GAME EQUILIBRIUM ANALYSIS ON AN AUCTIONING METHOD FOR AIRPORT CONGESTING RESOURCE ALLOCATION

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Abstract:

Flight delays cause lots of additional operational costs of airlines and travelers. Because airspace capacity is a scarce resource and airlines are self-interested, how to optimize the capacity allocation and avoid "the tragedy of the commons" is a hard problem for the Air Traffic Management authority. Through defining the marginal cost function and the opportunity cost function about the airlines, we introduce the first-price-sealed bid theory to realize the scare capacity allocation of congesting airport to the airlines which want them. Under the ATM authority's resource allocation policy, the airline will develop a set of scenarios to minimize the potential disruption to its schedule and implement the one that is most cost-effective through a competitive biding process with other airlines. Finally, Air Traffic Management authority could get the optimized global allocation result of airspace resources under the equilibrium condition.

Keywords:

air traffic management; ground delay program; first pricing sealed auction; traffic congestion

1. Introduction

During the last several years, airlines and passengers have been suffering from congestion at busy airports and airspaces, annual congestion delay costs airlines and travelers more than $20 billion in the world. Especially in china, with the greatly development of trades and economics in the last two decades, the air traffic have been growing rapidly and the flights delays have become a major public policy issue. It is predicted that there will be more than 3655000 flights operating in 2010. Though comparing with the European and USA, she has fewer than any of them, the distribution of flight flows in China is very uneven. More than 70% of flights have been operating on the eastern china. Nearly 30% of total flights have been flying between these three airports (including Beijing airport and Shanghai airport and Guangzhou airport) every year. and even the uneven trend is developing. So the Chinese air traffic operational system is fragile and is fluctuated easily. The air traffic congestion and delay problem is more and more prominent due to the military actions and bad weather conditions.

At present, traffic flow management(TFM) with the collaborative decision making aid(CDM) is applied widely to help to resolve the traffic congestion and balance demand and capacity when the airspace system was disrupted or the capacity was decreasing due to bad weathers or military events in the airspace system. A common condition in airline schedule planning (the process of generating the schedule with the greatest revenue potential) is that flight legs will be operated as planned according to the natural capacity of airspaces. But, when bad weather happened on the airspace, the capacity of airspace is decreased largely. The uncertain shortfall of capacity disrupts the planed schedules, and lots of affected flights need to be rescheduled. According to the schedule time, different traffic management initiatives such as reroutes, ground delay programs and miles-in-trail (MIT) restrictions can be used to revise the flight schedules and make the schedule demand adapt to the airspace system.

How to reschedule and allocate the limited airspace resources to these disrupted flights of different airlines equitably and efficiently under these three traffic management initiatives is the hard problem for the ATM authority. ATM authority aims to minimizing system delay time or cost under some certain fairness rules when it reschedules the disrupted flights of different users. Minimizing system delay time does not reflect the lowest total delay cost. Being incorporated into the collaborative decision making (CDM) process, the airlines could influence the rescheduling decisions to profit themselves[1, 2]. Because the total delay cost does not include the airlines' delay costs but also include the travelers'delay costs, the goals of different decision makers which include airlines and ATM authority may conflict and the available information for good decision makings varies among these decision makers. The airlines maybe hide the flight information that is disadvantageous to them, but is necessary to the optimal system decision. It is hard for ATM authority to get to the aim of the decision that is to reschedule these disrupted flights and allocate the limited airspace resources to airlines equitably and efficiently.

The auction is a resource rationing method of the market mechanism. The bid price is the reflection of the value of a scarce resource for the bidder. The successful use of auctions for telecommunication spectrum, energy and other commodities provide valuable insight into how to design auctions for the airspace resources [3-5].Due to the fast progress of network and web technologies, traditional trading systems can be operated welt on the internet. It unchained the technical barrier for the auction applied to the air traffic management[4, 6].

In this paper, we presents a first sealed auction method based on Dynamic Stackelberg equilibrium to realize the coincidence goal between the ATM authority and airlines. We make an attempt to set up the market-based, user self-decision Air Traffic Management mechanism. ATM authority sets up and announces the specific congestion toll schedules for the performance of the system that internalize the congesting external cost into the flight operational cost of airlines. ATM authority takes into consideration the global impact of dynamic congestion tolls that encourages the profit-oriented airlines to shift their low marginal profit flights to the non-peak traffic period or other legs which may be not charged by congesting fees or charged a little. Each airline is assumed to reschedule its disrupted flights by themselves according to the maximizing self-interested rule, while taking into consideration the pre-announced toll schedules and allocated capacity which is preferentially sold to the airline. Those elastic flights may be shifted from the congesting airspace to be delayed or to reroute other airspace.

