Collaborative Rerouting in the Airspace Flow Program

A Framework for User-cost Based Performance Assessment

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***Abstract*— The Airspace Flow Program (AFP) ground delays flights in order to control their flow through capacity constrained airspace regions. It has been successful in controlling traffic with reasonable delays, but the procedures must be improved upon to handle future projected demands. This paper explores a future AFP where centrally-managed rerouting and user input is incorporated into the initial resource allocations. A modeling framework was developed to evaluate and compare allocation strategies, under differing assumptions about traffic managers' knowledge about airline flight costs. It is used to quantify tradeoffs regarding the quality and timing of airlines' input information. Three allocation strategies were developed; they differ with respect to the input requested of airlines, and the resource allocation philosophy. They are assessed based on the total cost impact of the AFP initiative on flight operators. To this end, a flight cost function was developed to represent the cost of delay specific to each flight; it consists of deterministic components to represent what traffic managers know about the airlines, and a stochastic component to represent that which they do not. A numerical example demonstrates the situations under which better information quality could be more desirable than timeliness, and vice versa. Identifying these types of tradeoff points is a key contribution of this research effort.**

Keywords- delay; air traffic flow management (ATFM); Airspace Flow Program (AFP); Collaborative Decision Making (CDM); user cost; strategic planning.

I. Introduction

Adverse weather frequently and severely impacts flight operations in the National Airspace System (NAS). In addition, with the growth in demand projected over the next 20 years, weather and traffic-induced delays are also anticipated to increase under the current system. Air traffic flow management (ATFM) programs are used to reduce the scale and cost of disruptions to flight operators. One such initiative is the Airspace Flow Program (AFP), in which flights are held on the ground at departure airports in order to meter them through capacity constrained airspace regions. The AFP was first implemented in 2006 in the northeast region of the U.S., and has proven to be successful in controlling traffic with reasonable flight delays. However, as demands increase into the future, the benefits derived from the AFP process will become limited unless a procedure to better utilize airspace capacity is incorporated into the process.

This research addresses the need for a more comprehensive, centrally-managed, and user-input based resource allocation program for AFPs. We develop a modeling framework through which we formulate, evaluate, and compare strategies that employ rerouting combined with ground delay to minimize the impacts of AFP initiatives on users of the NAS. The assignment strategies differ with respect to the inputs requested of users, and the rules by which resource allocation decisions are made. This paper presents three strategies based on combinations of two resource allocations schemes and two forms of user input. The main objective of this paper is to investigate how these strategies perform in comparison to one another under different assumptions about airline utility. The model framework through which we can identify the tradeoff points between strategies is a key contribution of this research.

Throughout this paper, "operator" will be used to refer to NAS users such as commercial airlines and general aviation aircraft. "Traffic manager" will refer to traffic managers overseen by the Federal Aviation Administration (FAA). Section II describes the current system, and a literature review. Section III contains a problem overview while Section IV introduces the modeling framework and models. Section V presents an illustrative numerical example and Section VI concludes with a discussion and plans for future work.

II. Background

A. Constrained Airspace Rerouting

Flight rerouting due to severe en route weather and traffic congestion is performed in both strategic and tactical ATFM. It is manually intensive as it requires close coordination between several traffic management units. As a result, traffic managers select reroutes from a standard set compiled in the National Playbook, basically employing a "one size fits all" approach [1] without input from the operators. Airlines also have the option of rerouting their own flights before and after departure, subject to traffic managers' approvals. They often exercise this option to avoid assignment of undesirable routes and heavily delayed departure times.

Air traffic flow management initiatives, including centralized rerouting, can be inefficient without input from users, because resource allocations are made without knowledge about the value of the assignment to users. As a

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result, more collaborative approaches to rerouting have been proposed. Concepts that aim for more structured coordination between traffic managers and operators have existed since the early 2000s, but implementation has been difficult.

B. Collaborative Decision Making (CDM)

A significant improvement to NAS air traffic management began in the mid-1990s with the Collaborative Decision Making (CDM) program. CDM is a joint government and industry initiative that aims to improve both the technological and procedural aspects of air traffic management, through improved information exchange between government and industry. The first major application of CDM was to Ground Delay Programs (GDPs). When an airport has reduced arrival capacity due to severe weather either en route or near the airport, a GDP holds flights destined for that airport on the ground at their origin airports to meter demand. CDM information exchange between operators and the traffic manager drastically enhanced the effectiveness of GDPs in correcting demand/capacity imbalances and reducing delays, by ensuring that the traffic manager have up-to-date demand information and that "slots" vacated as a result of cancellations or other events could be used for other flights. GDPs are very effective in managing reduced arrival capacity when it is caused by inclement weather near the destination airport. However, GDPs can be inefficient, ineffective and inequitable in addressing en route constraints. As a result, the AFP was first implemented in 2006 to handle en route constraints.

C. Airspace Flow Program (AFP)

In an AFP, the constrained airspace region and the flights filed into this region during the time of reduced capacity are identified. The reduced capacity is then distributed by assigning delayed departure times to the impacted flights. Constrained airspace regions include those that are experiencing undesirable weather and/or heavy demands. Most AFPs begin after 2PM local time as airspace congestion and convective weather are more likely to occur after this time. They typically end after 10PM.

