En route charges for ANSP revenue maximization

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***Abstract*—In Europe, all Air Navigation Service Providers (ANSPs) finance their activities by charging airlines using their airspace. These 'en route charges' usually account for a significant part of the cost of a flight, and they can therefore influence the route choice: airlines may decide to fly longer routes to avoid countries with higher charges. If ANSPs want to maximize their revenues, they must choose the optimal charge to impose on their airspace. We show that this optimal charge can be identified through a Network Pricing Problem (NPP) formulation in the form of Bilevel Programming where the leader (i.e. the ANSP) owns a set of arcs (the airways in its national airspace) and charges the commodities (i.e. the flights) passing through them. As the en route charges are proportional to a Unit Rate value fixed by the ANSP, we are able to apply a similar methodology as in the case of a single toll arc for the NPP. By exploiting the structure of the problem, we propose an exact algorithm to compute the optimal Unit Rate and apply it to a case study relying on real air traffic data and realistic flight cost figures.**

I. Introduction

In accordance with European Commission Regulation 1794/2006 laying down a common charging scheme for air navigation services, every European Air Navigation Service Providers (ANSP) finances its activities by charging airlines to use its airspace through the mechanism of 'air navigation service charges', which are charges levied on all flights passing through the ANSP's airspace. These charges are composed of en route and terminal charges which are levied to finance costs for providing en route and terminal services, respectively. As en route charges linearly depend on a national Unit Rate which is fixed annually by the ANSP [1] and usually account for around 10-20% of the cost of a flight, the route choice can be influenced by them: airlines may decide to fly longer routes to avoid countries with high Unit Rates [2]. Currently, in most European states (except for the United Kingdom), the Unit Rate is set to allow the ANSP to completely recover all the costs it incurs to provide air navigation services, without making a profit. Over the next few years however many ANSPs, which nowadays are mostly not-for-profit corporations, are likely to move to more commercial approaches to the supply of air navigation services [3]. In this scenario, ANSPs would instead aim to fix their Unit Rates so as to maximize their revenues.

In this paper we propose a Network Pricing Problem (NPP) formulation to identify this optimal Unit Rate value, in the form of Bilevel Programming (see [4]) where the leader (i.e. the ANSP) owns a set of arcs (the airways in its national airspace) and charges the commodities (i.e. flights) passing through them. Flights are assumed to have a rational behavior and look for the minimum cost path through the network. We prove that the NPP approach to fix the charge on a single toll arc (e.g. see [4]) can be extended to our case where the charge on each arc is proportional to a constant. In fact, as the Unit Rate is unique for each country and the charge to be paid on an arc linearly depends on it, the leader has to decide on this single value only.

Our findings show that flight travel choices do depend on the Unit Rate value set by the ANSP, and we also identify the revenue-maximizing Unit Rate value.

The paper is organized as follows: the following section will briefly introduce the Network Pricing Problem and give some references to studies on this topic, and then third section will describe the structure of en route charges in Europe. Section four will present our model along with the computational procedure proposed to solve it, and finally in section five we present some results from a preliminary case study. The last section will summarize our findings and present some discussions of them.

II. The Network Pricing Problem

Consider a sequential game with two players, a leader L and a follower F. L plays first and decides his best strategy, taking into account the optimal strategy of F in reaction to his choice. F plays second, and so already knows L's choice of strategy when choosing his own. This is commonly known as a Stackelberg game [5] and has been widely studied in literature.

Bilevel programming (BP) provides an appropriate framework for modeling sequential games of this kind. A BP problem is a hierarchical optimization problem in which the constraints are defined by a second optimization problem. This formulation was introduced by [6], and several studies have followed. By setting x as the decision vector of the leader and y as the decision vector of the follower, the general formulation is:

$$\begin{aligned} \max_{x,y} \quad & F(x,y) \\ & (x,y) \leq 0 \\ y \in \arg\min_{y} \quad & f(x,y) \\ & g(x,y) \leq 0 \end{aligned}$$

This type of problem has been shown to be *NP*-hard even for linear objective functions or local optimality [7]–[9]. Several types of algorithms have been implemented in literature, and a literature review can be found in [9]–[11].

The Network Pricing Problem (NPP) is a type of Stackelberg game, which is based on a network, with an authority which owns a subset of arcs and imposes tolls on them, and users who travel on the network. The authority is the leader who wants to maximize his revenue, and network users are the followers who want to minimize their costs, and so will always travel on the minimum cost path.

