Bayesian Analysis of Accident Rate, Trend and Uncertainty in Commercial Aviation

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***Abstract*— An established approach in the evaluation of aviation accident statistics is to determine point estimates of the accident rate by dividing number of accidents by number of flights and to determine an uncertainty interval through evaluation of the underlying binomial distribution. The trend, however, is not estimated. Another established approach is to perform a regression analysis to estimate rate and trend, but then uncertainty is not estimated. In this paper we overcome these limitations of established approaches by studying the problem as one of Bayesian estimation of the joint conditional density function of accident rate and trend given accident and flight statistical data. Subsequently, a particle filter is used in order to perform numerical evaluations. The novel approach is shown to work well on commercial aviation accident data.**

Keywords-Bayesian estimation; accident statistics; particle filtering; uncertainty estimation

I. INTRODUCTION

Aviation accident data provides essential information to monitor aviation safety. If collected at large scale then this data provides insight into the progress made by aviation industry and they may indicate possible safety bottlenecks. Based on these insights, the aviation industry can set their strategy and priorities right.

A long standing problem in the evaluation of aviation accident statistics is the joint estimation of accident rate, trend and uncertainty. An established approach is to divide the number of accidents by the number of flights, and to determine a 95% uncertainty area by using the underlying binomial distribution [1]. However, this approach does not estimate trend. Another established approach in estimating rate and trend is to perform a regression analysis by which the relationship between a dependent variable and one or more independent variables is analyzed. Now the uncertainty is not estimated. Thus with established approaches, either rate and uncertainty or rate and trend are jointly estimated, but not all three.

The aim of this in this paper is to overcome the limitation of the established approaches by developing a Bayesian approach [2] towards the estimation of the joint probability density function of the accident rate and the trend. The numerical evaluation of such a Bayesian approach has become possible due to the development of powerful sequential Monte Carlo simulation techniques [3].

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For the problem of estimating the joint conditional probability density function of accident rate and trend given large scale aviation accident and flight statistics, an exact Bayesian characterization is being developed first. Subsequently, a particle filter approximation is introduced in order to perform numerical evaluations. This particle filter is then used to perform joint estimation of accident rate, trend and uncertainty from commercial aviation accident data.

The paper is organized as follows. Section II formulates the mathematical problem for piecewise constant rates per year. Section III derives a recursive Bayesian characterization of the joint conditional probability density function for the rate and trend. Section IV presents a particle filter approach towards the evaluation of this joint conditional probability density function. Section V presents the results for the particle filter applied to worldwide aviation accident data. Section VI compares the new results with classical estimation results. Section VII presents results for the case where the accident data has been split into two groups, namely air related accidents and ground related accidents. Section VIII draws conclusions.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

This section proposes a specific mathematical formulation of the problem addressed in this paper. In order to characterize a model for the number of accidents per flight in a year, we assume that the accident rates are piecewise constant. We also assume that observed data is available on the number of flights and on the number of accidents per year.

Let $\lambda_k \in [0,1]$ denote the accident rate per flight in year k and $a_k \in [1-\varepsilon, 1+\varepsilon]$ the accident trend in year k, $k \in [0,...,F]$ and assume that these two evolve according to the following model:

$$\begin{aligned} a_k &= a_{k-1} \\ \lambda_k &= a_{k-1}\lambda_{k-1} \end{aligned} \quad (1)$$

with $\lambda_0 \in [0,1]$ and $a_0 \in [1-\varepsilon, 1+\varepsilon]$. Furthermore we assume that the initial joint probability distribution p_{λ_0, a_0} is a Uniform distribution on $[0,1] \times [1-\varepsilon, 1+\varepsilon]$.