An AFP flight will receive a delayed departure time on its original filed route. It can either accept the assigned departure time, or reject and reroute around the constrained airspace (subject to traffic managers' approval). Slots to fly through the constrained region are vacated as flights are canceled or routed out, and the schedule is compressed such that remaining flights are moved up in time. Currently, the distribution of delayed departure times combined with airline-initiated rerouting and cancellation has proven to be adequate for handling capacity constraints. However, with growing demand, greater utilization of other available airspace capacity will be required. One strategy is to incorporate reroutes into the initial resource allocation, such that delayed departure times are combined with new route assignments. Flying a longer alternative route with less ground delay might be a desirable alternative to accepting a long ground delay on the original route[1]. Also, if neighboring routes could be more optimally utilized, the total delay cost of the AFP could be reduced. In order to offer resource assignments that are desirable to operators, however, the FAA will require a significant level of user input.

D. Literature Review

There has been much work in developing optimization models to support ATFM decisions. The objective of many such models is to minimize the system-wide cost of delay. They consider ground holding, air holding and rerouting decisions. The Bertsimas and Stock-Patterson model provide for flight-specific air and ground hold cost ratios in their model, but do not provide any information about them [2]. Goodhart's models provide a framework where ATFM decisions are made through information exchange between the FAA and operators [3].

Uncertainties in weather and capacity have been addressed in the single airport ground holding problem, which has been considered in deterministic and stochastic, static and dynamic formulations. The earliest work began with [4] and [5], and the problem was addressed in a collaborative context by [6]. Reference [7] formulated an algorithm to schedule, reroute and airhold flights flying into and around constrained airspace, imposing ordering schemes that align with CDM.

Much literature exists about resource rationing and equity in ATFM, specifically within the context of GDPs. Reference [8] describes a framework for equitable allocation, illustrating their operational impacts and use in reducing systematic biases. Reference [9] compares the efficiency of airspace resource allocation schemes as alternatives to GDP allocation schemes.

The assumption of continuously distributed VOT over flight populations has not been studied in the context of the ATFM problem. Value of time (VOT) was examined as a continuous distribution [10] over a vehicle population for a steady-state congestion pricing model. Comparing different methods of incorporating heterogeneous users' preferences into ATFM models has also not been studied extensively.

III. PROBLEM OVERVIEW

The AFP facilitates resource allocation decisions when en route demand/capacity imbalances exist. In addition to system capacity constraints, under the CDM philosophy decisions are shaped by the allocation and equity principles chosen for implementation, as well as the user information provided to the process. By altering these inputs, the resulting allocation structure can potentially look very different from another.

There are many resource allocation schemes that could be considered [11] and we list a few. Traffic managers may be instructed to meet system cost targets with or without certain equity constraints. Users could be allocated resources by order of information submission, the original schedule, or a random order. Airlines could also be assigned a proportion of the total available resources based on the number of flights they have scheduled. Priority may be given based on aircraft size.

Performance assessments are based on system efficiency measures as well as user satisfaction and cost considerations, which are part of the users' utility structure. The overall performance of an allocation scheme will improve when inputs that well represent users' utility are incorporated. User inputs can come in many forms, and we introduce two in this paper.

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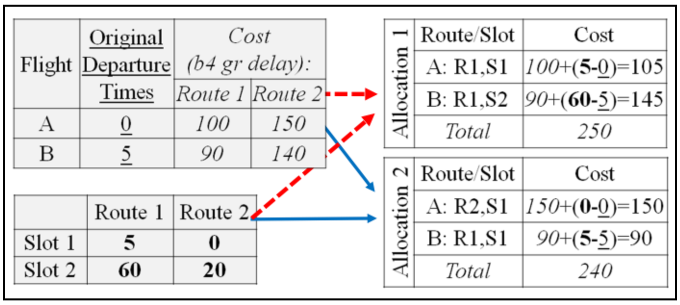
[1] This would be particularly true when there are critical downline flight and crew connections to be made.



Flight	Original Departure Times	Cost (b4 gr delay): Route 1	Route 2
A	0	100	150
B	5	90	140

	Route 1	Route 2
Slot 1	**5**	**0**
Slot 2	**60**	**20**

Allocation 1 Route/Slot	Cost
A: R1,S1	100+(**5**-0)=105
B: R1,S2	90+(**60**-5)=145
Total	250

Allocation 2 Route/Slot	Cost
A: R2,S1	150+(**0**-0)=150
B: R1,S1	90+(**5**-5)=90
Total	240

Figure 1. Allocation Illustration

Consider the simple example illustrated in Figure 1. Two flights (A and B) are planned to travel some nominal route with original departure times 0 and 5 minutes. The route is closed due to convective weather; to accommodate these flights, departure slots on two alternative routes are offered. Say that flights A and B offered their en route costs (in ground delay minutes) for each route, shown in the top left. The final cost is calculated based on the difference between the original departure time and the new slot time, plus the en route cost. If traffic managers are obliged to serve Flight A first (Allocation 1), then Flight A would be given Route 1 slot 1 as it is the lowest cost option available to it. Flight B would be left with Route 1 slot 2 as its best available option. The total cost of this allocation is 250. If the goal is to minimize total cost (Allocation 2) they would assign Flight A to Route 2 slot 1 and Flight B to Route 1 slot 1. The cost of this allocation is 240. Clearly the allocation results could change if airlines submitted different cost values.

