

# Fair Slot Allocation of Airspace Resources Based on Dual Values for Slots

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**Abstract**—Fair allocation of available resources among airlines is very challenging when there is a reduction in en-route resources. Each airline will typically place a different relative weight on delays, rerouting and cancelation. Whereas some airlines would like to preserve the on-time performance for certain flights and cancel or reroute many other flights, other airlines prefer to have less rerouting and cancelations while tolerating higher total delay. The value (or cost of delay) an airline associates with a particular flight may vary substantially from flight to flight. Airlines who wish to receive priority for certain flights usually are willing to pay more for specific time slots. To accommodate richer carrier preferences so that airlines can express the relative importance of delays, rerouting and cancelations, new concepts of slot values and dual pricing are introduced in this research. Unlike RBS, Ration by Schedule, the current algorithm in use for rationing airspace resources, that gives priority based on scheduled flight arrival times, our new allocation method provides flexibility to carriers to achieve their goals. Specifically, it also allows carriers to receive “premium” slots for an extra “charge”. In this paper, we describe a new rationing and randomized allocation method. We analyze the performance of the new method and compare it with RBS based on data derived from a real application. Our method has potential usefulness both in airspace flow program (AFP) planning and in the emerging System Enhancements for Versatile Electronic Negotiation (SEVEN).

**Keywords**—resource rationing; flow management; fairness; equitable allocation; AFP; Dual Price

## I. INTRODUCTION

When there is a capacity reduction due to the severe weather, rerouting flights is not sufficient to address extended capacity reductions in the airspace, and the need for additional tools has long been recognized. To meet that need the FAA (Federal Aviation Administration) introduced a new capability in the spring of 2006. The Airspace Flow Program (AFP) combines the power of Ground Delay Program (GDP) and Flow Constraint Area (FCA) to allow more efficient, effective, equitable, and predictable management of airborne traffic in congested airspace.

When TFM specialists at the Air Traffic Control System Command Center (ATCSCC), in consultation with FAA field managers and customer representatives, decide that the

weather conditions are appropriate they can plan and deploy an AFP. The first step is to use the Traffic Situation Display (TSD) to examine predicted weather and traffic patterns and identify the problem area by creating an FCA. An FCA is a user-defined volume of airspace along with associated flight lists and filters. FCAs are used to show areas where the traffic flow should be evaluated or where initiatives should be taken due to severe weather or volume constraints. Traffic managers or flight dispatchers define a geographic area of an FCA by drawing a polygon or a line on the display and defining the ceiling and floor of the FCA using a dialog box. FCAs are built by the ATCSCC and require a traffic management initiative (TMI).

The Enhanced Traffic Management System (ETMS) takes the FCA description and produces a list of the flights that are expected to pass through the FCA and the time they are expected to enter. This list, updated with fresh information every five minutes, is sent to the Flight Schedule Monitor (FSM), which displays the projected demand in a number of formats designed to support effective planning. FSM creates a common situational awareness among all users and service providers in the National Airspace System. All parties need to be aware of NAS constraints in order to make collaborative air traffic decisions. It is designed to effectively interact with existing FAA systems, FSM displays the Aggregate Demand List (ADL) information for both airport and airspace data elements for its users, which means everyone is looking at the same picture. The TFM specialists at the ATCSCC can enter the capacity of the FCA, expressed as the number of flights that can be managed per hour, and FSM will then assign each flight a controlled departure time so that the flow into the FCA does not exceed the declared capacity. These departure times are sent to the customers for flight planning and to the towers at the departure airports for enforcement.

The principal goal for the initial deployment of the AFP program is to better manage en route traffic during severe weather events. Compared to previous approaches, AFP’s reduce unnecessary delays while providing better control of demand, more equity, and more flexibility for customers [3].

