# The Air Traffic Flow Management Problem with Time Windows

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***Abstract*—This paper defines time windows of variable size through which flights execute their actions, e.g., enter an airspace sector, depart from or arrive at an airport. We propose a two-step approach based on a mixed integer programming formulation. The first step determines a set of time windows such that the overall cost of delay is minimized. As there might be different sets of time windows generating the same minimum delay, in the second step we choose the set of optimal time windows which also maximizes the overall time window size. In such a way, we provide to airlines, air navigation service providers and airports the largest degree of flexibility to perform their operations under the constraint that the minimum achievable delay is kept constant. We also gain information on the critical flights of the system: if the optimal width of a time window is equal to its minimum available value, any disruption that may cause the flight not to meet it may produce undesired downstream effects. Some computational experience for small-scale random instances is reported.**

## I. Introduction

The Air Transportation System both in Europe and in the United States is highly capacity constrained due to the limited availability of resources both on the ground and in en-route airspace. Capacity at airports is limited by the runway systems and the terminal airspace around them [1]. The capacity of en-route airspace sectors is limited by the maximum workload acceptable for air traffic controllers [2]. These capacity constraints are becoming a limiting factor in many regions of the world. In fact, as air traffic grows and/or capacity is reduced - mainly due to adverse weather conditions -, demand can exceed capacity at key points of the air transportation network and at critical times. These local overloads create delays which propagate to other parts of the air network, amplifying congestion as increasing number of local capacity constraints come into play. Air traffic flow management (ATFM) attempts to prevent local demand-capacity imbalances by adjusting the flows of aircraft on a national or regional basis [3]. In Europe, delays caused by ATFM measures in 2007 amounted to 21.5M minutes, producing an estimated cost of M€1300 to airlines [4].

The ATFM problem was first formalized in 1987 by Odoni [5]. Since then, a plethora of mathematical models have been developed. For instance, Bertsimas and Stock [6] provided a strong formulation of the ATFM problem. In 2007, Lulli and Odoni [7] illustrated the complex nature of the European ATFM system where congestion in the en-route airspace is an issue. They show that counter-intuitive solutions exist when assigning delays to the different phases of the flight. They also discuss the conflicts that may arise between the objectives of efficiency or equity. Recently, Bertsimas, Lulli and Odoni [3] presented an Integer Programming model for the ATFM problem. They provide a complete representation of all the phases of a flight, and modeled a wide set of control actions, including rerouting. Extensions of this work may be found in [8] and [9]. The interested reader may refer to [10] for a detailed relevant survey on ATFM models.

Most if not all of the models developed for the ATFM minimize the delay to be assigned to flights in order to resolve local demand-capacity imbalances. However, none of these models - to the best of our knowledge - explicitly considers the criticality of the flight. The execution of a flight - from the Air Traffic Control (ATC) point of view - requires a complex mix of capacitated resources which is negotiated between air traffic controllers and flight crews (or dispatchers) according to the air traffic conditions and is concretized in an approved flight plan. The operator of a flight is expected to adhere as precisely as possible to the flight plan, although some adjustments are possible. However, there are flights which have to be operated in strict accordance to the approved flight plan, since any delay assigned to them may have large downstream effects such as disruptions in the airline schedules or degradations of the ATC system performances. For these flights, there is no slack time in handling their operations and a limited number of recovery options is generally available.

Herein, we extend the Bertsimas, Lulli and Odoni model to detect "critical" flights. For each aircraft and for each phase of the flight the model identifies a time window. A time window is a period of time during which a certain phase of the flight, e.g., taking off, landing and entering sectors, has to be executed. The width of the window delineates the degree of flexibility to carry out a specific operation. The larger is the time window, the larger is the amount of slack time. The smaller is the time window, the more "critical" is the flight. For critical flights, it is important that all the activities executed in support of flight operations, e.g. maintenance, ground and flight crew activities and ATC clearances, are coordinated and executed on time.

We refer to the model herein presented as the Air Traffic Flow Management problem with Time Windows (ATFMTW). Our formulation lexicographically sets two objectives. The first step identifies the optimal sizes of the time windows to minimize the total cost of delay. Costs are defined by two super-linear functions, one for the departure and the other for the arrival delay costs, which ensure fairness of delay distribution between the various flights [7]. Given the minimum delay cost that can be assigned to the whole system, the second goal is to maximize the total width of the existing time windows. As some portions of the airspace may be less congested than others, a larger flexibility or room of maneuver can be given to airspace users, air navigation service providers and airports operating in such sparse areas without degrading the overall performance (i.e., the total cost of delay) of the entire system. On the other side, small-sized time windows impose strict limitations on the different elements of the system as any actor is urged to assist flights to respect them as much as possible (critical flights).