The transportation network is defined as a set of nodes linked by a set of arcs. A commodity is a network user who travels from an origin to a destination and has some fixed cost parameters. To avoid a trivial solution, an assumption is made that for each commodity there exists a toll free path, which does not pass through any of the arcs owned by the authority. This condition avoids the possibility of the authority imposing an infinite toll on its arcs, which would lead to infinite revenues.

The NPP can therefore be modeled using bilevel programming. The bilinear/bilinear [1] bilevel Network Pricing Problem was first introduced by [4] for a multicommodity network. We adopt the notation used by [12]:

- $i \in \mathcal{N}$ nodes
- $a \in \mathcal{A} \cup \mathcal{B}$ arcs ($\mathcal{A}$ is the set of toll arcs)
- $k \in \mathcal{K}$ commodities with demand η^k
- c_a travel cost of arc a, exclusive of toll
- $\forall k \in K$: (o^k, d^k) origin/destination of commodity k
- t_a toll on arc $a \in \mathcal{A}$ (imposed by the authority)
- x_a^k flow of commodity k on arc a ($x_a^k = 1$ if commodity k travels on arc a, 0 otherwise)

The multicommodity NPP has been formulated as follows:

[1]This means that the objective functions of both leader and follower are bilinear.

$$\max_{t,x} \quad \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}} \eta^k t_a x_a^k \tag{1}$$

$$t_a \geq 0 \qquad \forall a \in \mathcal{A}$$

$$x \in \arg\min_{x} \quad \sum_{k\in\mathcal{K}} \left(\sum_{a\in\mathcal{A}} (c_a + t_a) x_a^k + \sum_{a\in\mathcal{B}} c_a x_a^k \right) \tag{2}$$

$$\sum_{a\in i^-\cap\mathcal{A}} x_a^k + \sum_{a\in i^-\cap\mathcal{B}} x_a^k - \sum_{a\in i^+\cap\mathcal{A}} x_a^k - \sum_{a\in i^+\cap\mathcal{B}} x_a^k = \begin{cases} -1 & \text{if } i = o^k \\ 1 & \text{if } i = d^k \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \tag{3}$$

$$x_a^k \in \{0,1\} \qquad \forall k \in \mathcal{K}$$

where i^- and i^+ respectively denote the sets of arcs with i as tail or head.

In [4] the authors show that the lower level problem can be replaced by its primal dual constraints and primal dual optimality conditions, yielding a single-level problem. Many techniques have been applied to the NPP to obtain efficient algorithms and improved numerical results. For a deeper mathematical investigation and for a literature review of this problem, see [12], whose work concerns the particular case in which all toll arcs are connected and constitute a path, as occurs on motorways. Another interesting piece of work on this subject can be found in [13], which includes a large review of pricing in networks.

A. The case of a Single Toll Arc

As it will be useful later on in our description, we will now discuss the case of a Network Pricing Problem where the authority owns only one arc a. This is a relatively straightforward case, which can be solved using the parametric linear programming technique [4]. We define T as the tax value the leader can impose on arc a, and $\gamma_k(T)$ as the cost of the shortest path for the commodity k for a given value of T. We set the upper bound to the toll that can be imposed from the leader for commodity k as $\pi_k = \gamma_k(\infty) - \gamma_k(0)$. Then we sort all π_k quantities for all commodities in decreasing order. We assume that the order is $\pi_{k_1} \geq \pi_{k_2} \geq ... \geq \pi_{k_{|K|}}$. For any toll value T which is not equal to one of the values in this π_k sequence, we can increase the toll with $\epsilon > 0$ and achieve a higher revenue. Thus, every optimal value of T is equal to one of the π_k values. Moreover, for a toll value π_i ($i \in \{1...|K|\}$) only commodities $k \leq i$ (for which $\pi_k \geq \pi_i$) will choose the toll arc. The leader revenue function is:

$$\Pi(\pi_i) = \sum_{k\leq i} \pi_i \eta^k \tag{4}$$

where η^k is the demand for commodity k. The leader will choose the toll value that maximizes his revenue, so the

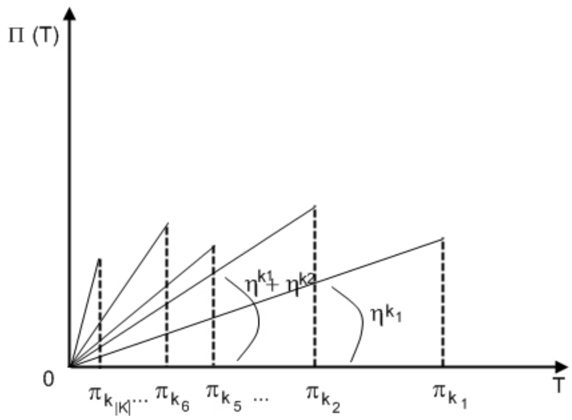


Figure 1. Leader revenue in the case of a Single Toll Arc

optimal solution will be:

$$T^{\star} = \pi_{i^{\star}} \text{ , such as } i^{\star} = arg \max_{i \in \{1 \dots |K|\}} \Pi(\pi_i) \qquad (5)$$

The leader revenue function is shown in the graph in Figure 1. It is a piecewise linear function, with discontinuities at π_i values. In each interval the function is described by a straight line which is linearly dependent on the cumulative demand of commodities which will choose the toll arc for that π_i value.