Today, AFP’s use GDP-like tools. However, there are important differences between resource allocation for GDP’s and enroute resource allocation. First, a GDP only applies delays to a subset of flights destined for a single airport while the AFP’s apply delays to a subset of flights predicted to fly through a designated FCA (GDP tools have been modified for AFP’s in this respect). Second, in the GDP setting, demand is established based on the set of flights scheduled to arrive at the GDP airport; GDP procedures implicitly assume all flights must be assigned an arrival slot. On the other hand, in the AFP or SEVEN setting, all flights on the demand list need not be granted access to the enroute resource. The flight operator has the prerogative to cancel flights not given access *or reroute such flights around the restricted airspace*. Thus, enroute resource allocation decision models should both determine which flights gain access and assign an access time (slot) for those flights that do gain access. The last related difference is the existence of a fixed flight schedule on which to base resource allocation for GDP’s. Ration-by-*schedule* uses, in a very fundamental way, the flight schedule as the basis for resource allocation. In concept this can be done for enroute problems by simply taking the schedule associated with the list of flights whose flight plans have been filed through the impacted enroute resource. A key difference, however, is that the filing a flight plan is a short-term action and, as a result, the possibility of flight operators trying to “game the system”, e.g. by filing unnatural flight plans, is a very real possibility.

In practice, each airline will typically place a different relative weight on delays, rerouting and cancelation. Whereas some airlines would like to preserve the on-time performance for certain flights and cancel or reroute other flights, other airlines prefer to have less rerouting and cancelation while tolerating higher total delay.

Using fairness principles as a basis for allocating scarce resources provides our research with a novel focus. In fact, some proposals address the rationing of airport arrival capacity in the long run. Using methods ranging from auctions [9] and congestion pricing [7] to bargaining schemes [1]. The allocation of slots under CDM is different, in that slots must be assigned on a daily basis due to fluctuation in airport or enroute capacity. The dynamic nature of the allocation process makes it more complicated and fairness plays an important role in this environment.

The method we propose applies to a general class of airspace resource allocation problems and, in fact, we have designed it to also be applicable to the emerging System Enhancements for Versatile Electronic Negotiation (SEVEN) [2]. While SEVEN should potentially have a broad range of application contexts, the key feature that it brings to bear, which is not present in AFP’s, is the ability for flight operators to express preferences among various options for the disposition of an individual flight. The ability for flight operators to express preferences is also a key feature of our proposed resource allocation method.

In this paper, we propose a new method for assigning AFP slots to flights and flight operators, which is fundamentally

different from the method currently used for GDP’s and AFP’s, ration-by-schedule (RBS). Our work uses as a starting point research on GDP’s [14] and the investigation of RBS as a basis for fair resource allocation [13].

## II. PROBLEM DESCRIPTION AND OVERVIEW OF PROCEDURE

In our research we assume that flights pass the boundary of FCA one at a time (this is consistent with current practice). Therefore we can express the capacity as the number of available time slots. We consider those flights that are “scheduled” to arrive at the boundary of FCA. Such a flight schedule can be derived based on each flights scheduled departure time and filed flight plan. Employing such a schedule can be problematic as it is not immune to gaming or strategic behavior on the part of flight operators.

The simple FCA capacity model employed allows the FCA to be characterized by a set of entry slots. Let  $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$  be the set of available slots and  $\mathcal{F} = \{f_1, f_2, \dots, f_n\}$  be the set of flights. However, in general  $n > m$ , i.e. the number of flights is greater than the number of slots during the AFP. The capacity,  $c_j$ , of each slot  $s_j$  is considered one which means each slot can be used by a single flight. Suppose there are  $K$  carriers  $\mathcal{A} = \{A_1, A_2, \dots, A_K\}$ , and  $\mathcal{F}_i$  is the set of flights of carrier  $A_i$ .  $a_f$  is the time flight  $f$  is scheduled to arrive at the boundary of FCA and  $t_j$  is the time of slot  $s_j$ . Flight  $f$  can be assigned to any slots  $s_j$  with  $t_j \geq a_f$ . As with GDP planning, although flights are assigned to slots, we view the flight-to-slot assignment as a slot-to-flight operator assignment.

We break the process down into two steps:

- Step 1a: Determine a fair share,  $FS_i$  for each flight operator,  $A_i$ .
- Step 1b: Obtain flight operator flight-slot priority lists.
- Step 2: Allocate flights to slots in a manner consistent with the fair share determined in Step 1a and their flight priorities obtained in step 1b.