This paper unfolds as follows: Section II presents the mathematical formulation of the model. Section III describes the computational experience to date and analyses the results. Finally, Section IV summarizes conclusions and indicates the next research steps.

## II. Model's mathematical description

Here we present the mathematical model for the ATFM with Time Windows (ATFMTW). As mentioned above this model can be envisioned as an extension of the model presented in [3]. As such, we define similar decision variables as described below.

For the sake of clarity, we first define the following notation:

$\mathcal{K} \equiv$ set of airports
$\mathcal{S} \equiv$ set of sectors
$\mathcal{F} \equiv$ set of flights
$f \in \mathcal{F} \equiv$ generic flight
$\mathcal{S}^f \subseteq (\mathcal{S} \cup \mathcal{K}) \equiv$ set of sectors that can be flown by flight $f$, including the origin and destination airports of $f$
$\mathcal{T} \equiv$ set of time periods
$\mathcal{C} \equiv$ set of pairs of flights that are continued
$\mathcal{P}_i^f \equiv$ set of sector $i$'s preceding sectors $(i \in \mathcal{S}^f)$
$\mathcal{L}_i^f \equiv$ set of sector $i$'s subsequent sectors $(i \in \mathcal{S}^f)$
$\mathcal{H} = [HI, HF] \equiv$ set of capacity periods
HI $\equiv$ initial instant of $\mathcal{H}$
HF $\equiv$ final instant of $\mathcal{H}$
NTH $\equiv$ number of time periods in a single capacity period
$\mathcal{D}_k(h) \equiv$ departure capacity of airport $k$ at capacity period $h$
$\mathcal{A}_k(h) \equiv$ arrival capacity of airport $k$ at capacity period $h$
$\mathcal{S}_j(h) \equiv$ capacity of sector $j$ at capacity period $h$
$s_f \equiv$ turnaround time of an airplane after flight $f$
$orig_f \equiv$ airport of departure of flight $f$
$dest_f \equiv$ airport of arrival of flight $f$
$l_{fjj'} \equiv$ minimum number of time periods that flight $f$ must spend in sector $j$ before entering in sector $j'$
$end_f \equiv$ maximum acceptable duration of flight $f$
$T_j^f = [\underline{T}_j^f, \bar{T}_j^f] \equiv$ set of feasible time periods for flight $f$ to to depart from $j = orig_f$ or arrive at $j = dest_f$ or enter sector $j$
$\underline{T}_j^f \equiv$ first time period in the set $T_j^f$
$\bar{T}_j^f \equiv$ last time period in the set $T_j^f$
MINTW $\equiv$ the minimum time window size
MAXTW $\equiv$ the maximum time window size

### A. Decision variables

As the width of the time windows is not a priori determined, we need to define for each flight $f$ that can cross sector $j \in \mathcal{S}^f$ two sets of decision variables: the beginning and the end of the time window. Therefore, we introduce the following binary decision variables:

$$wi_{j,t}^f = \begin{cases} 1, & \text{if time window for flight } f \text{ in sector } j \\ & \text{has been opened by time } t \\ 0, & \text{otherwise} \end{cases}$$

$$wf_{j,t}^f = \begin{cases} 1, & \text{if time window for flight } f \text{ in sector } j \\ & \text{has been closed by time } t \\ 0, & \text{otherwise} \end{cases}$$

For each flight $f$ and sector/airport $j$, the decision variables do not need to be defined for each $t \in \mathcal{T}$ but only on the set of feasible time periods $T_j^f$. Note that one variable for each flight-airport can be eliminated from the formulation. As flight's cancellation is not considered in the model, we can set variables $wf_{orig_f,\bar{T}_{orig_f}}^f$ and $wf_{dest_f,\bar{T}_{dest_f}}^f$ to 1 for each flight $f$, since flight $f$ has to leave from airport $orig_f$ and arrive to airport $dest_f$. Finally, we introduce additional decision variables to formulate the constraints associated to the utilization of the available capacity. The sector capacity is defined as the maximum number of flights that can be in a sector during a "capacity period" $h$ composed of a set of contiguous time periods. Then, to determine the capacity occupancy for a flight $f$ of sector $j$ at some capacity period $h$, we define the following binary decision variables:

$$co^f_{j,h} = \begin{cases} 1, & \text{if flight } f \text{ enters sector } j \text{ during} \\ & \text{capacity period } h \\ 0, & \text{otherwise} \end{cases}$$

We show next that the integrality condition can be relaxed.