III. En route charges in Europe

Although European Air Traffic Control is centrally coordinated, every country in Europe has an ANSP which manages flights within its national airspace. The air navigation service charges imposed by ANSPs to finance their activities are both a source of revenues for the ANSPs and costs for airspace users such as airlines.

For each flight, the en route charge is calculated using three basic elements [1]:

- Aircraft Weight Factor
- Distance Factor
- Unit Rate of en route charges (for each Charging Zone, i.e. each country)

The Weight Factor (expressed to two decimal places) is determined by dividing the maximum take-off weight (MTOW) of the aircraft (in metric tonnes, to one decimal place) by 50, and subsequently taking the square root of the result rounded to the second decimal, i.e. $w = \sqrt{MTOW/50}$.

The Distance Factor for each Charging Zone is obtained by taking the number of kilometers in the so-called 'Great Circle Distance[2]' between either the aerodrome of departure or the entry point of the zone and either the aerodrome of arrival or the exit point of the zone, and dividing it by 100. This operation is repeated for each Charging Zone which the flight passes through. The entry and exit points are the points at which the lateral limits of the Charging Zone are crossed by the route described in the last plan filed.

The Unit Rate of en route charges is fixed by each ANSP and is the charge imposed on a flight per 100km flown within a given charging zone, and per 50 metric tonnes of aircraft weight. The Unit rates are applicable from 1^{st} January of each year.

In literature, to the best of our knowledge, there are very few studies on air navigation charges in Europe. In [14] the congestion problem in European airspace is approached with the aim of applying a pricing solution. First an analysis of the formula used to calculate en route charges is performed and an explanation on why it is inefficient in preventing congestion is provided. Then a new formula is provided, with some congestion costs. Whilst the work is very interesting, it is not about the same type of analysis which we would like to develop on en route charges in Europe (how ANSPs should fix their Unit Rate value).

Another work on en route charges can be found in [15], where the authors provide a study on pricing schemes in the case of a unified upper airspace between certain countries. They propose some different scenarios of en route charges and analyze their impact on the actors involved (ANSPs and aircraft operators), but they do not propose a mathematical model to calculate an optimal charge.

In [16] the airport pricing models are analyzed and transposed for air carriers, whether they have market power or not. A pricing model for security charges on air travel is provided in [17]. Both of these works are interesting but they do not specifically deal with en route charges or with ANSPs' behavior when fixing their Unit Rate values, which is the central topic of our study.

In this paper we would like to analyze the choice of the Unit Rate to fix every year as a pricing problem for European ANSPs, with a mathematical model able to determine the optimal value. To assess the validity of this approach, we will now look at how much influence en route charges have in affecting the cost of a given flight.

In [2] the authors provide an interesting analysis of the degree to which en route charges condition airlines' choices of flight routes, compared with the influence of all other direct costs (such as fuel, crew and maintenance costs). The study provides both an experimental approach and a theoretical approach, and shows that there can sometimes be convenience in avoiding certain 'expensive' countries, with an analysis conducted on a sample of real data from 30 ECAC members from August 2002 (flights between almost 5000 origin/destination pairs during 5 days). In the study it is pointed out that, whilst en route charges are similar in magnitude to fuel costs, they are only around half the size of maintenance, crew, and fuel costs combined. Furthermore, when delays or en-route congestion occur, the impact of route charges becomes even weaker. Another aspect they reveal is the habit of airlines to always choose the same route between a given origin/destination pair, often only because they have always acted like this. Non-rational behavior such as this is difficult to take into account

[2]The Great Circle Distance is the shortest distance between any two points on the surface of a sphere measured along a path on this surface (as opposed to going through the sphere's interior).