The fair share for each carrier can be found in many different ways. A principal goal we seek is to provide equity among carriers. The allocation of homogeneous demands, when the total demand exceeds total available resources is addressed in [8], [15], [16] and, in the case of scheduling problems, is treated in [6], [4], [5], [12] (these models correspond to the situation in which all flights arrive at the beginning of the AFP). Vossen [12] uses a heterogeneous demand model to treat the different arrival times of flights. To allocate slots to flights, he uses “proportional random assignment” which randomly assigns slots to the carriers in proportion to the number of a carrier’s flights that can use a slot. In his method, slots sequentially are assigned to the carriers. The proportional random assignment method is a randomized allocation method. It is also time dependent. In the “proportional random allocation” method proposed by Moulin [4] there is no time dependency, which means that all agents can participate in the lottery at each time until their demand is met. In proportional random assignment, agents participate in the lottery as long as they

can use the slot under consideration. We will use this method as a way to determine a fair share to each flight [11] (and consequently each flight operator). However, we will *not* use it to actually allocate slots to flights.

The flight-to-slot assignment carried out in Step 2 is a type of randomized round-robin that employs flight-operator preferences. In step 1b, each flight operator specifies an ordered list of flight-to-slot assignments. The allocation procedure gives priority to the carriers who wish to maintain their on-time performance for certain key flights and in return receive fewer slots. At each iteration, when a flight operator has its “turn”, the the highest available assignment on that flight operator’s preference list is chosen. Here, by available, we mean the the associated flight has not yet been assigned a slot and the associated slot has not been assigned to a flight.

In Section III, we describe the procedure for determining  $FS_i$ , i.e. Step 1a and also explain the submission of flight priority list by each flight operators, Step 1b. We note that these procedures were previously described in [11] so this section is largely a review. Section IV covers Step 2, which is a new contribution. Section V provides our experimental results.

### III. FAIR SHARE AND FLIGHT PRIORITY LIST

#### A. Determining Fair Share of Each Carrier From Available Slots

As discussed above the goal of this section is to determine a fair share of available slots “owed” to each operator in expectation. Our procedure for determining this fair share requires as input a flight schedule. Vakili and Ball [11] explained how to determine the fair share of available slots. In this section, we just briefly explain the procedure, *Finding Fair Share based on Proportional Random Assignment*, FFS-PRA.

The availability of a schedule is characterized by knowing for each flight  $f$ , a scheduled arrival time  $a_f$ , which is interpreted as the time  $f$  is “scheduled” to arrive at the FCA boundary. Each slot  $s_j$  has an associated time  $t_j$  so that a flight  $f$  can be assigned to slot  $s_j$  if  $a_f \leq t_j$ .

We start by assuming there is an allocation that uses all slots (this almost always happens during congested periods – furthermore, this assumption can be dropped but doing so would complicate the presentation). We call an allocation that uses all slots a *complete* allocation. PRA, which underlies FFS-PRA is based on the following principles.

- Each flight can use *at most* one slot.
- All flights have equal share of each slot that they can use in any complete allocation.
- Each flight can be assigned to any slot later than its scheduled time of arrival.

We can now define the PRA procedure.

**PRA:**

- Step 1** : Set  $F_1 = \{f \in \mathbf{F} : a_f \leq t_1\}$  and  $i = 1$   
**Step 2** : Choose an  $f \in F_i$  with probability  $\frac{1}{|F_i|}$  and assign  $f$  to  $s_i$   
**Step 3** : Set  $i = i + 1$

**Step 4** : Set  $F_i = \{f \in \mathbf{F} : a_f \leq t_i\} - \{f\}$

**Step 5** : If  $i \leq m$  Then go to Step 2.

End.

We define for each flight  $f$  and slot  $j$ ,  $P_{fj}$  to be the probability that PRA assigns  $f$  to  $s_j$ . Also, define:

$$P_f = \sum_j P_{fj} = \text{PRA share for flight } f \quad (1)$$

$$FS_i = \sum_{f \in F_i} P_f = \text{PRA share for flight operator } A_i \quad (2)$$

Because of the structure of PRA,  $P_{fs}$  can be computed in polynomial time [11] as:

$$P_{fj} = \frac{\prod_{i=k}^{j-1} (n_i - i)}{\prod_{i=k}^j (n_i - (i - 1))} \quad (3)$$

where  $n_i$  is the number of flights that can be assigned to slot  $s_i$ ,  $k$  is the earliest slot,  $s_k$ , that flight  $f$  can use.