### B. Objective functions

The first objective is the minimization of the total delay cost. For a flight $f$ this total delay cost is defined as the sum of the departure and arrival delay costs. Thus we introduce two cost coefficients $ddc^f(t)$ and $adc^f(t)$ which represent the delay cost for flight $f$ when the departure and arrival time window is closed at time $t$, respectively:

$$ddc^f(t) = \begin{cases} 0 & \text{if } t \leq z_{orig_f} \\ (t - z_{orig_f})^{1+\epsilon_d} & \text{otherwise} \end{cases}$$

$$adc^f(t) = \begin{cases} 0 & \text{if } t \leq z_{dest_f} \\ (t - z_{dest_f})^{1+\epsilon_a} & \text{otherwise} \end{cases}$$

where $z_{orig_f}$ and $z_{dest_f}$ are respectively the last time periods in which flight $f$ can depart from its origin and arrive at its destination airport without causing a delay. If $DEL$ is the maximum allowed delay, it easily follows that $z_{orig_f} = \bar{T}_{orig_f} - DEL$ and $z_{dest_f} = \bar{T}_{dest_f} - DEL$. The values $\epsilon_d > 0$ and $\epsilon_a > 0$ are two positive parameters which make the cost coefficients super-linear. As proposed in [7], this choice grants a fair assignment of delay among different flights.

Therefore, the objective function minimizing the total cost of delay is formulated as follows:

$$Z_1 = Min \sum_{f \in \mathcal{F}} \sum_{t \in T^f_{orig_f}} \Big( (wf^f_{orig_f,t} - wf^f_{orig_f,t-1}) \cdot ddc^f(t) \Big) +$$

$$+ \sum_{f \in \mathcal{F}} \sum_{t \in T^f_{dest_f}} \Big( (wf^f_{dest_f,t} - wf^f_{dest_f,t-1}) \cdot adc^f(t) \Big)$$

Once the minimum cost of delay is attained, the second objective is to make the time windows as large as possible. In this way we may provide greater flexibility to the different stakeholders with no harm to the overall system performance. Time windows that cannot be enlarged allow a) to identify critical flights, i.e., flights whose operations need to be performed within the specified time windows, otherwise a delay would occur, and b) to spot the airspace bottlenecks, i.e., the portions of airspace which are most congested.

Mathematically, the second objective function searches its optimal solution on the polyhedron made of the optimal solutions of the first step. Then it maximizes the number of time periods composing each time window:

$$Z_2 = Max \sum_{f \in \mathcal{F},\, j \in \mathcal{S}^f} \sum_{t \in T^f_j} (wi^f_{j,t} - wf^f_{j,t-1}),$$

being subject to the additional constraint:

$$Z_1 = \sum_{f \in \mathcal{F}} \sum_{t \in T^f_{orig_f}} \Big( (wf^f_{orig_f,t} - wf^f_{orig_f,t-1}) \cdot ddc^f(t) \Big) +$$

$$+ \sum_{f \in \mathcal{F}} \sum_{t \in T^f_{dest_f}} \Big( (wf^f_{dest_f,t} - wf^f_{dest_f,t-1}) \cdot adc^f(t) \Big)$$

### C. Constraints

The model's constraints set is as follows:

$$co^f_{j,h} \geq wi^f_{j,\text{MIN}((h-HI+1)\cdot NTH+HI-1,\bar{T}^f_j)} - wf^f_{j,(h-HI)\cdot NTH+HI-1} \quad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f,\, \forall h \in \mathcal{H} \tag{1}$$

$$\sum_{f \in \mathcal{F}\,:\, orig_f = k} co^f_{k,h} \leq D_k(h) \qquad \forall k \in \mathcal{K},\, \forall h \in \mathcal{H} \tag{2}$$

$$\sum_{f \in \mathcal{F}\,:\, dest_f = k} co^f_{k,h} \leq A_k(h) \qquad \forall k \in \mathcal{K},\, \forall h \in \mathcal{H} \tag{3}$$

$$\sum_{f \in \mathcal{F}\,:\, j \in \mathcal{S}^f} co^f_{j,h} \leq S_j(h) \qquad \forall j \in \mathcal{S},\, \forall h \in \mathcal{H} \tag{4}$$

$$wi^f_{j,t+1} - wi^f_{j,t} \geq 0 \qquad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f, \forall t \in T^f_j \tag{5}$$