2. The auction equilibrium model for the certain capacity

We first define the usage cost of airspace r and the expectation delay cost of flight f_k and the opportunity cost of flight f_k .

Let the marginal usage cost of the certain capacity of airspace r as follow,

$$MC^1_{a_i}(f_k, t) = p^1_{a_i}(t) \tag{1}$$

Where $p^1_{a_i}(t)$ denotes the biding price of the certain capacity that airliner a_i submit for flight f_k

Let the lower one of the expectation delay cost and the rerouting cost of flight f_k as the opportunity cost of f_k using the certain capacity, as follow

$$OP_{f_k}(t) = \begin{cases} delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) + (1-prob_{gdp}(t)) * p_{a_i}(t+\Delta t_{f_k}) \\ if\ Edelay_{f_k}(t) < reroute_{f_k}(t), \\ reroute_{f_k}(t) \qquad if\ reroute_{f_k}(t) < Edelay_{f_k}(t). \end{cases} \tag{2}$$

Where

$$\begin{aligned} &Edelay_{f_k}(t) = prob_{gdp}(t) * [delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)] \\ &+ \overline{prob_{gdp}(t)} * [MC^1_{a_i}(f_k, t+\Delta t_{f_k}) + delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) \\ &= delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) + (1-prob_{gdp}(t)) * p_{a_i}(t+\Delta t_{f_k}) \end{aligned} \tag{3}$$

Where

The expectation delay cost at the future departing interval $(t+\Delta t_{f_k})$, we define in this paper, is the operational cost after receiving the Ground Delay Program Order, if the flight do not depart at the current interval. After a certain length $(t+\Delta t_{f_k})$ predicted by the airliner, if the congestion problem will not exist, the GDP will be canceled and the delay cost will be the ground delay cost from the current interval. But, if the situation has not become good, the cost should include the ground delay cost and the usage cost of airspace r_j at the future departing interval $(t+\Delta t_{f_k})$. $prob_{gdp}(t)$ is the probability that the GDP initiative will be canceled during the $(t+\Delta t_{f_k})$ th period, So, the delay cost of flight f_k is the expectation value including the ground delay cost with probability $prob_{gdp}(t)$ and the usage cost at the future departing interval $(t+\Delta t_{f_k})$ with probability $1-prob_{gdp}(t)$.

If the airliner wins the certain capacity biding game, the payoff utility of flight f_k is as follow,

$$OP_{f_k}(t)-MC^1_{a_i}(f_k,t)$$
$$=\begin{cases} Edelay_{f_k}(t)-p^1_{a_i}(t) & \text{if } delay_{f_k}(t+\Delta t_{f_k})-delay_{f_k}(t)<reroute_{f_k}(t) \\ reroute_{f_k}(t)-p^1_{a_i}(t) & \text{if } reroute_{f_k}(t)<delay_{f_k}(t+\Delta t_{f_k})-delay_{f_k}(t). \end{cases}$$

$$\approx\begin{cases} delay_{f_k}(t+\Delta t_{f_k})-delay_{f_k}(t)-prob_{gdp}(t)*p^1_{a_i}(t), & \text{if } delay_{f_k}(t+\Delta t_{f_k})-delay_{f_k}(t)<reroute_{f_k}(t), \\ reroute_{f_k}(t)-p^1_{a_i}(t) & \text{if } reroute_{f_k}(t)<delay_{f_k}(t+\Delta t_{f_k})-delay_{f_k}(t). \end{cases}$$
(4)

Where

Because the unknown expectation delay cost is the empirical data, for simplifying the problem we take the delay cost of flight instead of the expectation delay cost to the rerouting cost.

Only if is the marginal usage cost of the certain capacity lower than its opportunity operational cost, the airline will attend the auction for flight f_k. So, the payment utility value is always a positive number. Simply, we assume that $p_{a_i}(t+\Delta t_{f_k})$ is approximate to $p^1_{a_i}(t)$ if there will be still the congestion during the future departing interval $(t+\Delta t_{f_k})$.

Both opportunity operational cost $OP_{f_k}(t)$ of flight f_k and marginal cost of the uncertain capacity are the additional operational cost of the flight. We assume that the additional operational cost, caused by congestion-related events, of each flight that takes part in the auction are independent and uniform random variables on the same interval $(0, delay_A(T))$. Because all of users in set *A* are profit-oriented, we assume that in civil aviation industry there is the common maximum of the additional operational cost of flight -- $delay_A(T)$. Each of the bidders who auction the same resource submits a nonnegative biding price. The bidder submitting the highest bid price will win and pay his bid. Other bidders pay and receive nothing. Bidders are risk-neutral and all of this information is common knowledge. Bidder a_i's payoff, if wins and pays the biding price, is

$$bid_{a_i,f_k}(a_i,a_j,v_{a_i,f_k}(t))=\begin{cases} OP_{f_k}(t)-p^1_{a_i}(t), & \text{if } p^1_{a_i}(t)>p^1_{a_j}(t), \\ 0, & \text{or else.} \end{cases}, \qquad \forall_{i\neq j} a_j\in A \tag{5}$$