In this paper we consider a functional form to represent the cost of an AFP reroute to flights. This cost function has both deterministic and random components, to represent what information the FAA does and does not have about the operators of these flights. We use this function to assess the performance of several different resource allocation/user input combination models. We build models based on two different user input types – the parametric model and the stated route preference model. The parametric model requires users to supply parameters of the cost function, which traffic managers use to calculate costs of various reroute and ground delay options. The stated route preference model requires operators to supply more detailed, complete cost information about the route and ground delay options available in each AFP. It is based on the delay thresholds concept developed as part of the Flow Constrained Area Rerouting (FCAR) Decision Support Tool by Metron Aviation [12], which is discussed in further detail in the following section. Both models allocate resources based on system-optimal cost minimizations where equity is not considered. However, we also consider another version of the stated route preference model where flights are assigned resources by the order they submit their input data. In the first-submitted, first-assigned (FSFA) model, the earlier flights offer their input data, the more likely they are to receive a more desirable allocation. The FSFA allocation scheme is an easily understood and well-accepted rationale that has been adopted in various forms within CDM [9].

The main objectives of this research are to determine how models with these different resource allocation schemes and user inputs perform against one another under changing assumptions about flight utility. Performance will be measured using the total generalized flight cost of each models' optimal AFP resource allocation. The result is a framework through which user input and resource allocation combinations can be represented, evaluated, and compared.

IV. Model Framework

A. Evaluation Scenario

We introduce a simple model context in Figure 2. A nominal route (Route 1) connects two fixes in en route airspace. Flights enter Route 1 at entry fix "A" and leave at exit fix "B". Route 1 has sufficient capacity to serve the scheduled demand $D_0(t)$, until a capacity constraint develops at a fixed location along its path and lasts for duration T. The capacity of Route 1 is reduced, and an FCA is created. The total scheduled demand must be reassigned to observe this reduced capacity. All N flights originally scheduled to use Route 1 are either given delayed departure times, rerouted to an alternative route, or both. Each alternate route r is characterized by its travel time air_r and capacity $S_r(t)$. We assume that fixes A and B are not bottlenecks, and for the purpose of this analysis they are considered the flights' origin and destination.

As mentioned previously, FAA traffic managers have limited access to the details of how airlines make flight cost calculations and subsequent routing decisions. This analysis is unconcerned with the airlines' actual costs for the original scheduled flight plans, as it is assumed that these flight plans were those most preferred under ideal conditions. We are concerned with evaluating the additional cost of greater en route time and ground delay due to AFP.

We can assume that $c_{n,r}$, the additional cost of the n^{th} departing flight taking route r due to an AFP, is a function of the increased air time (compared to the nominal route, and assuming that aircraft fly at fuel-efficient speeds) and time spent in ground delay. The additional en route time and ground delay account for many direct costs such as additional fuel, crew time, equipment maintenance, and indirect costs such as passenger satisfaction, gate time, flight coordination, and the airline's satisfaction with their own particular objectives. We assume air holding is not necessary because we have perfect information about the capacity constraint duration T, scheduled demand $D_0(t)$, and all route capacities $S_1(t), \ldots, S_R(t)$. As such, all anticipated delay is incurred on the ground.

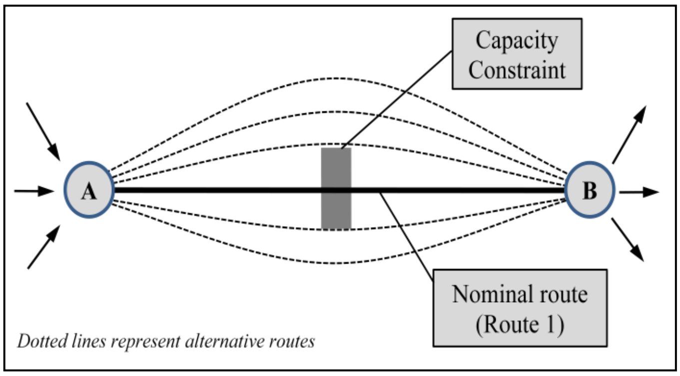


Figure 2. Model Framework

The generalized flight cost function is specified such that the air time, ground delay, and error components do not interact with one another. It is also a linear function of inputs, and is quantified in units of ground delay minutes.

$$c_{n,r} = c_{n,r}^{air} + c_{n,r}^{grdelay} + \varepsilon_{n,r}, \ \ \varepsilon_{n,r} \sim P \tag{1}$$

Each cost component can be further identified as follows:

$$c_{n,r} = \alpha_n \cdot (h_r - h_0) + d_{n,r} - s_n + \varepsilon_{n,r}, \qquad \varepsilon_{n,r} \sim P \tag{2}$$

where α_n is a ratio for converting additional AFP-related en route time to ground delay minute units for flight n, h_r is the newly assigned en route time for route r, h_0 is the en route time for the original (scheduled) route, $d_{n,r}$ is the new departure time for flight n on route r at fix A under the AFP, s_n is the original scheduled departure time for flight n at fix A, and $\varepsilon_{n,r}$ is the random error term for the cost of the AFP, and follows distribution P.

The quantity $(h_r - h_0)$ is non-negative because it is likely that the nominal route had the shortest flying time under an optimal speed, hence its status as the nominal route. Here we assume the effects of tactical control are insignificant compared to the delay cost of the AFP. Ground delay is non-negative because aircraft cannot depart before their original scheduled time, such that $\left(d_{n,r} - s_n\right) \geq 0$.