Let h_k denote the number of flights in year k. The accident rate Λ_k in year k is then given by

$$\Lambda_k = h_k \lambda_k \tag{2}$$

Let κ_k denote the number of accidents in year k. We assume that κ_k given Λ_k has a Poisson distribution:

$$p_{\kappa_k|\Lambda_k}(\kappa \mid \Lambda) = \begin{cases} \dfrac{\Lambda^{\kappa}}{\kappa!}\exp(-\Lambda) & \kappa = 0,1,2,... \\ 0 & \text{else} \end{cases} \tag{3}$$

Bayesian estimation problem

Let $\mathcal{H}_k = \{\underline{h}_0, ..., \underline{h}_k\}$ and $\mathcal{K}_k = \{\underline{\kappa}_0, ..., \underline{\kappa}_k\}$ where $\underline{h}_k$ and $\underline{\kappa}_k$ are the observed realizations of h_k and κ_k. Given the flight statistics $\mathcal{H}_F$ and the accident statistics $\mathcal{K}_F$, the problem is to characterize the joint conditional density of λ_k and a_k:

$$p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)$$

This joint conditional density, defines estimates $\hat{\lambda}_k \triangleq E\{\lambda_k \mid \mathcal{H}_k, \mathcal{K}_F\}$ and $\hat{a}_k \triangleq E\{a_k \mid \mathcal{H}_k, \mathcal{K}_F\}$ as follows:

$$\hat{\lambda}_k = \int\int \lambda\, p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)\, da\, d\lambda \tag{4a}$$

$$\hat{a}_k = \int a \left(\int p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)\, d\lambda \right) da \tag{4b}$$

From the joint conditional density, 95% uncertainty intervals for λ_k can also be determined by the values $\hat{b}_{\lambda_k}^{\text{lower}}$ and $\hat{b}_{\lambda_k}^{\text{upper}}$ such that

$$\int_0^{\hat{b}_{\lambda_k}^{\text{lower}}} \left(\int_{1-\varepsilon}^{1+\varepsilon} p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)\, da \right) d\lambda = 0.025$$

$$\int_{\hat{b}_{\lambda_k}^{\text{upper}}}^{1} \left(\int_{1-\varepsilon}^{1+\varepsilon} p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)\, da \right) d\lambda = 0.025$$

Similarly, 95% uncertainty intervals for a_k can be determined by the values $\hat{b}_{a_0}^{\text{lower}}$ and $\hat{b}_{a_0}^{\text{upper}}$ such that

$$\int_{1-\varepsilon}^{\hat{b}_{a_k}^{\text{lower}}} \left(\int_0^1 p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)\, d\lambda \right) da = 0.025$$

$$\int_{\hat{b}_{a_k}^{\text{upper}}}^{1+\varepsilon} \left(\int_0^1 p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)\, d\lambda \right) da = 0.025$$

III. Characterization of Joint Conditional Density

In this section a recursive Bayesian characterization of the joint conditional density $p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda, a)$ is derived.

Applying Bayes' rule yields:

$$\begin{aligned} p_{\lambda_0,a_0|\mathcal{K}_k,\mathcal{H}_k}(\lambda, a) &= p_{\lambda_0,a_0|\kappa_k,\mathcal{K}_{k-1},\mathcal{H}_k}(\lambda, a \mid \underline{\kappa}_k) \\ &= \frac{1}{c_k} p_{\kappa_k|\lambda_0,a_0,\mathcal{K}_{k-1},\mathcal{H}_k}(\underline{\kappa}_k \mid \lambda, a)\, p_{\lambda_0,a_0|\mathcal{K}_{k-1},\mathcal{H}_k}(\lambda, a) \end{aligned}$$

where c_k denotes a normalising constant.

Since $p_{\lambda_0,a_0|\mathcal{K}_{k-1},\mathcal{H}_k}(\lambda, a)$ is independent of the number of flights in year k, thus

$$p_{\lambda_0,a_0|\mathcal{K}_{k-1},\mathcal{H}_{k-1},h_k}(\lambda, a) = p_{\lambda_0,a_0|\mathcal{K}_{k-1},\mathcal{H}_{k-1}}(\lambda, a),$$

it follows that

$$p_{\lambda_0,a_0|\mathcal{K}_k,\mathcal{H}_k}(\lambda, a) = \frac{1}{c_k} p_{\kappa_k|\lambda_0,a_0,\mathcal{K}_{k-1},\mathcal{H}_k}(\underline{\kappa}_k \mid \lambda, a)\, p_{\lambda_0,a_0|\mathcal{K}_{k-1},\mathcal{H}_{k-1}}(\lambda, a) \tag{5}$$