Let us now compare this method of computing fare shares with the implicit fare shares allocated by RBS. RBS, of course, is a deterministic procedure that either assigns a slot to a flight or does not. Thus, the RBS “fare share” for a flight is either zero or one. Since PRA employs randomization and since it employs the principles described earlier, any flight that appears in *any* complete allocation will have a positive fare share. Therefore, all flights included in an AFP will have a positive share of available slots.

This is a very important point. While RBS will give zero share to later flights, FFS-PRA will give such flights a positive share. We should note that flights that are scheduled earlier will typically receive a higher share than later scheduled flights. Therefore, FFS-PRA implicitly gives a higher share to earlier scheduled flights and so it gives some weight to the basic RBS principle. However, it balances this principle with the principle that each flight included in the AFP has a claim to a portion of the available capacity.

Further, (see [10]) we can show that PRA meets the fundamental fair allocation principles, which are *impartially*, *consistency* and *equal treatment of equals* and *demand monotonicity* (see [15]).

Impartiality states that allocation rule should not discriminate among the flights *except* insofar as they differ in type. In other words, if two flights are indifferent in type and in the feasible set, they will receive the same fair share. The consistency property states that the expected fair shares should be independent of the order in which flights are assigned to the slots. *Equal treatment of equals* states that if two flight operators have the same schedule, they will receive the same fair share. *Demand monotonicity* says that an increase in carrier  $i$ ’s total number of flights (with other flights remaining unchanged), can not deteriorate carrier  $i$ ’s fair share.

#### B. Flight Priority List

As discussed earlier our slot allocation procedure requires airline flight-slot preference information. There are two types of preference lists.

In the first type, carriers submit to the FAA an ordered list of flight-to-slot assignments. For example, carriers submit an

ordered list of  $(f_i, s_j)$  pairs. This type of list can be very long when the number of slots is large. The second type of list is a compact version of the first type. Instead of submitting an ordered list of  $(f_i, s_j)$  pairs separately, carriers submit the pair of flights and an interval of slots. For example, if a carrier ordered preference list is  $(f_i, s_j), (f_i, s_{j+1}), (f_i, s_{j+2}), (f_l, s_k)$  then it can be expressed as  $(f_i, s_j : s_{j+2}), (f_l, s_k)$ .

Suppose, carrier  $A$  has three flights  $A101, A102$  and  $A103$  and also assume there are six available slots,  $s_1, \dots, s_6$ . The earliest slots,  $a_f$ , that each flight can be assigned could be:

Slot:	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$
Flights:	A101	A102	A103			
$a_f$	$s_1$	$s_4$	$s_6$			

The following table illustrates a possible flight priority list.

**Preference List for  $A$**

Rank	(Flight,Slot)	Rank	(Flight,Slot)
1	(A103, $s_6$ )	6	(A101, $s_3$ )
2	(A101, $s_1$ )	7	(A101, $s_4$ )
3	(A101, $s_2$ )	8	(A101, $s_5$ )
4	(A102, $s_4$ )	9	(A101, $s_6$ )
5	(A102, $s_5$ )	10	(A102, $s_6$ )

For simplicity, the flight priority list can be shown as:

Rank	(Flight,Slot)	Rank	(Flight,Slot)
1	(A103, $s_6$ )	4	(A101, $s_3 : s_4$ )
2	(A101, $s_1 : s_2$ )	5	(A101, $s_5 : s_6$ )
3	(A102, $s_4 : s_5$ )	6	(A102, $s_6$ )

In this example, for carrier  $A$ , the highest priority is to insure that flight A103 leaves on time. Thus, Carrier A would prefer to receive slot  $s_6$ , before several earlier slots, in order to insure the ontime departure for flight A103.