Table I
BREAKDOWN OF THE TYPICAL ROUTE-DEPENDENT COSTS OF A FLIGHT

	AEA 2003	**AEA 2007**	**Ryanair 2008**	**Ryanair 2009**
Fuel and Oil	35 %	50 %	57 %	65 %
Maintenance	28 %	23 %	4 %	3 %
Staff	22 %	17 %	20 %	17 %
En route charges	15 %	10 %	19 %	15 %
Total	100 %	100 %	100 %	100 %

in a mathematical model. Finally in these kinds of studies the data availability is often a problem, as airlines are often unwilling to make their cost values available (for instance airline companies generally have specific contracts with fuel suppliers, and as these can differ greatly between airlines, it is difficult to consider a significant average value). However, with some analysis and calculations, the study reveals that en route charges may play a significant role in defining the routes flown by an aircraft when a given origin and destination must be connected, and the way of charging flights for Air Navigation Services could manage the demand in the European airspace. The complexity in estimating the exact impact that en route charges have on overall flight costs, and then on route choice, should not be neglected, even though the way a flight is charged is relatively easy to compute.

To have a more precise idea of the impact of en route charges on the cost of a flight with some numerical values, we can consider the typical breakdown of flight costs. We only consider costs that change with route choice, so these are en route charge costs, fuel costs, staff costs and maintenance of the aircraft. Airport charges, depreciation, marketing and other costs are not useful for our analysis as they are independent of route choice. In Table I we report some data taken from the Annual Report of Ryanair (one of the so-called ‘low cost’ airlines, which in total have a share of around 25% of the European market [18]) and other data from the Summary Report of the Association of European Airlines (AEA) which counts national airlines and others, but excludes low cost airlines (and represents around the 50% of the European market [18])[3]. By setting the sum of en route charge costs, fuel costs, staff costs and maintenance costs equal to 100, we can calculate the percentage contributed by each factor. One can clearly see that en route charge costs have a significant impact. They range from 10% to 19% of the route-dependent costs of a flight, with their impact being lessened when the cost of oil is high (as was the case in 2007 for instance).

IV. BEST REACTION OF AN ANSP

We apply the Network Pricing Problem (NPP) to the case of a single ANSP, which wants to determine the charges to impose on its arcs for the next year, and which knows other ANSPs’ charges. This means finding the best reaction of the ANSP to the behavior of the system (ie. the actions of other countries’ ANSPs and network users), in order to maximize its

[3]These data can be found on their web sites, in [19], [20] and [18]

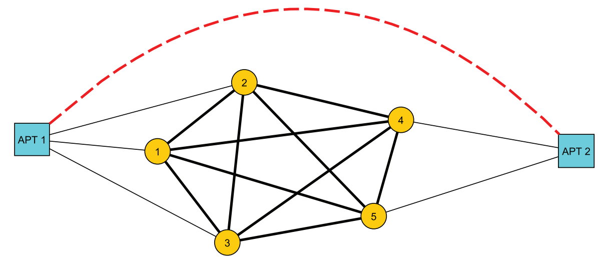


Figure 2. Example of the network

revenue. We therefore consider just one leader who wants to determine the best charge to impose on his toll arcs. All other arc costs are known. The followers are the flights which move on the network by choosing the minimum cost path. Every flight is a different commodity, and as we saw previously the relevant characteristics are not only the origin and destination, but also the operational costs, which are different for each type of aircraft and airline.

According to [1], we describe the air network for en route charges with nodes at airports and at crossing points between countries. Some considerations about the structure of this network have to be made. First of all, a country is not, in general, a convex set, as national borders tend to be highly irregular, so it may occur that a flight enters a country, exits and then re-enters it. This non-convexity property means that it may not be easy to define *a priori* an upper bound on the number of toll arcs to be used by a given flight. However it is always true that a flight does not pass through consecutive toll arcs (arcs own by the same country). Another property of the graph which is not always valid is the completeness. The air space is divided into ‘airways’ and there may not exist an airway between every pair of nodes, both for nodes at national boundaries and for airports. During this first step, we relax these properties and make the assumptions that countries are convex and that the graph is complete. The convexity assumption allows us to say that for each flight only one toll arc can be chosen (this remains true if we consider internal airports and internal flights). The completeness assumption allows us to describe possible paths from an origin to a destination by considering all pairs between entry/exit points of a given country. Figure 2 reports an example of a network: all paths from APT1 to APT2 can be identified with all the pairs of nodes which delimit a country (e.g. one path is the pair $(1, 4)$ which means the path APT1-1-4-APT2).

To maintain the existence of a toll free path for each commodity we should consider only over flights, because if we choose a commodity that lands or takes off from the country of the ANSP which we are considering, it would be obliged to pass one toll arc on entry or exit. The leader could therefore impose a very high (infinite) charge on his toll arcs and have a very high (infinite) revenue from this commodity. In Figure 2 the toll free path is represented with a red dashed line.