Using equations (1), (2) and (3) to evaluate (5) yields

$$\begin{aligned} p_{\lambda_0,a_0|\mathcal{K}_k,\mathcal{H}_k}(\lambda, a) = \frac{1}{c_k} \frac{\left(\underline{h}_k \lambda a^k\right)^{\underline{\kappa}_k}}{\underline{\kappa}_k!} \exp\left(-\underline{h}_k \lambda a^k\right) \cdot \\ \cdot p_{\lambda_0,a_0|\mathcal{K}_{k-1},\mathcal{H}_{k-1}}(\lambda, a) \end{aligned} \tag{6}$$

Repeated substitution of (6) for $k = F, k = F-1, ..., k = 0$, and subsequent evaluation yields

$$\begin{aligned} p_{\lambda_0,a_0|\mathcal{K}_F,\mathcal{H}_F}(\lambda, a) = \frac{1}{c} \left(\prod_{k=0}^{F} \frac{\left(\underline{h}_k \lambda a^k\right)^{\underline{\kappa}_k}}{\underline{\kappa}_k!} \exp\left(-\underline{h}_k \lambda a^k\right) \right) \cdot \\ \cdot p_{\lambda_0,a_0}(\lambda, a) \end{aligned} \tag{7}$$

where $c = c_0 \cdot c_1 \cdot ... \cdot c_F$.

Next we characterize $p_{\lambda_k,a_k|\mathcal{K}_F,\mathcal{H}_F}(\lambda, a)$ in terms of $p_{\lambda_0,a_0|\mathcal{K}_F,\mathcal{H}_F}(\lambda, a)$:

$$\begin{aligned} p_{\lambda_k,a_k|\mathcal{K}_F,\mathcal{H}_F}(\lambda, a) = \int p_{\lambda_k,a_k|\lambda_0,a_0}(\lambda, a \mid \lambda', a') \cdot \\ \cdot p_{\lambda_0,a_0|\mathcal{K}_F,\mathcal{H}_F}(\lambda', a')\, d\lambda' da' \end{aligned} \tag{8}$$

Due to (1),

$$p_{\lambda_k,a_k|\lambda_0,a_0}(\lambda,a\mid\lambda',a')=\delta_{\left[(a')^k\lambda',a'\right]}(\lambda,a)$$

Substituting this in (8) yields:

$$p_{\lambda_k,a_k|\mathcal{K}_F,\mathcal{H}_F}(\lambda,a)=\int\delta_{\left[(a')^k\lambda',a'\right]}(\lambda,a)p_{\lambda_0,a_0|\mathcal{K}_F,\mathcal{H}_F}(\lambda',a')\,d\lambda'da' \quad (9)$$

IV. Particle Filtering towards Estimation of Rate, Trend and Uncertainty

In this section a particle filter approach towards the numerical evaluation of $p_{\lambda_k,a_k|\mathcal{H}_F,\mathcal{K}_F}(\lambda,a)$ is presented. This particle filter is able to provide an arbitrarily close approximation of the true Bayesian solution by increasing the number of particles. The main idea behind this particle filter is to approximate the joint conditional density of λ_0 and a_0 given $\mathcal{H}_k$ and $\mathcal{K}_k$ by an empirical density that is defined by a set of particles, i.e. samples from the joint conditional density with corresponding weights.

Particles are randomly drawn from the initial distribution p_{λ_0,a_0}, each with same weights. Next the particles evolve and are updated according to the underlying stochastic model and the new measurements, where for each particle its weight is adapted based on the likelihood of the measurements for that particle. For the problem at hand, the underlying stochastic model is given by equations (1), (2) and (3), the measurements are given by $\mathcal{H}_k$ and $\mathcal{K}_k$, and the weights are adapted based on equation (6). With this particle filter, estimates $\hat{\lambda}_0$ and $\hat{a}_0$ of the model parameters λ_0 and a_0 can be obtained by simply taking the weighted average over all particles. A formal description of this particle filter reads as follows:

A *particle* is defined as a triplet $(\mu^j,\lambda_0^j,a_0^j)$, $\mu^j\in[0,1]$, $\lambda_0^j\in[0,1]$, $a_0^j\in[1-\varepsilon,1+\varepsilon]$, $j\in[1,...,N]$, where N denotes the number of particles, j refers to the j^{th} particle, μ^j denotes the weight of particle j, λ_0^j denotes the expected number of flights in year 0 of particle j, and a_0^j denotes the trend parameter of particle j. With these particles the joint conditional density of λ_0 and a_0 given $\mathcal{H}_k$ and $\mathcal{K}_k$ can be approximated by

$$p_{\lambda_0,a_0|\mathcal{K}_k,\mathcal{H}_k}(\lambda,a)\approx\sum_{j=1}^{N}\mu_k^j\delta_{\left[\lambda_0^j,a_0^j\right]}(\lambda,a)$$

The particle filter starts with an initiation step after which it cycles through the measurement update step and output equations:

Step 1: Initiation

Start with a set of N particles in $[0,1]\times[0,1]\times[1-\varepsilon,1+\varepsilon]$ i.e.

$$\left\{(\mu_{k-1}^j,\lambda_0^j,a_0^j);\ j\in[1,N]\right\}$$

with $\mu_0^j=1/N$, λ_0^j independently drawn from $p_{\lambda_0}(\lambda)$ and a_0^j independently drawn from $p_{a_0}(a)$ for each $j\in[1,...,N]$, e.g. both λ_0^j and a_0^j independently drawn from Uniform distributions on $[0,1]$ and $[1-\varepsilon,1+\varepsilon]$ respectively.

Step 2: Measurement processing

Perform for $k=0,...,F$ cycling through equations (10) and (11) below:

Using measurement $\underline{\kappa}_k$, determine new weights per particle,

$$\left\{(\mu_k^j,\lambda_0^j,a_0^j);\ j\in[1,N]\right\} \quad (10)$$

with for the new weights, using equation (6) for $k=0,1,2,...$:

$$\mu_k^j=\frac{1}{c_k}\frac{\left(\underline{h}_k\lambda_0^j\left(a_0^j\right)^k\right)^{\underline{\kappa}_k}}{\underline{\kappa}_k!}\exp\left(-\underline{h}_k\lambda_0^j\left(a_0^j\right)^k\right)\mu_{k-1}^j \quad (11)$$

with c_k a normalising constant such that $\sum_{j=1}^{N}\mu_k^j=1$.

Step 3. Joint conditional density at year k:

The particle filter outputs the joint conditional density of λ_0 and a_0 given $\mathcal{H}_F$ and $\mathcal{K}_F$ in the form of empirical density

$$p_{\lambda_0,a_0|\mathcal{K}_F,\mathcal{H}_F}(\lambda,a)\approx\sum_{j=1}^{N}\mu_F^j\delta_{\left[\lambda_0^j,a_0^j\right]}(\lambda,a) \quad (12)$$

Substitution of (12) into (9) and subsequent evaluation yields the joint conditional density of λ_k and a_k given $\mathcal{H}_F$ and $\mathcal{K}_F$ in the form of the empirical density

$$p_{\lambda_k,a_k|\mathcal{K}_F,\mathcal{H}_F}(\lambda,a)\approx\sum_{j=1}^{N}\mu_F^j\delta_{\left[\left(a_0^j\right)^k\lambda_0^j,a_0^j\right]}(\lambda,a) \quad (13)$$

Estimates $\hat{\lambda}_k$ and $\hat{a}_k$ are obtained by calculating the weighted average over all particles:

$$\hat{\lambda}_k\approx\sum_{j=1}^{N}\mu_F^j\lambda_0^j\left(a_0^j\right)^k \quad (14)$$

$$\hat{a}_k\approx\sum_{j=1}^{N}\mu_F^j a_0^j \quad (15)$$

Step 4. In this step we determine the probability density of κ_k given $\mathcal{H}_F$ and $\mathcal{K}_F$.

