated to triangular distributions, similar to the distribution used for modeling the transition CAS variability, but balanced to the right (contrary to the input distribution). For higher/lower transition CAS, the distance and time until reaching the target altitude at the end of the trajectory decreases/ increases for a fixed TOD.

3) **Sensitivity to TOD location**

The location of the TOD is basically constrained by the cruise altitude, the altitude at the metering fixed point, the descent speed and the wind. With this information it is possible to accurately predict the time at which the aircraft will start the descent [13]. Additional data such as the aircraft mass would improve those predictions though. In this particular case, the TOD location can be obtained univocally from the ADS-B tracks. The optimization process defined for obtaining the best trajectory prediction let the position fluctuates a bit around such nominal instant in order to obtain the minimum vertical error. This flexibility is translated into a better overall prediction of the descending part of the trajectory.

The variability of the TOD location is defined by the following interval and probability density function,

- \( \text{TOD} \in [\text{TOD}^{\text{ref}} - 20 \text{sec}; \text{TOD}^{\text{ref}} + 20 \text{sec}] \)
- triangular distribution \((\bar{X} = 949 \text{sec}; \sigma = 8.3 \text{sec})\) defined within the above described interval whose central value is the reference TOD location calculated in III.A. Fig. 7 shows the stochastic distribution used in the sensitivity analysis.

**Figure 9.** Distributions of Δd and Δt at the end of the trajectory

The behaviors of the flown distance and elapsed time variables replicate the input triangular distribution. For earlier TOD locations, the flown distance and flight duration are shorter, while for later TOD locations, the behavior is the opposite.

4) **Sensitivity to initial mass, transition CAS and TOD location errors**

The most generic sensitivity analysis takes into account the aforementioned intervals and distributions for the initial mass, the transition CAS and the TOD location at the same time. Based on those distributions, the distributions of the studied outputs are depicted in following Fig. 9.

The behaviors of the flown distance and elapsed time variables can be assimilated to triangular distributions. In this case the correlation with the distribution used for modeling the transition CAS is clear, which leads to conclude that this is the most influencing input when the AI is known and defined as explained in previous sections.
IV. EVALUATION OF PREDICTION ERRORS

In previous section, it has been concluded that the effect of the uncertainty on the transition CAS produces the highest impact on the trajectory predictions. This statement is true once the following assumptions are valid:

- the lateral path can be predicted with a very high accuracy, i.e., the lateral profile do not add uncertainty to the process
- the location of the TOD is known or at least is contained in a small time interval
- the descent is performed strictly at idle throttle conditions since the beggining of the descent
- a Mach/CAS transition is executed after the TOD
- the descent does not include level-off segments

Following Table IV summarizes the maximum deviations (difference between the maximum and minimum values) referred to the reference trajectory obtained in the sensitivity analyses presented above.

<table>
<thead>
<tr>
<th>CASE</th>
<th>ΔFlown distance [m]</th>
<th>ΔElapsed time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,845</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>25,783</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>9,199</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>31,337</td>
<td>199</td>
</tr>
</tbody>
</table>

The highest flown distance and elapsed time deviations from the reference trajectory are produced by the high variability of the transition CAS. Although the other two factors also produce deviations regarding the reference, their effects are not comparable.

To reduce such relevant impact on the predictions, it is required to have a pre-knowledge of the intended Mach/CAS profile to be flown and to limit the variations from such scheme within a controlled interval. Otherwise, the uncertainty of the predicted trajectory would make impossible an efficient and safe arrival traffic management.

V. CONCLUSIONS AND FURTHER WORK

Trajectory prediction is a key capability which is essential for an efficient ATM system. However, DSTs show some limitations that impact the fidelity of the trajectory predictions available on ground. These limitations are basically related to the accuracy of the information used for predicting the aircraft trajectories. Even considering the most sophisticated aircraft performance models and most precise weather forecasts, the lack of up-to-date information about the aircraft intent leads to high predictions errors.

The proposed methodology aims at reducing the impact of the AI uncertainties by an initial adaptation of the prediction capabilities based on recorded flight data. Once the AI which best fits the real data is found, the process of analyzing the impact of the AI uncertainties is straightforward.

The main conclusion of this work is that in CDA procedures the key parameter which produces the highest impact on the prediction of CDAs is the transition CAS. Hence, the strategic air-ground synchronization of the Mach/CAS schedule to be flown by the aircraft would reduce significantly the prediction errors in time and flown distance at which the aircraft reaches a target altitude.

The main drawback of the approach presented in this paper is that the limitations assumed for describing the AI constrain the capability of predicting the majority of the real procedures. The most common operational procedures include a deceleration from the transition CAS to 250kn, which is the speed restriction at 10,000ft. Future works will explore the effect of considering more sophisticated TPs capable of describing the AI by means of the altitude law which can be deducted from the ADS-B tracks. The analysis of such alternative will allow to capture the deceleration procedures more precisely, and therefore, any sensitivity analyses based on that will be more representative.

REFERENCES

[12] http://www.ecmwf.int/