$$wf^f_{j,t+1} - wf^f_{j,t} \geq 0 \qquad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f, \forall t \in T^f_j \tag{6}$$

$$wi^f_{j,t} - wf^f_{j,t} \geq 0 \qquad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f, \forall t \in T^f_j \tag{7}$$

$$wi^f_{j,\bar{T}^f_j} = wf^f_{j,\bar{T}^f_j} \qquad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f \tag{8}$$

$$wf^f_{j,\bar{T}^f_j} \leq \sum_{j' \in \mathcal{L}^f_j} wf^f_{j',\bar{T}^f_{j'}} \qquad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f \;:\; j \neq dest_f \tag{9}$$

$$wf^f_{j,\bar{T}^f_j} \leq \sum_{j' \in \mathcal{P}^f_j} wf^f_{j',\bar{T}^f_{j'}} \qquad \forall f \in \mathcal{F},\, \forall j \in \mathcal{S}^f \;:\; j \neq orig_f \tag{10}$$

$$\sum_{j' \in \mathcal{L}^f_j} wf^f_{j',\bar{T}^f_{j'}} \leq 1 \qquad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}^f \;:\; j \neq dest_f \tag{11}$$

$$\sum_{j' \in \mathcal{P}^f_j} wf^f_{j',\bar{T}^f_{j'}} \leq 1 \qquad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}^f \;:\; j \neq orig_f \tag{12}$$

$$wi^f_{j,t} \leq \sum_{j' \in \mathcal{P}^f_j} wi^f_{j',t-l_{fj'j}} \qquad \forall f \in \mathcal{F}$$
$$\forall j \in \mathcal{S}^f,\ \forall t \in T^f_j\ :\ j \neq orig_f \qquad (13)$$

$$wf^f_{j,t} \geq \sum_{j' \in \mathcal{L}^f_j} wf^f_{j',t+l_{fjj'}} - (1 - wf^f_{j,\bar{T}^f_j}) \qquad (14)$$
$$\forall f \in \mathcal{F},\ \forall j \in \mathcal{S}^f,\ \forall t \in T^f_j\ :\ j \neq dest_f$$

$$wi^{f'}_{orig_{f'},t+s_f} \leq wf^f_{dest_f,t} \qquad \forall (f,f') \in \mathcal{C},$$
$$\forall t \in T^f_{dest_f} : t + s_f \in T^{f'}_{orig_{f'}} \qquad (15)$$

$$wi^f_{orig_f,t} \leq wf^f_{dest_f,t+end_f-1} \qquad \forall f \in \mathcal{F},$$
$$\forall t \in T^f_{orig_f} : t + end_f - 1 \in T^f_{dest_f} \qquad (16)$$

$$wf^f_{j,t+\text{MINTW}-1} \leq wi^f_{j,t} \qquad \forall f \in \mathcal{F},\ \forall j \in \mathcal{S}^f, \forall t \in T^f_j \quad (17)$$

$$wi^f_{j,t-\text{MAXTW}+1} \leq wf^f_{j,t} \qquad \forall f \in \mathcal{F}, \forall j \in \mathcal{S}^f, \forall t \in T^f_j \quad (18)$$

$$wi^f_{j,t} \in \{0,1\} \qquad \forall f \in \mathcal{F},\ \forall j \in \mathcal{S}^f,\ \forall t \in T^f_j \qquad (19)$$

$$wf^f_{j,t} \in \{0,1\} \qquad \forall f \in \mathcal{F},\ \forall j \in \mathcal{S}^f,\ \forall t \in T^f_j \qquad (20)$$

Constraints (1) make sure that the decision variable $co^f_{j,h}$ represents the entrance of flight $f$ in sector $j$ at capacity period $h$, with its value being 1 in case of entrance, 0 otherwise, as required by the definition of that decision variable. The $co^f_{j,h}$ variable may have a value different from 0 or 1, in fact in those cases where sector $j$ is not congested at period $h$, the decision variable may have a value of 2 or more. This fact, however, does not make the model get wrong results, as the sector's capacity will still be respected, and the capacity occupancy variables will get to their right value if they are forced to do so by capacity restrictions.