#### IV. SLOT ALLOCATION PROCEDURE

Vakili and Ball [11] proposed a randomized allocation procedure, PBPR, that uses fair share and flight priorities as exogenous input to allocate slots to flight operators. PBPR guarantees that each carrier receives between the ceiling and floor of its fair share. However, in PBPR, it was implicitly assumed that all slots had equal values. Specifically, when measuring an allocation against a carrier's fair share, they only considered the total number of slots a carrier received. The priority list allows carriers to express a preference among slots, however, it does not allow a carrier to trade off the number of slots received with which slots are received. For example, carriers that would like to maintain on time performance for key flights, may be willing to pay more than others for particular slots. We wish to allow carriers to "pay more" for earlier slots when they wish to do so.

Our objective here is somehow to distinguish between those carriers who want to maintain on-time performance for certain flights and in return receive fewer slots and those carriers who can tolerate more delay but would like to receive more slots.

Consider the example of carrier  $A$  who prefers to receive priority for certain flights in exchange for receiving fewer

slots in total. The algorithm employs a parameter which is the "value" of the higher priority slots distributed. Suppose that value was set at 2 "slot units" and that carrier  $A$ 's fair share is 5.5. Then carrier  $A$  could receive two "high-priority" slots based on 2 ( $\lfloor \frac{5.5}{2} \rfloor$ ). The remainder of its fair share is 1.5, which can then be used to receive later slots. It is very important to notice that only those carriers that can afford this trade off (have a fair share  $\geq 2$ ) are considered. If a small carrier with a small fair share prefers to receive good slots and it does not have enough budget to give up a second flight, it can not be considered.

#### A. Slot Values

For illustration purposes, suppose we have two sets of airlines. Let  $A_1$  be the set of airlines that prefer less delay and  $A_2$  the set of airlines that prefer to receive more slots. In our allocation algorithm we initially give priority to the airlines in  $A_1$ . Therefore, they must pay more for each slot they initially receive because of the priority. Let us assume the price of each slot they receive is  $P_H$ . Since airlines in  $A_1$  receive priority in the allocation process their exogenous fair share must be greater than  $P_H$ .

The FAA acts as an independent, fair moderator. The FAA announces the value of priority slots. This value must be greater than one. The process operates so that the total value of slots given away equals the number of slots available. Since the value of each slot for the airlines in set  $A_1$  is  $P_H$ , we can compute the value of remaining slots. Thus, later (less preferred) slots will have a value less than one. Suppose there are  $m$  slots available, to compute the value of remaining slots, we need to find the number of slots that are assigned to airlines in  $A_1$ . Let us call this number  $m_1$ :

$$m_1 = \left( \sum_{a \in A_1} \lfloor FS_a / P_H \rfloor \right) / P_H \quad (4)$$

Where  $FS_a$  is the fair share of carrier  $a$ . Then the value of remaining slots can be computed as:

$$P_L = \frac{m - P_H \times m_1}{m - m_1} \quad (5)$$

As we can see the value of the remaining slots is less than one. Note that higher  $P_H$  values result in a smaller  $m_1$ . We will show the effect of varying  $P_H$  in our simulation results.

#### B. Dual Price Proportional Random Allocation

Dual Price Proportional Random Assignment (DP-PRA) is a new algorithm that considers the carriers' tradeoff between delay and rerouting (or cancellation). DP-PRA contains two phases: First phase allocates slots to the flights in the set  $A_1$  and in the second phase all remaining slots are allocated from the earliest available to the latest available. The second phase can use the PBPR [11].

We define two policies: under *Policy  $P_1$*  carriers prefer to prioritize certain flights, i.e. receive fewer slots but less delay; under *Policy  $P_2$*  carriers wish to treat all flight equally (and receive more slots).

We formally define DP-PRA below:

**DP-PRA:**

**Step 0a:** Inputs: Set of flights  $\mathcal{F}$ , set of carriers  $A$ , set of available slots  $\mathcal{S}$ , Carriers' preference lists:  $PList_1, PList_2, \dots, PList_K$  also  $P_H$  and carriers set  $A_1 = \{a \in A : P_1 \succ_a P_2, FS_a \geq P_H\}$ .