If the AFP capacity of each alternative route r is $S_r(t)$, it then follows that the instantaneous minimum headway at time t is $S_r^{-1}(t)$. Now assume that $S_r(t)$ is constant over the duration of the AFP, and aircraft on route r are scheduled with constant headways. We have established that the n^{th} flight (out of a total AFP population of N) is scheduled to depart at $d_{n,r}$. If we instead tabulate flights by route, the departure time of flight i on route r (of total flights X_r assigned to r) can be expressed as a linear function of i with slope $g_r = \frac{1}{S_r(t)}$. We also assume that original scheduled demand $D_0(t)$ is constant, and s_n can be expressed as a linear function of n with slope $g_0 = \frac{1}{D_0(t)}$. It then follows that the total estimated cost of an AFP (without accounting for unknown cost components) is expressed as:

$$\hat{C} = \sum_{r=1}^{R} \sum_{i=1}^{X_r} \alpha_i \rho_r + g_r i - g_{0,i,r} \tag{3}$$

where $\hat{C}$ is the total estimated cost of an AFP, α_i is the cost ratio of additional AFP-related en route time for the i^{th} flight on route r, ρ_r is the additional en route time if a flight is reassigned to route r ($\rho_r = h_r - h_0$), g_r is the new AFP departure headway on route r, and $g_{0,i,r}$ is the original (before AFP) scheduled departure time for flight i on route r. Also, if X_r is the total number of flights assigned to route r, then $\sum_{r=1}^{R} X_r = N$.

This paper focuses on the case where all flights were originally scheduled to depart at the same time (i.e. $g_{0,i,r} = 0$). However, this analysis has been extended to a case where flights are originally scheduled to depart at different times.

B. *Parametric Reroute Model*

1) *Concept*

In the parametric reroute model, the FAA allocates AFP resources using the cost function shown previously (1-3) with parameters supplied by operators. If specified well, the model can provide a good reflection of operator utility, and the resource allocation can be very efficient. If specified poorly, resource allocations can be inefficient. We would like to ascertain how this approach performs in comparison to the stated route preference strategies under increasingly errant specifications.

We envision that an FAA mandate would require airlines to provide cost parameters for their domestic flights to a central database. Airlines would be encouraged to update these parameters as desired. When an AFP is announced (typically several hours prior to the start time [12]), the parameters are used to determine route and ground delay assignments for the AFP-affected flights. We assume that airlines are implicitly incentivized to provide their most up-to-date cost parameters to maximize their likelihood of obtaining desirable flight plans in the AFP. This model does not employ means of providing additional incentives or equity in resource rationing.

This model is formulated as a route assignment problem with a system optimal solution objective. The outcome of this model will be the number of flights, X_r, to assign to each route, r, to minimize the total cost of AFP to operators. It is assumed that AFP flights are in competition for the available resources of lowest cost. As the AFP departure time increases for each subsequent flight i assigned to route r, and $g_{0,i,r} = 0$, the ground delay of flights on a route is monotonically increasing.

2) *Model Specification*

The N total flights originally scheduled to fly nominal Route 1 (Figure 3) in T are reassigned to one of R routes with new departure times. We assume that the flight operators submit different en route cost parameter values to traffic managers, such that $\alpha_1 \neq \alpha_2 \neq \cdots \neq \alpha_N$. We assume that cost parameters are distributed over the flight population according to a probability distribution, and the N AFP flights are a representative population sample. Furthermore, if N flights are ordered by increasing α, we define $\alpha(n) = \alpha_n$ to be the en route cost parameter for the n^{th} flight. The value of α_n is determined as shown in the left graph of Figure 3.

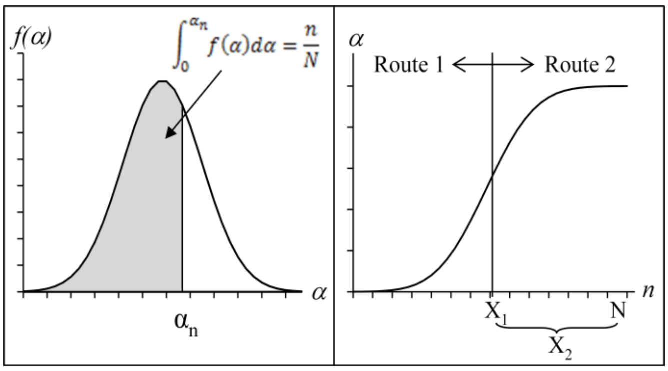


Figure 3. PDF of En Route Cost Parameter α across Flights

Given a set of routes, flights with the highest α values should be assigned to the routes with lowest en route times, and vice versa, if the unique minimum cost solution is to be obtained. The right graph of Figure 3 shows α plotted over n (shown as a continuous variable). For instance, if there are two routes such that $\rho_1 > \rho_2$, flights with lower α should be assigned to Route 1 such that those with higher α can take Route 2. If there are more than two route options, we order them according to decreasing en route times $\rho_1 > \rho_2 > \cdots > \rho_R$, and aircraft can be ordered and assigned by increasing α.

We assume that α is uniformly distributed in $(\alpha_{\min}, \alpha_{\max}]$. Then α is a linearly increasing function of n such that:

$$\alpha = \alpha_{min} + \left(\frac{\alpha_{max} - \alpha_{min}}{N}\right) \cdot n \quad (4)$$

The model is defined as follows:

Decision variable: $X_r\ \forall r$ (total flights assigned to route r)

Objective function (as per Equation (3), with $g_0 = 0$):

$$\min_{X_1,\dots,X_R} \hat{C} = \sum_{r=1}^{R}\sum_{i=1}^{X_r}\left(\rho_r \cdot \left[\alpha_{min} + \theta \cdot \left(\sum_{j=1}^{r} X_{j-1} + i\right)\right] + g_r i\right) \quad (5)$$

where $\theta = \frac{\alpha_{max} - \alpha_{min}}{N}$.