Constraints (2), (3) and (4) define respectively the departure, arrival and sector capacity limits, ensuring that the number of flights which may depart from (or arrive at) airport $k$, or enter sector $j$ at capacity period $h$ will not exceed the departure, arrival or sector capacity for the given capacity period. Constraints (5), (6) and (7) define the time connectivity of the decision variables, with the decision variables $wi^f_{j,t}$ and $wf^f_{j,t}$ being monotonic increasing (which is specified by the first two constraints), while $wi^f_{j,t}$ must always be greater or equal than $wf^f_{j,t}$ as the first decision variable represents the first instant of possible entering of flight $f$ in sector (or airport) $j$, while the latter represents the last instant of possible entering of flight $f$ in sector/airport $j$. Constraints (8) ensure the fact that if a time window for some flight $f$ and some sector $j$ ever opens, then it will also have to close by the last time period of definition. Constraints (9) and (10) represent the continuity of a flight. The flight path, from the airport of origin to the airport of destination is given by a sequence of sectors, which are contiguous one to the other. Constraints (9) ensure the continuity of a flight from a sector j to some following sector $j' \in \mathcal{L}^f_j$, while constraints (10) ensure that a flight $f$ can reach some sector $j$ if and only if one of its preceding sectors $j' \in \mathcal{P}^f_j$ was in flight $f$'s path. Constraints (11) and (12) ensure the uniqueness of the flight's path for each flight $f$, respectively by guaranteeing that a flight that has reached some sector $j$ will reach only one of its following sectors $j' \in \mathcal{L}^f_j$, and making sure that a flight that has reached some sector $j$ can come only from one of its preceding sectors $j' \in \mathcal{P}^f_j$. Constraints (13) and (14) stipulate that a flight cannot enter the next sector on its path until it has spent at least $l_{fjj'}$ time periods (the minimum possible) travelling through one of the preceding sectors on its current path. To guarantee that the turnaround between continued flights is respected, constraints (15) are implemented. Continued flights are those flights for which the aircraft of flight $f$ will be used for a following flight $f'$, with $s_f$ being the minimum amount of time needed to prepare flight $f'$ for departure, following the landing of flight $f$. Constraints (16) impose that the total flight time does not exceed the maximum duration of the flight. Constraints (17) and (18) define the minimum and maximum size for a time window respectively. Finally, constraints (19) and (20) set the decision variables $wi^f_{j,t}$ and $wf^f_{j,t}$ as binary.

## III. Computational experience

Our preliminary computational experiments show that the proposed two-step approach gives novel insights on which actions are appropriate to cope with local demand and capacity imbalances. The first step determines a set of time windows such that the overall cost of delay is minimized. As there might be different sets of time windows generating the same minimum delay, i.e., multiple optimal solutions may exist, in the second step we choose the set of optimal time windows which also maximizes the overall time window width. In such a way, we provide to airlines, ANSPs and airports the largest degree of flexibility to perform their operations under the constraint that the minimum achievable delay is kept constant. Equivalently, we gain information on the bottlenecks or critical flights of the system. For instance, if at the end of the second step the optimal width of a time window is equal to its minimum available value any disruption that may cause the flight not to meet this time window may produce downstream effects which lead to an increase of the overall delay.

We tested our model on 30 random instances of 300 non-continued flights, 25 en-route sectors, and 5 airports (3 of which are hubs). The size of the time periods in which no delay is assigned for each flight-airport are all set equal to 15 minutes, while the minimum and maximum allowed time window sizes are all equal to 5 and 15 minutes, respectively. The time periods are 5 minutes large, and the time horizon $\mathcal{T}$ considered is 3 hours. According to these settings, time windows can only have three different sizes: 5, 10 or 15 minutes. Moreover, the size of a single capacity period $h$ is defined by the number of time periods included in it. The capacity for airports (both for arrival and the departure of flights) and sectors is initially set at 11 flights every 15 minutes. Other key parameters which are set equal to all flights are: the time to traverse a sector, the maximum delay in the time window assignment, the departure and arrival delay cost coefficients that are used to ensure fairness between flights, and the maximum extra-duration for the flight time. Then the maximum acceptable duration $end_f$ for flight $f$ is computed by adding the maximum extra-duration to the minimum flying time calculated between the airport of origin and the airport of destination.

This airspace is modeled as a grid of squared cells. This choice allows to accommodate sectors of arbitrary shape. As the instances should represent realistic cases, airports should not be too close to each other, and they should be distributed in space. Airports are therefore randomly distributed, but the minimum distance between two airports is equal to 3 cells. Airports are also randomly subdivided between regional airports and hubs, the former only connected to hubs, while the latter connected both to regional airports and other hubs.