From the law of total probability we have:

$$p_{\kappa_k|\mathcal{K}_F,\mathcal{H}_F}(\kappa) = \int p_{\kappa_k|\lambda_k,\mathcal{K}_F,\mathcal{H}_F}(\kappa|\lambda)\, p_{\lambda_k|\mathcal{K}_F,\mathcal{H}_F}(\lambda)\, d\lambda \qquad (16)$$

Together with (13) this yields

$$p_{\kappa_k|\mathcal{K}_F,\mathcal{H}_F}(\kappa) \approx \int p_{\kappa_k|\lambda_k,h_k}(\kappa|\lambda,\underline{h}_k) \sum_{j=1}^{N} \mu_F^j \delta_{\left[(a_0^j)^k \lambda_0^j\right]}(\lambda) d\lambda \quad (17)$$

Thanks to (3) this becomes for $\kappa = 0,1,2,...$:

$$p_{\kappa_k|\mathcal{K}_F,\mathcal{H}_F}(\kappa) \approx \int \frac{(\lambda \underline{h}_k)^{\kappa}}{\kappa!} \exp(-\lambda \underline{h}_k) \sum_{j=1}^{N} \mu_F^j \delta_{\left[(a_0^j)^k \lambda_0^j\right]}(\lambda) d\lambda \, (18)$$

Subsequent evaluation yields for $\kappa = 0,1,2,...$

$$p_{\kappa_k|\mathcal{K}_F,\mathcal{H}_F}(\kappa) \approx \sum_{j=1}^{N} \mu_F^j \frac{\left(\left(a_0^j\right)^k \lambda_0^j \underline{h}_k\right)^{\kappa}}{\kappa!} \exp\left(-\left(a_0^j\right)^k \lambda_0^j \underline{h}_k\right) \quad (19)$$

which is the equation to be used in step 4.

V. APPLICATION OF THE PARTICLE FILTER TO WORLDWIDE AVIATION ACCIDENT DATA

The particle filter of section IV is now applied to worldwide commercial aviation accident and flight data. First we explain which input data is used. Subsequently the particle filtering results are presented.

A. *Input data*

The aviation accident and flight data used in this paper are from [4]. The table below specifies the criteria that have been used for the selection from this database.

TABLE I. DATA SELECTION CRITERIA

Selection Criteria of accidents and flights	
Time period	1/1/1990 – 31/12/2008
Occurrence Class	Accident
Aircraft Category	Fixed Wing
Aircraft Mass group	> 5700 kg
Location of occurrence	Worldwide (not filtered)
Operation Type	Scheduled Commercial Air Transport

In figures 1 and 2 the resulting $\mathcal{H}_F$ and $\mathcal{K}_F$ data is visualised. Figures 1 and 2 provide the number of flights $\underline{h}_k$ and the number of accidents $\underline{\kappa}_k$ for $k = 0,1,...,18$, where $k = 0$ corresponds with year 1990. Figure 3 provides for each year the ratio between the number of accidents $\underline{\kappa}_k$ and the number of flights $\underline{h}_k$.

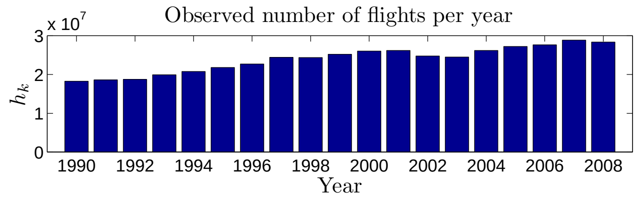

Figure 1. Observed number of flights $\underline{h}_k$ for $k = 0,1,...,18$, corresponding with years 1990 – 2008.

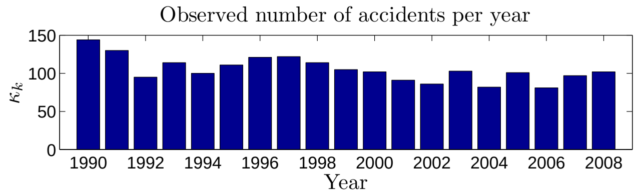

Figure 2. Observed number of accidents $\underline{\kappa}_k$ for $k = 0,1,...,18$, corresponding with years 1990 – 2008.

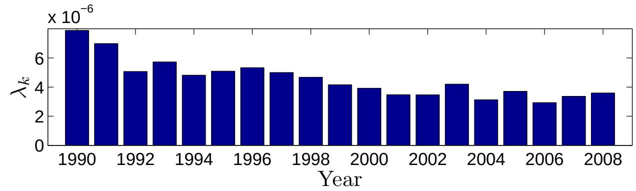

Figure 3. Ratio $\underline{\kappa}_k / \underline{h}_k$ for $k = 0,1,...,18$, corresponding with years 1990 – 2008.