Constraints: $\sum_{r=1}^{R} X_r = N;\ X_r \geq 0, \forall r$

The first part of the objective function represents the cost of additional en route time for flight i on route r, while the last term represents the ground delay for flight i on route r. The terms within the square brackets represent the α value for i on r. The choice of which flights to assign to which routes is based on the ordering described before Figure 3. The X_1 flights with the highest α values are assigned to route 1; the X_2 flights with the next highest α values are assigned to route 2; and so on. One can see that given the X_r values, this will yield the lowest cost assignment.

The first constraint ensures that all the flights caught in the AFP will be assigned to an available route and departure slot. The second constraint ensures that all route counts are non-negative. The objective function was checked for convexity. X_r is an integer variable, but this was relaxed to find a solution. Even if solutions are not integer, rounding (to preserve N) will still produce acceptable results because the headways on each route should be designed include some buffer space [5]. Also, if by rounding up X_r the route capacities were slightly exceeded occasionally, it would not be catastrophic.

If $X_n < 0$ or $X_n > N\ \forall n$, then interior solutions to the objective function of (6) do not exist, and solutions lie at the boundaries. In these cases, $X_n^* = 0$ and $X_n^* = N$ respectively.

Recall that the resulting resource allocation scheme is based on the estimated costs to operators. If $\varepsilon_{i,r}$ represents the unknown cost component for a flight, the "true" cost of the scheme is calculated by adding an error term to the total cost.

$$C = \sum_{r=1}^{R}\sum_{i=1}^{X_r}(\alpha \cdot \rho_r + g_r i + \varepsilon_{i,r}) = \hat{C} + \sum_{r=1}^{R}\sum_{i=1}^{X_r}\varepsilon_{i,r}, \quad \varepsilon_{i,r} \sim P \quad (6)$$

If $\varepsilon_{i,r}$ are iid Gumbel with parameters (a, b), then according to the central limit theorem their sum ε is asymptotically distributed normal with mean $a - 0.5772b$ and standard deviation $\frac{\pi}{\sqrt{6}} bN$. We use $E[\varepsilon] = a - 0.5772b$ in the analytical solution. For simulated solutions we sample $\varepsilon_{i,r}$ N times to find C.

The Gumbel distribution has several important properties that make it analytically convenient to use in the specification of choice probabilities and expected cost [13], which we utilize for one of the stated route preference models that are discussed next. Also, the Gumbel distribution is reasonably similar to the normal distribution.

C. *Stated Route Preference Model*

1) *Concept*

The stated route preference models utilize the FCAR delay threshold concept [12]. FCAR was developed in order to give operators flexibility in identifying the best reroute options for their AFP-impacted flights.

In the FCAR process, operators of impacted flights are asked to submit route preference information to the traffic managers. For each route r, the operator of flight n submits a delay threshold value, $\Delta_{n,r}$, which is the cost at which flight n should be switched from route r to another. The quantity $\Delta_{n,r}$ contains the airlines' complete cost information about route r, relative to the original flight plan, before ground delays are assigned. $\Delta_{n,r}$ is expressed in units of ground delay minutes such that airline costs are not explicitly revealed. Once the FAA receives the delay threshold values, they will rank flights route/departure time slot combinations based on some adopted resource rationing scheme [12]. For each sequential flight they choose a feasible departure time slot on each route, and based on the delay thresholds, determine the flight's minimum cost route.

An example is shown in the figure below. Suppose a flight n had three route options, and the flight operator submitted a delay threshold value for each route r ($\Delta_{n,r}$). Once it is that flight's turn for allocation, traffic managers check the slot availability on each route and determine the ground delay that flight n must take on each route: $GD_{n,1}, GD_{n,2}$, or $GD_{n,3}$. The route assigned to flight n is $min(\Delta_{n,1} + GD_{n,1},\ \Delta_{n,2} + GD_{n,2},\ \Delta_{n,3} + GD_{n,3})$, which is route 3 according to the figure.

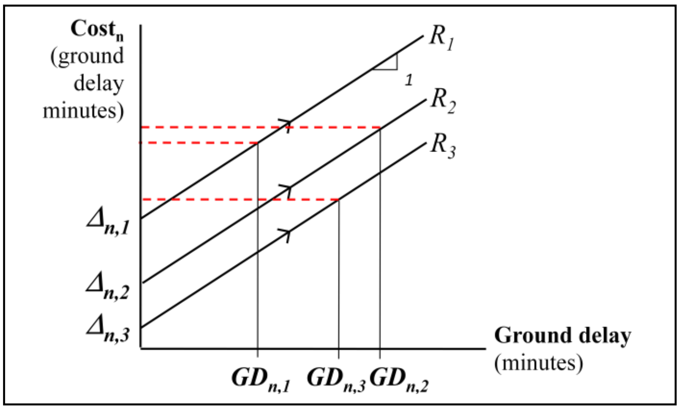


Figure 4. Delay Thresholds

We consider two stated route preference model scenarios. In the first, an AFP has been announced, and FAA traffic managers request flight operators to submit their delay threshold inputs by a deadline. Resources are allocated only after this deadline, when traffic managers have presumably received most or all flights' information. To represent this procedure we employ a model where the entire set of inputs is considered simultaneously in making allocations. We then consider a second system where flight operators are allocated their preferred resources on a first-submitted, first-assigned (FSFA) basis. It is envisioned that operators would be incentivized to submit their inputs as soon as they are able.