Flights are generated randomly choosing both the departure and destination airports; imposing, however, that there are no direct connections between regional airports. The flight path unfolds among adjacent cells.

there is a delay in a few instances the average delay per flight is between 10 and 31 seconds, being lower in all other cases. Thus this example depicts a situation with moderate congestion. The forth and fifth columns respectively indicate the total number of time windows existing in the system for every instance and the associated total width (in minutes), i.e., the optimal solution of the second step. The following three columns describe the percentage size (or width) distribution of such time windows. We observe that in the instances under study a share from 20% to 40% of the time windows cannot reach the maximum size of 15 minutes. In particular, a share between 4% and 17% of time windows needs to remain at the minimum width of 5 minutes. These time windows are responsible for the final total delay, and thus represent the bottlenecks of the system. The flights associated to them are the critical flights as no slack time is available in case of unforeseen events. Finally, the last column shows the computational time (in seconds) needed to solve every instance on a Intel Core 2 Duo CPU at 3.00 GHz and with 3.23Gb of Ram.

| Inst. | Delay Cost | Delay Time | Time Windows Nr. | Time Windows Max Width | TW size (%) 5 | TW size (%) 10 | TW size (%) 15 | Comp. Time |
|---|---|---|---|---|---|---|---|---|
| 0 | 2,0 | 10 | 1515 | 19660 | 9 | 22 | 69 | 146 |
| 1 | 0,0 | 0 | 1437 | 18970 | 7 | 22 | 71 | 10 |
| 2 | 0,0 | 0 | 1625 | 20360 | 12 | 26 | 62 | 69 |
| 3 | 0,0 | 0 | 1504 | 19625 | 6 | 27 | 67 | 51 |
| 4 | 31,5 | 155 | 1436 | 17665 | 17 | 21 | 63 | 134 |
| 5 | 3,0 | 15 | 1577 | 20010 | 10 | 25 | 64 | 507 |
| 6 | 0,0 | 0 | 1409 | 19120 | 6 | 17 | 77 | 17 |
| 7 | 0,0 | 0 | 1664 | 21670 | 9 | 21 | 70 | 53 |
| 8 | 0,0 | 0 | 1590 | 20955 | 5 | 26 | 69 | 58 |
| 9 | 0,0 | 0 | 1427 | 19385 | 4 | 21 | 75 | 27 |
| 10 | 0,0 | 0 | 1585 | 20200 | 12 | 20 | 67 | 77 |
| 11 | 1,0 | 5 | 1512 | 20390 | 5 | 20 | 75 | 69 |
| 12 | 4,0 | 20 | 1659 | 21015 | 10 | 27 | 63 | 251 |
| 13 | 2,0 | 10 | 1575 | 20405 | 6 | 28 | 65 | 382 |
| 14 | 4,0 | 20 | 1666 | 21415 | 8 | 26 | 65 | 217 |
| 15 | 2,0 | 10 | 1510 | 19550 | 7 | 28 | 66 | 476 |
| 16 | 2,0 | 10 | 1680 | 21760 | 8 | 25 | 67 | 431 |
| 17 | 2,0 | 10 | 1540 | 20870 | 6 | 18 | 77 | 103 |
| 18 | 0,0 | 0 | 1548 | 19825 | 11 | 21 | 68 | 41 |
| 19 | 18,2 | 90 | 1624 | 20130 | 15 | 23 | 63 | 308 |
| 20 | 0,0 | 0 | 1616 | 21710 | 6 | 19 | 75 | 59 |
| 21 | 10,0 | 50 | 1630 | 20350 | 12 | 26 | 62 | 1283 |
| 22 | 4,0 | 20 | 1576 | 20420 | 9 | 23 | 68 | 137 |
| 23 | 0,0 | 0 | 1563 | 20655 | 8 | 20 | 72 | 33 |
| 24 | 0,0 | 0 | 1477 | 19785 | 7 | 18 | 75 | 17 |
| 25 | 7,0 | 35 | 1634 | 20685 | 8 | 32 | 61 | 1206 |
| 26 | 4,0 | 20 | 1487 | 19165 | 10 | 23 | 68 | 92 |
| 27 | 22,4 | 110 | 1635 | 22160 | 6 | 16 | 78 | 876 |
| 28 | 1,0 | 5 | 1518 | 19125 | 14 | 20 | 66 | 185 |
| 29 | 4,0 | 20 | 1634 | 21650 | 7 | 20 | 72 | 400 |