**Step 0b:** Calculate the fair share of each airline  $FS_a$  based on PRA

**Step 0c:** Calculate  $P_L$  based on 4 and 5

**Step 1: PHASE 1** while  $A_1 \neq \emptyset$  Do:

**Step 1a:**  $\forall a \in A_1$ , Randomly choose an  $a^* \in A_1$  in proportion to  $FS_{a^*}$ .

**Step 1b:** From  $PList_{a^*}$ , assign the best slot available to the highest priority flight  $(f^*, s^*)$

**Step 1c:**  $FS_{a^*} = FS_{a^*} - P_H$ ,  $PList_{a^*} = PList_{a^*} - \{f^*\}$  and  $\mathcal{S} = \mathcal{S} - \{s^*\}$  and  $A_1 = \{a \in A_1 : FS_a \geq P_H\}$ .

end while

**Step 2: PHASE 2**

**Step 2a:**  $A = \{a \in A : FS_a > 0\}$ .

**Step 2b:** for all  $a$  in  $A$ ,  $FS_a = FS_a/P_L$ .

**Step 2c:** Run PBPPRA.

In the first phase of algorithm we consider just carriers in  $A_1$  who can afford a slot with value of  $P_H$ . A carrier will be chosen randomly based on its fair share,  $FS_{a^*}$ . Then from  $PList_{a^*}$  we assign the best slot available to the highest priority flight,  $f^*$ . Assign  $f^*$  to  $s^*$  then remove  $f^*$  from  $PList_{a^*}$  and  $s^*$  from  $\mathcal{S}$ . We reduce the fair share of  $a^*$  by  $P_H$ . We repeat this phase until  $A_1$  becomes empty. Now, we move to the second phase.

In the second phase of the algorithm all airlines with positive fair share will be considered. The value of each slot in the second phase is  $P_L$ . We make the value of each slot one and increase the fair share of all airlines by  $1/P_L$ . Then, we execute PBPPRA. A carrier will be chosen randomly in proportion to its fair share. From  $PList_a$  the highest priority flight from carrier  $a$  will be chosen. Carrier  $a$ 's fair share will be reduced by one.

We can show that DP-PRA satisfies some desirable properties:

1) **The value of slots allocated to a carrier  $A_i$  should be close to  $FS_i$ .** Let us consider the fair share as an exogenous budget each carrier has. This property, which is a version of "equal treatment of equals" is probably the most fundamental to consider. It says that each flight operator should get its fare share (within a tolerance). The actual total slot value for a carrier after using the procedure can be calculated based on whether slot value is  $P_H$  or  $P_L$ , and based on actual total number of slots received by that carrier. After applying DP-PRA, then for any two carriers with equal fair share the difference in actual total slot value for two carriers with the same fair share will be less than an upper bound of  $2P_L$ . To be more precise, if two carriers with equal fair share belong to the same set, then the difference in actual total slot

value for each carrier is less than  $P_L$ . And if two carriers with equal fair share belong to two different sets, then the difference in actual total slot value for each carrier is less than  $2P_L$ .

2) **Each flight operator should be motivated to submit a "truthful" preference list.** This is considered a fundamental property of allocation methods, more formally known as *strategy proofness*. If the "dominant" strategy for each flight operator is to submit its true priority list, then flight operators need not seek to "game the system" and so the problem they face is relatively straight forward. Further, the overall system will be more stable in the sense that there should not be claim that certain operators gained an unfair advantage. It can be shown that DP-PRA is strategy-proof if in each step of allocation procedure there are more than one carrier to compete for a slot.

## V. EXPERIMENTAL RESULTS

In our experiment, we use the same test data set as used in [11]. This data set that had been employed by the CDM Future Concepts Team to perform human-in-the-loop experiments related to SEVEN. It contained 386 flights with 38 flight operators. The data included scheduled arrival times at an FCA boundary. The FCA duration was from 18:00 pm to 21:00 pm. As we explained in [11] a flight cost function can be generated as:

$$C(x, P) = \begin{cases} 0 & x \leq 15 \\ (32 + 0.1P)(x - 15) & 15 < x \leq M_p \\ (32 + 0.1P)(M_p - 15) & x > M_p \end{cases}$$

Where  $M_p$  is flight specific max delay. Given the cost function, we generated the priority list for each flight operator based on all available flights that could use a slot; and the assumption is that the flight operator preferred allocating the slot to the flight with the highest marginal cost of delay. The flight operators are randomly assigned to set  $A_1$ , the ones who prefer to receive better slots, or  $A - A_1$ , flight operators who prefer to receive more slots.