A. Mathematical Model

We consider the set A of toll arcs, i.e., a flight is charged by the ANSP when passing through any arc of A. Let N be the set of all endpoints of the arcs in A. We denote as $(i, j) \in A$ the generic toll arc where both i and j belong to N. If K is the set of all the commodities, the charge or toll to be paid by the generic flight $k \in K$ is equal to the product of the Unit Rate T fixed by the ANSP, the distance $l_{i,j}$ of the arc (i, j) and the factor w^k depending on the Maximum Take-Off Weight of the aircraft performing the flight. If o^k and d^k are the origin and destination points of flight $k \in K$, respectively, we denote as $d(o^k, i)$ the minimum cost path from origin o^k to node i for all $i, k \in N \times K$ and as $d(j, d^k)$ the minimum cost path from node j to destination d^k for all $j, k \in N \times K$. In this way we represent the portion of flight which is performed outside the airspace controlled by the ANSP. In addition we consider the possibility for each flight to reach its destination without crossing any arc in A. This toll free path should exist for each commodity to guarantee an upper bound of the Unit Rate that the leader can impose on its arcs. We denote as r^k the cost of the minimum cost toll free path. We finally denote as c^k the unit cost of flight k which takes into account all other flight-related costs (e.g., fuel, maintenance and crew costs) besides the en route charges.

The Route Charges Pricing Problem (RCCP) can be written as:

$$\max_{T,x} \quad T * \left[\sum_k \sum_{i,j} x^k_{i,j} \; l_{i,j} \; w^k \right] \tag{6}$$

$$T \geq 0$$

$$\arg\min_{x,y} \quad \sum_k \left\{ \sum_{i,j} \left[d(o^k, i) + l_{i,j}(c^k + T w^k) + d(j, d^k) \right] x^k_{i,j} \right\} + \sum_k r^k y^k \tag{7}$$

$$\sum_{i,j} x^k_{i,j} + y^k = 1 \qquad \forall k \in K \tag{8}$$

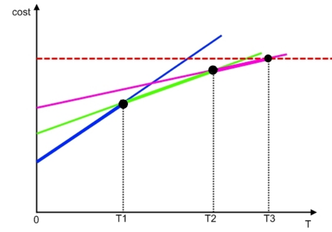
$$x^k_{i,j} \in \{0, 1\} \qquad \forall i, j, k \in N \times N \times K$$

$$y^k \in \{0, 1\} \qquad \forall k \in K$$

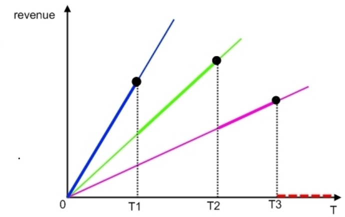
where T is the non-negative decision variable representing the Unit Rate fixed by the leader and holding on all toll arcs, $x^k_{i,j}$ is a set of binary variables equal to 1 if arc (i, j) is chosen by commodity k and 0 otherwise, and y^k is a set of binary variables equal to 1 if the toll free path with cost r^k is chosen by commodity k, 0 otherwise. The leader chooses the Unit Rate value T which maximizes its revenue (Equation 6), and knows the reaction of the followers: each commodity considers all possible paths between its origin and destination, and chooses the minimum cost path (Equations 7 and 8).

B. Computational Procedure

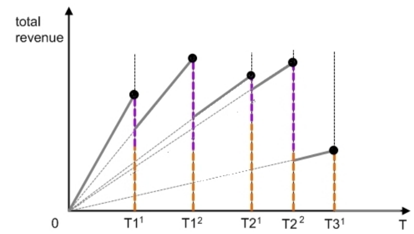
As there is just one decision variable T at the leader level, the bilevel problem can be solved through a procedure similar to the one described for the case of a single toll arc for the NPP, which is the following one:



(a) Cost for one commodity



(b) Leader's revenue for one comm



(c) Total leader's revenue

Figure 3. Computational procedure - Functions on T

1) For each commodity, we calculate the costs for all possible paths between their origin and destination (Equation 7). As the costs of the toll arcs depend on T, we identify the values of T for which the commodity has convenience in changing its path choice. We obtain a piecewise linear concave function, bounded at the upper limit by the toll free path r^k, Figure 3(a).
2) The leader's revenue for a single commodity is a non-continuous function, linear in each interval of T previously determined, Figure 3(b).
3) The above steps are repeated for each commodity to find all significant T values. Finally for each T, the leader's total revenue is determined as the sum of the revenues from each commodity, Figure 3(c). It is then straightforward to identify the Unit Rate value which maximizes the leader's revenues.