Table I
COMPUTATIONAL RESULTS FOR THE ATFMTW MODEL. CAPACITY EQUAL TO 11 FLIGHTS EVERY 15 MINUTES

| Inst. | 5:5 | 5:10 | 5:15 | 10:5 | 10:10 | 10:15 | 15:5 | 15:10 | 15:15 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 15 | 11 | 12 | 20 | 46 | 22 | 12 | 36 | 126 |
| 1 | 3 | 12 | 16 | 15 | 35 | 36 | 13 | 42 | 128 |
| 2 | 19 | 12 | 17 | 10 | 62 | 23 | 15 | 22 | 120 |
| 3 | 5 | 6 | 4 | 10 | 31 | 42 | 19 | 37 | 146 |
| 4 | 48 | 1 | 13 | 8 | 56 | 18 | 15 | 25 | 116 |
| 5 | 21 | 4 | 8 | 14 | 55 | 29 | 16 | 22 | 131 |
| 6 | 7 | 6 | 12 | 11 | 30 | 30 | 16 | 20 | 168 |
| 7 | 8 | 16 | 21 | 23 | 30 | 33 | 17 | 26 | 126 |
| 8 | 6 | 11 | 10 | 16 | 55 | 23 | 15 | 28 | 136 |
| 9 | 1 | 2 | 13 | 6 | 38 | 25 | 16 | 34 | 165 |
| 10 | 25 | 4 | 10 | 11 | 44 | 30 | 25 | 32 | 119 |
| 11 | 8 | 4 | 14 | 13 | 44 | 32 | 13 | 48 | 124 |
| 12 | 23 | 3 | 6 | 11 | 75 | 23 | 17 | 28 | 114 |
| 13 | 9 | 6 | 18 | 11 | 45 | 25 | 15 | 37 | 134 |
| 14 | 12 | 6 | 11 | 17 | 52 | 35 | 16 | 26 | 125 |
| 15 | 12 | 4 | 16 | 7 | 47 | 39 | 12 | 40 | 123 |
| 16 | 9 | 17 | 5 | 15 | 56 | 18 | 27 | 26 | 127 |
| 17 | 6 | 10 | 9 | 10 | 35 | 36 | 20 | 28 | 146 |
| 18 | 23 | 9 | 12 | 13 | 38 | 16 | 17 | 37 | 135 |
| 19 | 38 | 11 | 5 | 9 | 52 | 25 | 15 | 21 | 124 |
| 20 | 8 | 10 | 18 | 9 | 45 | 28 | 16 | 25 | 141 |
| 21 | 28 | 11 | 7 | 11 | 48 | 19 | 16 | 27 | 133 |
| 22 | 22 | 6 | 5 | 6 | 53 | 11 | 11 | 25 | 161 |
| 23 | 7 | 8 | 12 | 25 | 27 | 31 | 11 | 28 | 151 |
| 24 | 5 | 17 | 10 | 15 | 28 | 29 | 22 | 27 | 147 |
| 25 | 13 | 9 | 7 | 8 | 70 | 28 | 30 | 38 | 97 |
| 26 | 13 | 12 | 18 | 10 | 47 | 40 | 23 | 29 | 108 |
| 27 | 7 | 9 | 7 | 22 | 25 | 32 | 27 | 25 | 146 |
| 28 | 30 | 5 | 13 | 16 | 40 | 20 | 10 | 23 | 143 |
| 29 | 16 | 12 | 10 | 13 | 52 | 23 | 16 | 22 | 136 |

Table II
NUMBER OF FLIGHTS' BREAKDOWN PER TIME WINDOW SIZE AT DESTINATION AND ARRIVAL AIRPORTS. CAPACITY EQUAL TO 11 FLIGHTS EVERY 15 MINUTES

The main results obtained so far are illustrated in Table I. The first column indicates the instance number. The second column shows the solution of the first step, i.e., the minimum cost of delay whereas the third column shows the total delay in minutes. We observe that in 40% of the cases all the flights' requests can be accommodated as no delay is produced. When

Additional information on the degree of flexibility granted to flights is available from Table II. Here for each instance, column $d : a$ represents the number of flights (out of 300) with the time window at the departure airport of size $d$ and at the arrival airport of size $a$, respectively. As an example, in instance 4, 48 flights (i.e., 16%) have both departure and destination time windows of 5 minutes. Thus these are really constrained flights whose operations need to be tightly executed at both airports. On the other side, a large share of flights (from 97 to 168 depending on the instance) enjoys the largest flexibility as they have both departure and arrival time windows of 15-minute size (see the last column of Table II).