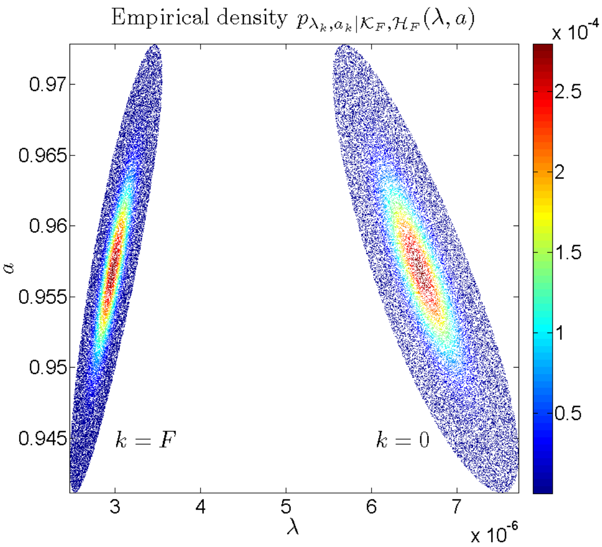

Figure 4. Joint conditional density of $p_{\lambda_k,a_k|\mathcal{K}_F,\mathcal{H}_F}$ for $k = 0$ and $k = F$

B. *Particle filtering results*

We use the particle filter equations of section IV with one million particles (i.e. $N = 10^6$). Particle filter based numerical evaluation of $p_{\lambda_k, a_k | K_F, H_F}(\lambda, a)$ is depicted in the form of the empirical joint conditional densities in figure 4. From the orientation of the empirical joint conditional densities in Figure 4 it can observed that estimation of accident rate λ_k and accident trend a_k involves strong correlation.

Figures 5 and 6 provide marginal empirical densities for $p_{\lambda_F | \mathcal{K}_F, \mathcal{H}_F}(\lambda)$ and $p_{a_F | \mathcal{K}_F, \mathcal{H}_F}(a)$ as these resulted from the empirical joint conditional density $p_{\lambda_F, a_F | \mathcal{H}_F, \mathcal{K}_F}(\lambda, a)$. Corresponding 95% uncertainty intervals are also given (dotted vertical lines). From the shapes of the empirical densities in these figures it can be observed that they closely resemble Gaussian densities.

Figure 7 provides the marginal empirical density for $p_{\kappa_F | \mathcal{K}_F, \mathcal{H}_F}(\kappa)$ which follows from particle filter step 4.

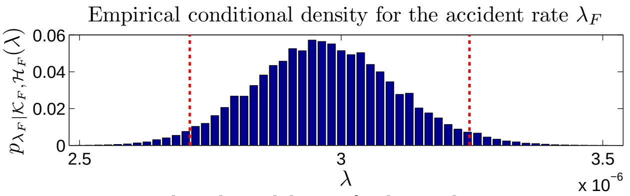

Figure 5. Empirical conditional density for the accident rate at year 2008, $\lambda_F : p_{\lambda_F | \mathcal{K}_F, \mathcal{H}_F}(\lambda)$ with 95% uncertainty interval.

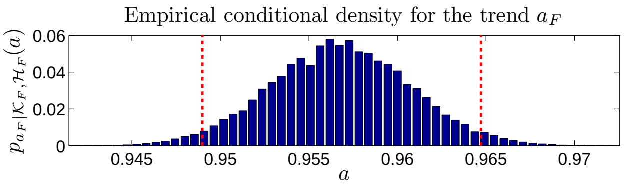

Figure 6. Empirical conditional density for the trend at year 2008, $a_F : p_{a_F | \mathcal{K}_F, \mathcal{H}_F}(a)$ with 95% uncertainty interval.

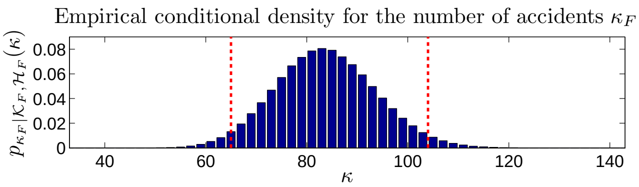

Figure 7. Empirical conditional density $p_{\kappa_F | \mathcal{K}_F, \mathcal{H}_F}(\kappa)$ for κ_F at year 2008, with 95% uncertainty interval.