In the stated route preference model we assume that each airline would calculate the additional cost of a reassignment option using (2). However, airlines do not know what slots the FAA has available for their flight(s) on each route, and therefore have no information about the amount of ground delay that will be assigned to their flights. As a result airlines submit delay thresholds ($\Delta_{n,r}$) that are calculated as follows. Traffic managers use these to compare the cost of route options combined with different ground delay slots.

$$\Delta_{n,r} = \alpha_n \rho_r + \varepsilon_{n,r}, \qquad \varepsilon_{n,r} \sim P \tag{7}$$

Our specification assumes that a delay threshold is the airline's "true" and complete generalized cost for a flight n to fly route r before ground delay is known. Traffic managers will allot resources to each flight through a particular allocation scheme using these delay thresholds. The delay thresholds ensure that under any combination of ground delay slots that could be assigned to their flight, the airlines have informed the FAA about which resources are of maximum utility to them.

2) Batch Model

In the batch model it is assumed that traffic managers receive delay thresholds from all airlines with AFP-impacted flights, before allocating resources, such that there still is no reward for submitting delay thresholds earlier than others. Airlines do not optimize or choose any resource options by offering delay thresholds; they simply offer the requested information about each of their choices to the FAA for use in the optimization. As a result the model remains a route assignment problem with a system optimal solution. The batch model is formulated identically to the parametric models except that the error term is included in the objective function, to represent the fact that airlines submit complete information about their preferences through their delay thresholds.

Again assume we have the situation of Figure 2 where N identical flights are to be reassigned to one of R routes with departure slots $d_{n,r}$. We want to know how many flights should be assigned to each route to minimize total user cost.

Decision variables: $X_r,\ \forall r$

Objective function:

$$\min_{X_1,\dots,X_R} C = \sum_{r=1}^{R} \sum_{i=1}^{X_r} (\Delta_{i,r} + g_r i), \quad \varepsilon_{i,r} \sim P \tag{8}$$

where $\Delta_{i,r} = [\alpha_{min} + \theta(\sum_{j=2}^{r} X_{j-1} + i)] \cdot \rho_r + \varepsilon_{i,r}$, and $\theta = \frac{\alpha_{max} - \alpha_{min}}{N}$.

Constraints: $\sum_{r=1}^{R} X_r = N;\ X_r \geq 0, \forall r$

Condition: Order routes such that $\rho_1 > \rho_2 > \cdots > \rho_R$; order flights by increasing α.

Because this model contains random variables unique to each flight and route (i.e. the error term is contained in the objective function), Equation (8) cannot be solved analytically. However, we can treat each flight as an individual entity, where the decision variables are binary indicators of the route that each flight chooses. The model is formulated as binary integer quadratic program (BIQP) where the CPLEX solver is used to obtain a solution using the branch and bound algorithm. The results of this model tell us what route each individual flight is assigned to. Let's say that

$$x_{n,r} = \begin{cases} 1 & if\ route\ r\ is\ chosen\ for\ flight\ n \\ 0 & otherwise \end{cases}$$

Decision variables: $x_{n,r}\quad \forall n, r$

Objective function:

$$\min_{x_{n,r}, \forall n,r} C = \sum_{n=1}^{N} \sum_{r=1}^{R} \left[\Delta_{n,r} + g_r \cdot \sum_{k=1}^{n} x_{k,r} \right] \cdot x_{n,r} \tag{9}$$

where $\Delta_{n,r}$ and θ are as defined previously.

Constraints: $x_{n,r}\ in\ \{0,1\}\ \forall n, r,\ \sum_{r=1}^{R} x_{n,r} = 1\ \forall n$

Constraint 1 restricts $x_{n,r}$ to be binary; constraint 2 ensures that each flight has been assigned to one route. The matrix for $\varepsilon_{n,r}$ was built from NxR random draws of the Gumbel distribution.

3) First-submitted, First-assigned (FSFA) Model

In the first-submitted, first assigned (FSFA) model, FAA traffic managers receive delay thresholds from operators in a sequence unknown beforehand. Each time an operator submits their delay thresholds for a flight, they are allocated the best possible resources available at the time, without considering future requests. This is identical to each flight choosing the minimum cost route and slot combination available. As a result, the FSFA process can be represented using the log-sums concept of the logit discrete choice model [13]. When the unknown portions of the utilities are assumed to be iid Gumbel with location parameter a and scale parameter b, the expected minimum cost and choice probabilities associated with a set of alternatives can be found. According to [14] and [15], the probability of agent n choosing an alternative r is:

$$P(V_{n,r}) = \frac{\exp\left(\frac{1}{b} \cdot V_{n,r}\right)}{\sum_{j=1}^{R} \exp\left(\frac{1}{b} \cdot V_{n,j}\right)} \tag{10}$$

where $V_{n,r}$ is the deterministic utility of r to agent n. In choice modeling we are typically concerned with the cost difference between two alternatives. If $E[W_n]$ is the expected cost of an alternative to n and $E[c_n^0]$ is that of another, then the difference between the two is:

$$E[c_n] = E[W_n] - E[c_n^0]$$
$$= \frac{1}{\gamma_n}\left(b \cdot ln\left[\sum_{r=1}^{R} exp\left(\frac{V_{n,r}}{b} + a\right)\right] - b \cdot ln\left[exp\left(\frac{V_n^0}{b} + a\right)\right]\right) \quad (11)$$

where W_n is the cost to operator n, γ_n is the (constant) marginal utility of income, and a and b are the Gumbel distributional parameters.