We compared the results of DP-PRA against ration-by-schedule (RBS), which is currently used to allocate FCA access during airspace flow programs. In our experiment, we considered 40%, 50%, 60%, 70% and 80% en-route capacity reduction for the FCA. We performed 2000 repetitions of the procedure since both procedures are random. In first part of all of our experiment we set  $P_H = 2$ . We will show later the effect of changing  $P_H$ . For each capacity reduction, the number of carriers that can participate in the first phase of algorithm is different. It is clear that as capacity increases the fair share of each airline increases, consequently the number of airlines that can participate will increase as well. Airlines 1, 3, 5, 6, 7, 9, 17, 19, 20, 21, 25, 26, 28, 29, 30, 31, 34, 35 have the second policy. Table I shows the airlines and number of flights (or slots) that are assigned in the first phase for each capacity reduction. Table II shows the percentage of cost savings for DP-PRA compared to RBS.

% Capacity reduction	List of Airlines	Number of slots
40	{1,3,5,6,21,25,28,29,34}	45
50	{1,5,6,21,25,28,29,34}	37
60	{1,5,6,21,25,29,34}	27
70	{1,5,6,21,25,29,34}	21
80	{1,21,29}	12

TABLE I  
LIST OF AIRLINES THAT CAN PARTICIPATE IN THE FIRST PHASE AND THE NUMBER OF SLOTS ARE ASSIGNED

% Capacity reduction	DP-PRA
40	18.19
50	15.72
60	11.69
70	9.71
80	6.78

TABLE II  
COMPARISON OF COST REDUCTION FOR DP-PRA AND MDP-PRA VS. RBS

The main advantage of DP-PRA compared to PBPR is to meet carriers' preference better. Figure 1 shows the average number of slots carriers in  $A_1$  receives compare to previous procedures for 60% capacity reduction.

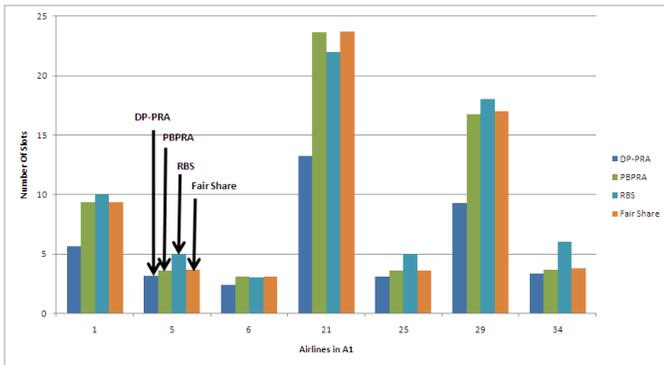


Fig. 1. Comparison of number of slots received for airlines in  $A_1$

### A. Effect of $P_H$

We have used  $P_H = 2$  in our experiments. Here we want to investigate the effect of  $P_H$  in overall performance of DP-PRA. Choosing the right  $P_H$  is a challenge for the FAA. There can be many different performance criteria; for example, deviation from carriers' fair share, total internal cost, how many slots should be assigned in the first phase. Minimizing the total internal cost is very hard for the FAA to measure because each carrier's cost information is private. Here we explain the effect of  $P_H$  on one performance criteria. In our examples we consider 40% capacity reduction in enroute resources.

The FAA can consider the deviation of total slot value received from fair share as a one criteria. Figure 2 shows the total define Minimum Square Error (MSE) of slot values from carriers' fair share. As can be seen, a minimum occurs at  $P_H = 2.75$  and  $P_H = 3.5$  for the procedure.