For year 2008, the estimates of the accident rate $\hat{\lambda}_F$ and accident trend $\hat{a}_F$ are given by

$$\hat{\lambda}_F = 2.97 \times 10^{-6}$$
$$\hat{a}_F = 0.957$$

The corresponding standard deviations $\hat{\sigma}_{\lambda_F}$ and $\hat{\sigma}_{a_F}$, and correlation coefficient $\hat{\rho}_{\lambda_F, a_F}$ are given by

$$\hat{\sigma}_{\lambda_F} = 0.14 \times 10^{-6}$$
$$\hat{\sigma}_{a_F} = 0.004$$
$$\hat{\rho}_{\lambda_F, a_F} = 0.87$$

Hence, standard deviations amount some 5% of the estimated means, and there is strong correlation.

VI. Comparison with Classical Estimation Results

An established, approach, e.g. [1], is to determine for each year the ratio between the number $\underline{\kappa}_k$ of accidents and the number $\underline{h}_k$ of flights as an indication of estimated accident rate for each year, and to use the underlying binomial distribution to determine a 95% uncertainty area around this point estimate.

Now we compare the classical estimated 95% uncertainty intervals with our new 95% uncertainty intervals that apply to $\kappa_k / \underline{h}_k$ with $p_{\kappa_k | \mathcal{K}_F, \mathcal{H}_F}(\kappa)$ as is illustrated in Figure 7 for $k = F$. Figure 8 shows for each year the particle filtering based estimation of mean rate and 95% uncertainty interval of $\kappa_k / \underline{h}_k$ versus the classical point estimates and 95% uncertainty interval of the accident probability. The dashed line represents the mean, and the dotted lines represent the 95% uncertainty interval of our new estimation results. The classical point estimates of the accident probability with 95% uncertainty intervals are depicted as circles with corresponding error bars.

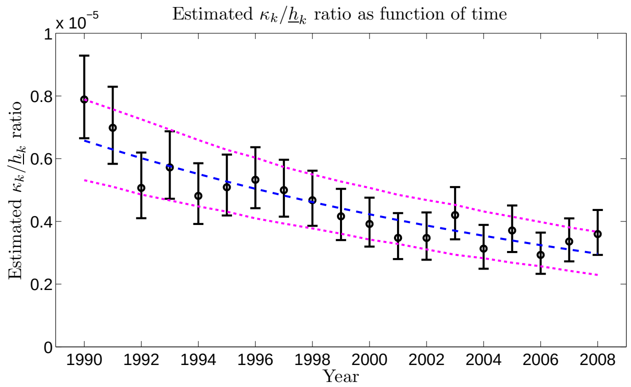

Figure 8. Newly estimated $\kappa_k / \underline{h}_k$ ratio (— — — = mean) with 95% uncertainty interval (▪▪▪▪) versus classical point estimates (● = mean) and 95% uncertainty interval (I).

Figure 8 shows that the sizes of the 95% areas of the $\kappa_k / \underline{h}_k$ ratios are quite similar for both approximations. However the new approach yields a much smoother estimate of the evolution of the mean over time.

VII. RATE, TREND AND UNCERTAINTY FOR AIR AND GROUND RELATED ACCIDENTS

We also estimate rate, trend and uncertainty separately for air related accidents and ground related accidents. The resulting estimates for the $\kappa_k / \underline{h}_k$ ratio are depicted in Figure 9. This shows that since 2003 the ground related accident ratio tends to overtake lead from the air related accident ratio.

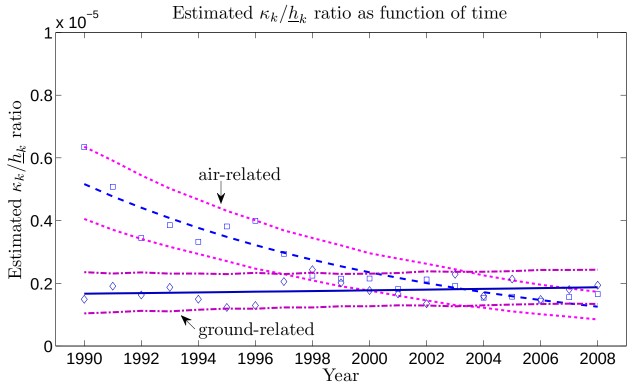

Figure 9. Newly estimated air related $\kappa_k / \underline{h}_k$ ratio (— — = mean) with 95% uncertainty interval (•••••) versus ground related $\kappa_k / \underline{h}_k$ ratio (—— = mean) with 95% uncertainty interval (— • —), with measured air related $\kappa_k / \underline{h}_k$ ratios (□) and ground related $\kappa_k / \underline{h}_k$ ratios (◊).