In the context of the AFP assignment, $E[c_n]$ represents the additional expected cost for flight n due to the AFP. We represent the deterministic utility using the cost function for a flight in the AFP such that

$$V_{n,r} = -(\alpha_n \cdot \rho_r + d_{n,r}), \qquad V_n^0 = 0, \forall n, r \quad (12)$$

Recall that $d_{n,r}$ is the departure time (and ground delay, since scheduled departure times are $t \approx 0$ for the formulations introduced in this paper) for flight n assigned to r. Since the utility function $V_{n,r}$ is represented directly by the cost equation, $\gamma_n = 1$. We rewrite Equation (11):

$$E[c_n] = b \cdot ln\left[\sum_{r=1}^{R} exp\left(-\frac{\alpha_n \cdot \rho_r + d_{n,r}}{b}\right)\right] \quad (13)$$

The location parameter a cancels out of the equation due to its inclusion in the AFP cost and in the original cost. We now describe the recursive procedure by which the expected minimum cost is calculated for each flight.

1) Assign α_n value to each flight n. Randomly order the flights to simulate their unknown submission order.
2) For flight $n = 1$, we calculate $V_{1,r}$, $P_1(r)$, and $E[c_1]$ using (12), (10), and (13) respectively, for all r.
3) For $n = 2, 3, \dots, N$:
   a. Determine the expected ground delay $E[d_{n,r}]$ on each route r for flight n. $E[d_{n,r}]$ is calculated based on the conditional probability that the previous flight $(n-1)$ took r. Event "$(n-1)$ took route r" is represented by B; event "$(n-1)$ did not take route r" is represented by $(1-B)$. $E[d_{n,r}]$ then becomes:

$$E[d_{n,r}]$$
$$= E[d_{n,r}|B] \cdot P(B) + E[d_{n,r}|(1-B)] \cdot (1 - P(B)) \quad (14)$$
$$= (E[d_{n-1,r}] + g_r) \cdot P(B) + E[d_{n-1,r}] \cdot (1 - P(B))$$

   $P(B)$ is the probability of agent $n-1$ taking route r, and was calculated in step 2 using (10).

   b. Find the expected utility of each alternative route for n, expressed as $E[V_{n,r}] = \alpha_n \rho_r + E[d_{n,r}]$.
   c. Calculate the expected cost $E[c_n]$ using (13), using $E[V_{n,r}]$ calculated in (b).
   d. Find the route choice probabilities $P(V_{n,r})$ as in (10), using $E[V_{n,r}]$.
   e. Repeat (a) through (d) until $n = N$.
4) Find $\sum_{n=1}^{N} E[c_n]$.

We can perform the above calculations for different values of the Gumbel scale parameter b, where increasing b increases the variance of the Gumbel-distributed error term $\varepsilon_{n,r}$.

V. Numerical Example

When the FAA has perfect information ($\varepsilon_{n,r} = 0 \ \forall n, r$), the parametric (P1) and batch stated route preference (SP1) models are identical and hence yield identical resource allocations and total costs. As the traffic managers' uncertainty about the airlines increases, the P1 cost result should remain the same, as resource allocations do not take the (changing value of) error into account. The SP1 model uses complete information to do a system-optimal resource allocation; as such, it will always yield the minimum total cost solution under any error variance. For this reason the SP1 solution is the baseline result. Under a zero error assumption, the FSFA stated preference model (SP2) solution will be equal or inferior to the other models because it does not offer a system-optimal solution. With greater uncertainty we might expect the total cost of the SP2 solutions to decrease like that of SP1.

To obtain insight into the performance of the three models under increasing uncertainty, which we model using increasing error variance, we present a numerical example. Suppose $N = 200$ flights must be reassigned routes and departure times as part of the AFP. The nominal route remains open but with reduced capacity. There are a total of 5 routes to which flights can be reassigned; the details are contained in Table 1. We consider the scenario where air cost ratios α are evenly distributed between $(1,25]$ across the N flights.

As the interest is in relative rather than absolute performance, Figure 5 shows the cost differences of P1 and SP2 against the cost of SP1. SP1 requires simulation of the error term, and the results shown below are for 10 iterations.

There are three important conclusions to make from Figure 5. Firstly, as the FAA knows less and less about the airlines, the parametric (P1) model solutions degrade in comparison to those of SP1 and SP2. Secondly, the cost difference between the SP1 and SP2 results is consistent over increasing error variance. This is due to the fact that the error is known in both the SP1 and SP2 decision making processes. Finally, one can observe that the P1 solution is superior to the SP2 solution when the traffic managers know more about the operators (i.e. at small variance levels). However, after a certain error level (a standard deviation of about 10% of the zero error cost solution) the SP2 solution is more cost efficient. This result is intuitive; when traffic managers have plentiful and accurate information

TABLE I. Scenario for Numerical Example

Route	Capacity (aircraft per hour)	Departure Headway, g_r (min)*	En Route Time, h_r (min)	ρ_r (min)
1	30	2	125	25
2	12	5	120	20
3	7.5	8	110	10
4	6	10	107	7
5 (nominal)	4	15**	100	0

* This is the arrival (and departure) headway at Fix A.