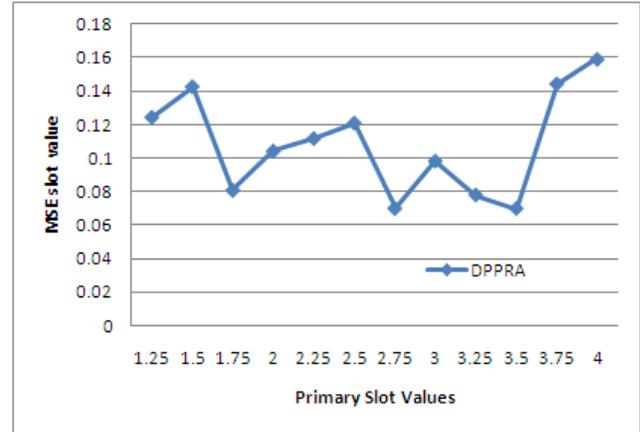


Fig. 2. Effect of primary slot values on MSE of slot values.

## VI. CONCLUSIONS

In this paper a new procedure for slot allocation has been proposed. Unlike PBPR that implicitly assigns the same value to all slots, under DP-PRA, we allow carriers to "pay more" to receive high priority slots. The main goal is to address carriers' preferences better. Our experiments show that, when using DP-PRA, carriers can better optimize their internal cost functions. Our procedure meets certain fair allocation principles, including equal treatment of equals and strategy proofness. A related challenge is to set (exogenously) the "price" for the high value slots. We have provided experiments to lend insight into this decision.

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## REFERENCES

- [1] M. Adams, S. Kolitz, J. Milner, and A. Odoni. Evolutionary concept for decentralized air traffic management. *Air Traffic Control Quarterly*, 4:281-306, 1997.
- [2] Gaertner, N, M. Klopfenstein and G. Wilmouth, Updated Operational Concept for System Enhancements for Versatile Electronic Negotiation (SEVEN), September 7, 2007 available at <http://cdm.fly.faa.gov/index.html>
- [3] Metron Aviation. [www.metronaviation.com/airspace-programs.php](http://www.metronaviation.com/airspace-programs.php)
- [4] H. Moulin. The proportional random allocation of indivisible units. *Social Choice and Welfare*, 19:381-413, 2002.
- [5] H. Moulin and R. Stong. Fair queueing and other probabilistic allocation methods. *Mathematics of Operations Research*, 27:1-30, 2002.

- [6] H. Moulin and R. Stong. Filling a multicolor urn: an axiomatic analysis. *Game and Economic Behavior*, 45:242-269, 2003.
- [7] A. Odoni and T.P. Fan. The potential of demand management as a short-term means of relieving airport congestion. In *ATM-2001 EUROCONTROL-FAA Seminar, Santa Fe, NM*. 2001.
- [8] B. O'Neill. A prolem of rights arbitration from the talmud. *Mathematical Social Sciences*, 2:345-371, 1982.
- [9] S. Rassenti, V. Smith, and R. Buffin. A combinatorial auction mechanism for airport time slot allocation. *Bell Journal of Economics*, 13:402-417, 1982.
- [10] N. Vakili Pourtaklo, *Preference-Based Fair Allocation of Limited Airspace Resources*, PhD - Electrical Engineering, U of Maryland, December, 2010.
- [11] N. Vakili Pourtaklo, M. Ball. Equitable allocation of enroute airspace resources. *Eighth USA/Europe Air Traffic Management Research and Development Seminar*, 2009.
- [12] T.W.M. Vossen *Fair Allocation Methods in Air Traffic Management*. PhD Thesis, University of Maryland, R.H. Smith School of Business, 2002.
- [13] Vossen, T., M. Ball. 2006. Optimization and mediated bartering models for ground delay programs. *Naval Research Logistics*, 53:75-90.
- [14] M. Wambsganns. Collaborative decision making through dynamic information transfer. *Air Traffic Control Quarterly*, 4:107-123, 1996.
- [15] H.P. Young. Distruptive justice in taxation. *Journal of Economic Theory*, 48:321-335, 1988.
- [16] H.P. Young. Progressive taxation and equal sacrifice. *American Economic Review*, 80:253-266, 1990.

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