For year 2008, the estimates of air related accident rate $\hat{\lambda}_{F,\text{air}}$ and accident trend $\hat{a}_{F,\text{air}}$ become:

$$\hat{\lambda}_{F,\text{air}} = 1.25\times 10^{-6}$$
$$\hat{a}_{F,\text{air}} = 0.924$$

and the estimated standard deviations $\hat{\sigma}_{\lambda_F,\text{air}}$ and $\hat{\sigma}_{a_F,\text{air}}$, and correlation coefficient $\hat{\rho}_{\lambda_F,a_F,\text{air}}$ become :

$$\hat{\sigma}_{\lambda_F,\text{air}} = 0.08\times 10^{-6}$$
$$\hat{\sigma}_{a_F,\text{air}} = 0.005$$
$$\hat{\rho}_{\lambda_F,a_F,\text{air}} = 0.90$$

Hence, standard deviation for the estimated rate amount some 6% of the estimated means, and there is strong correlation. For air related accidents this means that the estimated trend value is significant.

For year 2008, the estimates of ground related accident rate $\hat{\lambda}_{F,\text{ground}}$ and accident trend $\hat{a}_{F,\text{ground}}$ become:

$$\hat{\lambda}_{F,\text{ground}} = 1.87\times 10^{-6}$$
$$\hat{a}_{F,\text{ground}} = 1.007$$

and the estimated standard deviations $\hat{\sigma}_{\lambda_F,\text{ground}}$ and $\hat{\sigma}_{a_F,\text{ground}}$, and correlation coefficient $\hat{\rho}_{\lambda_F,a_F,\text{ground}}$ become

$$\hat{\sigma}_{\lambda_F,\text{ground}} = 0.12\times 10^{-6}$$
$$\hat{\sigma}_{a_F,\text{ground}} = 0.007$$
$$\hat{\rho}_{\lambda_F,a_F,\text{ground}} = 0.84$$

Again, standard deviations amount some 6% of the estimated means, and there is strong correlation. For ground related accidents this means that the estimated trend value does not deviate significantly from being at the steady value of 1.0.

The above results show that the estimated mean of ground related accident rate is in 2008 about 50% higher than the air related accident rate. Moreover, the estimated trends are: 0.65% per year increase in ground related accident rate and 7.5% per year decrease in air related accident rate. And this difference in estimated trends for air and ground related accident rates is statistically significant.

VIII. CONCLUDING REMARKS

In this paper a novel approach towards joint estimation of accident rate, trend and uncertainty in aviation accident data has been developed. This novel approach is based on the exact Bayesian estimation of the joint conditional density of the accident rate and trend given observed accident and flight data. For numerical evaluation a particle filter approximation has been developed.

Numerical evaluations and comparison to classical approach shows the validity of the novel approach for joint estimation of accident rate, trend and uncertainty.

The novel method also shows that estimated air related accident rate has a decreasing trend whereas ground related accident rate has a slowly increasing trend. As a result of this, since 2003 estimated ground related accident rate tends to overtake lead from estimated airborne related accident rate. This clearly shows that there is an urgent need in developing measures that are effective in bending the trend in ground related accident rate from a yearly increase to a yearly decrease.

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REFERENCES

[1] G. van Es. "A Review of Civil Aviation Accidents Air Traffic Management Related Accidents:1980-1999," Proc. USA/Europe R&D Seminar on Air Traffic Management, Santa Fe, USA, 2001.

[2] J.M. Bernardo and A.F.M. Smith, Bayesian Theory, Wiley, 1994.

[3] A. Doucet, N. de Freitas, and N. Gordon, Sequential Monte Carlo Methods in Practice, Springer-Verlag, 2001.

[4] NLR Air Safety Data Base, version 2009.