It is interesting to note that this procedure can be carried out even if the country is not convex. The important point for the computation described above is to know all possible paths for each commodity; once we know these, even if there is more than one toll arc in any one of them, we know the total distance flown over toll arcs by the commodity, and so we can proceed as described.

It may not be very easy however to find all the possible paths, as they could be great in number in a complete graph. The first stage of our algorithm requires us to enumerate all possible paths in the network for a given origin/destination pair, and in general this problem has a high degree of complexity (NP-sharp). In a real case however, due to the particular

Table II
ORIGIN/DESTINATION PAIRS OF THE EXAMPLE

Origin		Destination		Paths outside Switz.	Paths through Switz.
EDDM	Munich	LFBO	Toulouse	2	1
EGLL	London	LIPZ	Venice	1	1
EDDM	Munich	LEBL	Barcelona	2	2
EDDM	Munich	LIMC	Milan	1	3
EBBR	Brussels	LIRF	Rome	3	2
EDDF	Frankfurt	LIRF	Rome	2	2
EHAM	Amsterdam	LIRF	Rome	2	2
EIDW	Dublin	LIPZ	Venice	1	1
EIDW	Dublin	LIRF	Rome	4	2
LFPG	Paris	LOWW	Wien	3	3

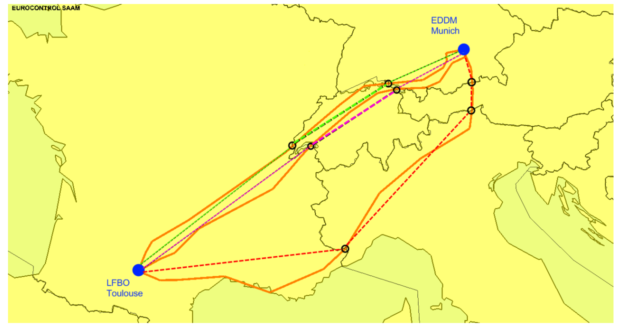


Figure 4. Routes between Munich and Toulouse

Table III
ROUTES BETWEEN MUNICH (EDDM) AND TOULOUSE (LFBO)

	DIST. FLOWN (km)	TIME FLOWN (min)	CRCO DIST. (km)
Green path	**1.002,21**	**86,00**	**963,52**
ED (Germany)	224,18	19,33	202,44
LS (Switzerland)	287,17	22,32	279,74
LF (France)	490,85	44,35	481,34
Pink path	**996,41**	**83,48**	**970,45**
ED (Germany)	218,54	24,40	204,23
LS (Switzerland)	248,19	17,33	244,41
LF (France)	529,69	41,75	521,81
Red path	**1.171,26**	**100,76**	**1.110,69**
ED (Germany)	84,23	8,15	82,75
LO (Austria)	68,99	5,15	68,87
LI (Italy)	469,30	36,80	451,77
LF (France)	548,75	50,65	507,30

Table IV
COSTS OF COMMODITIES BETWEEN MUNICH AND TOULOUSE

Commodity	Path	Total Fixed Costs (EUR)	Variable Cost (SU)
11: A319 flight	**Green**	3272,55	3,06
	Pink	3258,88	2,68
	Red	4060,67	0,00
12: B744 flight	**Green**	7011,04	6,69
	Pink	6991,02	5,84
	Red	8707,76	0,00
13: AT72 flight	**Green**	1417,10	1,72
	Pink	1413,79	1,51
	Red	1788,77	0,00

topology of the air network, the problem would not be relevant as there are only a limited number of possible airway paths that a flight can use for each origin/destination pair (and likewise for each commodity). This non-completeness property of the real network helps to reduce the number of possible paths between two origin/destination points, and to avoid a large increase in the computational time required.

V. PRELIMINARY CASE STUDY

We delineate a preliminary case study considering a sample of commodities flying over Switzerland, a central European country. In order to build the model we first need data about the network (the number and locations of entry/exit points of Swiss airspace and the length of the arcs inside and outside of Switzerland) and about the aircraft used (the aircraft weight is needed to calculate en route charges, and cost parameter values such as fuel and others are needed to calculate all route-dependent costs). We then need to know the Unit Rate values of all of the countries bordering Switzerland.