** Headways after capacity is reduced due to AFP.

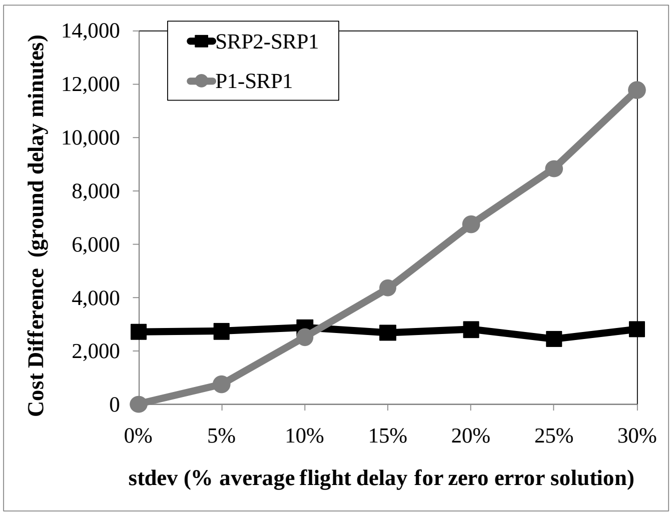


Figure 5. Total Cost, Numerical Example

about the airlines, the system optimal resource allocation will be superior to the FSFA allocation using complete information. However, when traffic managers have less information about the airlines, it becomes better to do a FSFA allocation with complete information rather than a system optimal allocation with incomplete information. Identifying these types of trade-off points is the core of this research.

Numerical checks demonstrated that the parameter values $(\alpha, \alpha_n, g_n, \rho_r)$ have little effect on the solutions' relative positions to one another. Formal sensitivity tests will be performed as part of future work.

VI. DISCUSSION & FUTURE WORK

In this paper we propose a modeling framework through which we can investigate the many issues involved with incorporating user inputs in allocating constrained airspace capacity. We develop, evaluate and compare three user input and resource allocation schemes, under differing assumptions about how much traffic managers know about airline flight costs. The numerical example demonstrated the situations under which better information quality could be more desirable than timeliness, and vice versa. Building a model framework through which we can identify these types of tradeoff points is a key contribution of this research effort.

There are several important questions that are, and will continue to be, addressed. How much are flight operators willing to sacrifice input quality in order to submit their inputs faster? How does the timing of traffic managers' decisions affect the quality of their decisions to the operators? Also, airlines update their information constantly in the GDP and AFP databases. Given that their objectives and goals change so continually and rapidly, how will this affect decision-making when the goal is to maximize their utility? Addressing these questions is central to this research effort. As a result, it is important to continue discussions with practitioners, in order to better understand and represent airline behavior within the modeling framework of this research.

As part of on-going work, we are developing an additional stated route preference model, consisting of a hybrid between the system-optimal and FSFA resource allocation schemes. The advantage of this model is that it preserves the FSFA reward structure but potentially offers greater cost efficiency. We would like to develop a performance assessment procedure that combines user cost metrics with traditional operational performance metrics and emissions metrics. We would also like to improve the user cost specification by including missed connections, to account for downstream effects of flight delay.

This research investigates the interaction and information exchange between flight operators and the FAA. The ultimate goal is to provide insight into the potential mechanisms of collaborative resource allocation within the context of the AFP, in order to guide future AFP policy decisions.

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REFERENCES

[1] Taber, N. J. (2005). *Operational Concept for Integrated Collaborative Rerouting (ICR).* McLean, VA: The MITRE Corporation, MTR05W000053.

[2] Bertsimas, D., & Stock Patterson, S. (1998). The Air Traffic Flow Management Problem with Enroute Capacities. *Operations Research , 46* (3), 406-422.

[3] Goodhart, J. (2000). *Increasing Airline Operational Control in a Constrained Air Traffic System.* University of California, Berkeley: PhD Dissertation.

[4] Andreatta, G., & Romanin-Jacur, G. (1987). Aircraft Flow Management Under Congestion. *21*.

[5] Richetta, O., & Odoni, A. R. (1993). Solving Optimally the Static Ground-Holding Policy Problem in Air Traffic Control. *Transportation Science* , 228-238.

[6] Ball, M. O., Hoffman, R., Odoni, A., & Rifkin, R. (2003). A Stochastic Integer Program with Dual Network Structure and its Application to the Ground-Holding Problem. *51* (1).

[7] Jakobovits, R., Krozel, J., & Penny, S. (2005). Ground Delay Programs to Address Weather within En Route Flow Constrained Areas. San Francisco, CA: AIAA Guidance, Navigation and Control Conference and Exhibit.

[8] Vossen, T., Ball, M., Hoffman, R., & Wambsganss, M. (2003). A General Approach to Equity in Traffic Flow Management and its Application to Mitigating Exemption Bias in Ground Delay Programs.

[9] Hoffman, R., Burke, J., Lewis, T., Futer, A., & Ball, M. (2005). Resource Allocation Principles for Airspace Flow Control. San Francisco: AIAA Guidance, Navigation, and Control Conference and Exhibit.

[10] Mayet, J., & Hansen, M. (2000). Congestion Pricing with Continuously Distributed Values of Time. *Journal of Transport Economics and Policy, 34, Part 3,* 359-370.

[11] Ball, M., Futer, A., Hoffman, R., & Sherry, J. (2002). Rationing Schemes for En-route Air Traffic Management (CDM Memorandum).

[12] Hoffman, R., Lewis, T., & Jakobovits, R. (2004). Flow Constrained Area Rerouting Decision Support Tool, Phase I SBIR: Final Report.

[13] Train, K. (2003). *Discrete Choice Methods with Simulation.* Cambridge University Press.

[14] Domencich, T. A., & McFadden, D. (1975). *Urban Travel Demand: A Behavioral Analysis.* Amsterdam: North-Holland.

[15] Ben-Akiva, M., & Lerman, S. R. (1994). Discrete Choice Analysis: Theory and Application to Travel Demand. MIT Press.