The available capacity at airports and sectors obviously influences the time window size and consequently the overall delay. To analyze such capacity effects on the system, we consider again the same 30 random instances of Tables I and II by augmenting the capacity of all airports and sectors to 12, 13 and 14 flights every 15 minutes, respectively. As expected, we observe that as the capacity raises the cost of the delay decreases and the total width of the time windows increases (see Table III where the average values over the 30 instances are shown).

| Capacity | Delay Cost | MAX TW Width (min) | Comp. Time (sec) |
|---|---|---|---|
| 11 | 4,14 | 20289,50 | 257,23 |
| 12 | 0,43 | 21087,17 | 91,09 |
| 13 | 0,03 | 21637,17 | 33,94 |
| 14 | 0,00 | 22024,67 | 26,08 |

Table III
AVERAGE VALUES OF THE $1^{st}$ AND $2^{nd}$ STEP OF THE ATFMTW MODEL, AND AVERAGE COMPUTATIONAL TIME, W.R.T. CAPACITY

Analogously, the share of 15-minute time windows monotonically increases with the capacity (see Figure 1). We also notice that in situations where there is practically no congestion (as in the case with capacity 14 where all 30 instances have no delay), a non negligible share of minimum size time windows still exists. Thus we cannot rule out the presence of bottlenecks and critical flights even when the system is apparently not under pressure.

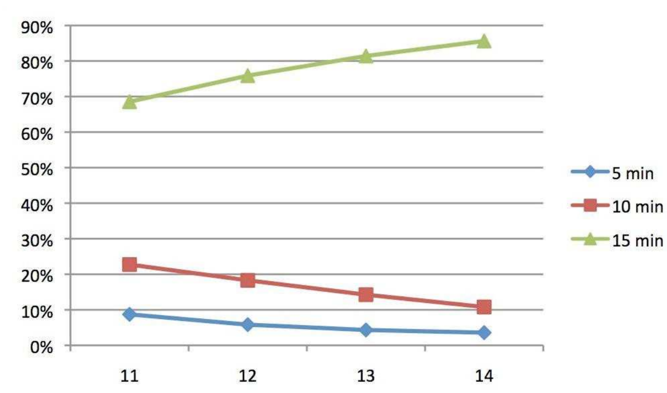


Figure 1. Time Window size w.r.t capacity

## IV. Conclusions

This paper introduces time intervals of variable width, the so called time windows, where flights are allowed to depart, arrive or enter a sector. Each flight is expected to plan and execute its operations to comply with the sequence of its time windows (which might be of various size) from the departure to the destination airports. Analogously, the same airport or sector may have time windows of different size associated to different flights.

We present a mixed-integer programming formulation to determine the largest cumulative size of the time windows provided that the minimum total cost of delay for all flights is attained. This approach indicates to each flight the maximum available degree of flexibility to perform all its operations without providing any degradation to the system performances in terms of cost of delay. Additionally, by detecting the time windows of the smallest size (i.e., 5 minutes in our setting) it is possible to identify which flights are more constrained than others, and which airports or sectors impose limitations on the remainder of the system.

Our preliminary results, based on small-scale random instances with moderate or low congestion, confirm the potentialities of the proposed approach. The flexibility granted to flights monotonically increases with the capacity while the system delay simultaneously decreases. We also show that apparent kind situations with no congestion may contain non negligible shares of minimum size time windows, thus indicating the existence of bottlenecks and critical flights.

In the continuation of this study we plan to further analyze how the size of time windows is distributed within the system, e.g., extending the focus on the whole sequence of time windows for a flight and not only at the departure and destination airports as in Table II. The evaluation of the spatial distribution of the time window size would also provide additional insight: airports or sectors with a large number of small time windows would be identified as critical resources or bottlenecks for the system.

An additional contribution of this work may arise by comparing the total delay costs from the ATFMTW model and some other classical formulations of the ATFM problem (see Section I). All these latter models aim to minimize the overall cost of the delay that must be assigned to flights. To attain this goal, they identify the time period for each flight to arrive in every sector that can be flown by it, including its origin and destination airports. The width of these time periods is fixed for all flights and all sectors and is usually rather large (15 minutes).

The mathematical model presented in this paper overcomes this limitation as it defines time windows of variable sizes for flights to execute their actions. Thus we introduce a degree of flexibility into the system that can be exploited to reduce the overall amount of delay. In fact, the time windows of fixed size become a feasible solution, but not necessarily optimal, for the ATFMTW problem.

Finally, some modeling effort is also required to strengthen the proposed formulation and solve instances of larger scale together with more congested configurations.

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