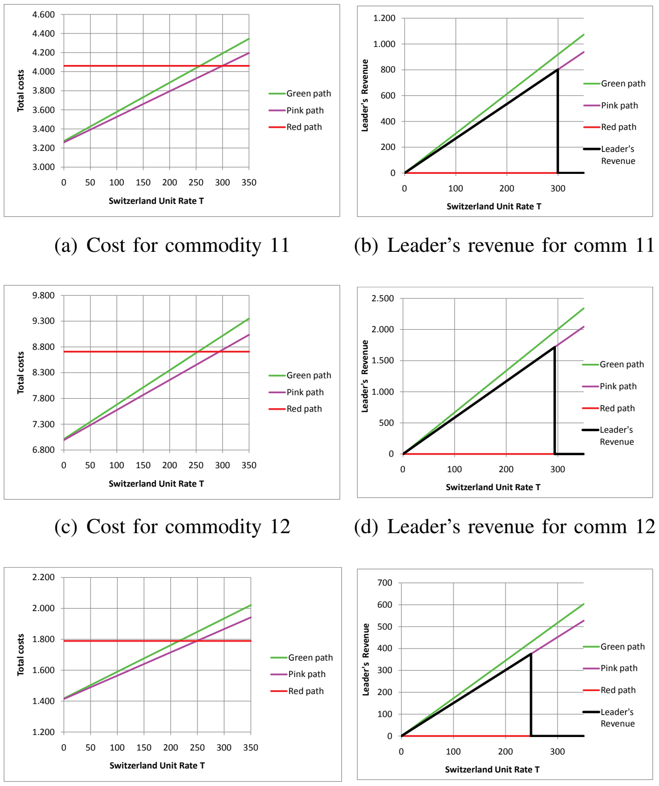
The network topology and arc distances for 10 Origin/Destination pairs have been extracted with the aid of the 'System for traffic Assignment and Analysis at a Macroscopic level' (SAAM) software relying on actual flight data from 29 June 2007. The pairs and the number of paths for each of pair are reported in Table II.

We choose seven different types of aircraft, which are commonly used for European flights, such as Airbus, Boeing and ATR, and then derive all their flight cost data from standard figures publicly available.

The official Unit Rates values valid in June 2007 can be easily found on the EUROCONTROL CRCO web site.

To determine the Unit Rate value which maximizes the revenues of the Swiss ANSP, we solve the RCCP with the procedure previously described. Due to lack of space, we now report in detail only the steps made for the first Origin/Destination pair (Munich-Toulouse). Figure 4 shows the map of all the paths for flights between Munich and Toulouse, and Table III reports their distance data.

Using these data, we are able to calculate the cost of each path as a function of the Unit Rate value T. The cost of a path is composed of a fixed part and a variable part: the fixed part is composed of fuel, maintenance and crew costs for all the distance or time flown plus the cost of en route charges for the distance flown in all countries except Switzerland; the variable part is represented by the cost of en route charges for the distance flown over Switzerland (this is the product of the Weight Factor and the Distance Factor, which are fixed for that given commodity and path, multiplied by the Unit Rate value T, which can ideally vary between zero and infinite). The paths which do not pass over Switzerland do not have a variable cost component. The product of the Weight Factor and the Distance Factor is defined as a Service Unit (SU). In Table IV we report these cost values calculated for three commodities which flew between Munich and Toulouse.



(a) Cost for commodity 11 (b) Leader's revenue for comm 11

(c) Cost for commodity 12 (d) Leader's revenue for comm 12

(e) Cost for commodity 13 (f) Leader's revenue for comm 13

Figure 5. Commodities cost and Leader revenues - Functions on T

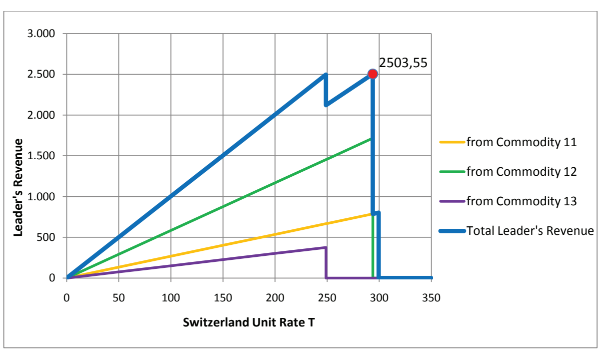


Figure 6. Leader's revenues from commodities 11-12-13

These values are reported graphically in Figures 5(a), 5(c) and 5(e) for these commodities, where it is possible to see the dependence on T. From these values it is therefore possible to derive the leader's revenues, which are reported in Figures 5(b), 5(d) and 5(f).

To calculate the leader's revenue from these three commodities, the contributions of each of them must to be summed, as reported in Figure 6: the blue function is constructed as the sum of the black functions of Figures 5(b), 5(d) and 5(f).

These calculations have been repeated for commodities flying over all the routes chosen for the example (three different types of aircraft for each pair). For certain routes it

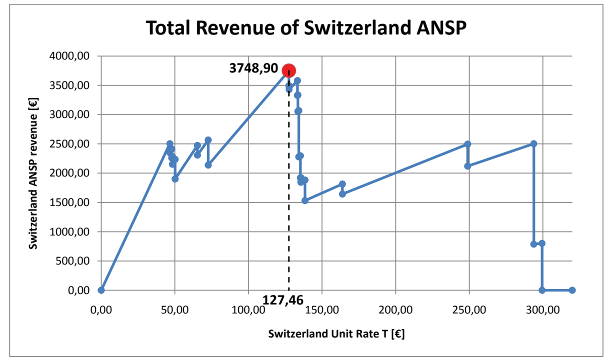


Figure 7. Preliminary example - Revenue of the ANSP

can be seen that taking a path which avoids Switzerland will always be better, for any value of T, including for $T = 0$, the minimum possible value (meaning that a path outside Switzerland is always cheaper than the fixed part of the paths inside Switzerland); in this case it clearly makes no sense for commodities to fly over Switzerland. In our formulation none of these situations provide any revenue for the Swiss ANSP, and so they are not relevant. Thus in our example we have 18 commodities which can choose to fly or not over Switzerland, in dependence on the Unit Rate value T fixed by the Swiss ANSP, and the leader's revenue has been calculated for all of them, depending on the Unit Rate value T.

Finally the leader's total revenue is the sum of the contributions for each commodity. Based on the available commodities of our preliminary example, Figure 7 shows the revenue function for the Swiss ANSP and highlights the Unit Rate value which maximizes its revenues. In this case the ANSP should fix $T = 127,46$ to achieve the maximum revenue from this sample of commodities. As we made all the calculations on a small set of commodities, by no means representative of the level of traffic over Europe, the result cannot be considered significant for its value, but it is interesting to gain a better understanding of the procedure and to reveal its potential on real data.

VI. Conclusion

This paper formulates a Network Pricing Problem addressing the case where an authority controlling a set of arcs fixes a unique value such that any commodity traversing these arcs has to pay a toll proportional to this common value. This framework depicts the way most European ANSPs are likely to behave in the near future when they determine the Unit Rate values which maximize their revenues. This is because an airline flying through an airway under the responsibility of a given ANSP has to pay it the so-called en route charges, and these charges are proportional to the Unit Rate set by the ANSP. By exploiting the structure of the problem, we propose an exact algorithm to compute the optimal Unit Rate relying on real air traffic data and realistic flight cost figures. The algorithm is polynomial except for the first step, which enumerates all possible paths in the network for a given origin/destination pair. However, the air network has a fairly simple topology, meaning there are only a few different routes possible for each flight. Our results also suggest that the Unit Rate can indeed be an instrument for an ANSP to modify the path choice of commodities. Further investigations should be carried out on a larger data set.

As an unavoidable hypothesis of the NPP is to have a toll free path for each commodity, we were obliged to consider only flights over the country of study, as any flight which takes off or lands in it will not have a toll free path. However, as these flights have a significant share of the total number of flights, they are an important source of revenue for the ANSP, and it is important to consider them. Further studies should be carried out to look for a way to include flights taking off and landing in the country considered. An idea could be to fix an upper bound to the Unit Rate value, but it would not be trivial to decide the value of this upper bound.

We considered all parameters as deterministic, meaning that their precise values are known a priori. This may not always be true however: the ANSP aims to fix the Unit Rate that will be applicable to flights for the following year. It is therefore reasonable to suppose that some parameters, such as the level of traffic or the cost values, will have a degree of uncertainty around the values of the previous year. In this case a robust optimization approach could be used.

More generally, it could be interesting to consider the whole European system and the 'competition' between more than one ANSP, as they simultaneously fix their Unit Rates. In this case we would face a bilevel problem with multiple leaders, and thus with a game theory approach it could be possible to see if there are Nash equilibria or not, and if cooperation could bring advantages.

Finally, the model we proposed for en route charges in Europe could also be generalized for other transportation problems with a similar structure, where a leader wants to fix a charge per kilometer and commodities travel on the network. We saw that this particular kind of NPP can be solved using a relatively simple procedure. It could be interesting to conduct a deeper mathematical analysis of this particular case of an NPP, to prove the computational complexity and to see if there are some useful properties. Moreover, a sensitivity analysis could be conducted against some parameters (for example, fixed arc costs), to quantitatively investigate the stability of the model. To our knowledge, no studies have yet been carried out to perform a sensitivity analysis on a NPP.

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