Because the bid game is peer to peer, we just need to analyzing the equilibrium strategy of a_i : $p^1_{a_i}(t)=p^{1*}_{a_i}(OP_{f_k}(t))$. Given the equilibrium solution $OP_{f_k}(t)-p^{1*}_{a_i}(t)$,the expected payoff function is as follow,

$$Ebid_{a_i,f_k}(p^1_{a_i}(t))=(OP_{f_k}(t)-p^1_{a_i}(t))*[\prod_{j\neq i}\text{probability }(p^1_{a_j}(t)<p^{1*}_{a_i}(t))] \tag{6}$$

Where the first part before the multiplicative sign is the payoff of a_i , the second part is the probability that a_i wins all of each other. The probability that bidder a_i wins bidder a_j is

$$\begin{aligned} &probability(p^1_{a_j}(t)<p^{1*}_{a_i}(t))=probability(p^{1*}_{a_j}(OP_{f_k}(t))<p^{1*}_{a_i}(t)) \\ &=probability(OP_{f_k}(t))_{a_j}<\Phi(p^{1*}_{a_i}(t))=\Phi(p^{1*}_{a_i}(t))/delay_A(T) \end{aligned} \tag{7}$$

Where $\Phi(p^{1*}_{a_i}(t))$ is the inverse function of $p^{1*}_{a_i}(t)$, which denotes additional operational cost saving is $\Phi(p^{1*}_{a_i}(t))$ if airline a_i submitted bid price $p^{1*}_{a_i}(t)$ ..

So, we get

$$Ebid_{a_i,f_k}(p^{1*}_{a_i}(t)) = (OP_{f_k}(t) - p^{1}_{a_i}(t)) * [\Phi(p^{1*}_{a_i}(t)) / delay_A(T)]^{n-1}$$

$$Ebid_{a_i,f_k}(p^{1*}_{a_i}(t)) = \begin{cases} [delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) - prob_{gdp}(t) * p^{1}_{a_i}(t)] \\ *[\Phi(p^{1*}_{a_i}(t)) / delay_A(T)]^{n-1}, \\ [reroute^{*}_{f_k}(t) - p^{1*}_{a_i}(t)] * [\Phi(p^{1*}_{a_i}(t)) / delay_A(T)]^{n-1} \\ 0, \qquad \qquad or\ else. \end{cases} \quad (8)$$

For maximizing the expected payoff, we get

$$\frac{\partial Ebid_{a_i,f_k}(p^{1*}_{a_i}(t))}{\partial p^{1*}_{a_i}(t)} = 0$$

If $delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) < reroute_{f_k}(t)$, we get

$$\frac{\partial Ebid_{a_i,f_k}(p^{1*}_{a_i}(t))}{\partial p^{1*}_{a_i}(t)} = -prob_{gdp}(t) * [\Phi(p^{1*}_{a_i}(t))]^{n-1}$$
$$+[delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) - prob_{gdp}(t) * p^{1}_{a_i}(t)] * (n-1)\Phi^{n-2}\Phi^{'}(p^{1*}_{a_i}(t)) = 0.$$

If $reroute_{f_k}(t) < delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)$, we get:

$$\frac{\partial Ebid_{a_i,f_k}(p^{1*}_{a_i}(t))}{\partial p^{1*}_{a_i}(t)} = -[\Phi(p^{1*}_{a_i}(t))]^{n-1} + [reroute^{*}_{f_k}(t) - p^{1*}_{a_i}(t)] * (n-1)\Phi^{n-2}\Phi^{'}(p^{1*}_{a_i}(t)) = 0$$

Due to $\Phi(p^{1*}_{a_i}(t)) = \min(OP_{f_k}(t), reroute_{f_k}(t))$, we get the equilibrium bid price of a_i, as follow,

$$p^{1*}_{a_i}(t) = \begin{cases} \frac{n-1}{n * prob_{gdp}(t)}[delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)] \\ -\frac{1}{n}[delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)]^{1-n}, \\ \qquad if\ delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) < reroute_{f_k}(t), \\ \frac{n-1}{n} reroute_{f_k}(t),\ if\ reroute_{f_k}(t) < delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t). \end{cases} \quad (9)$$

Where the equilibrium price of this bid game relies on the number of bidders and the value $v^{*}_{a_i,f_k}(t)$ and their own estimations about the GDP delay situation. Each airline's bidding price is determined by the value from the bidding resource. At the equilibrium condition, the airline who gets the highest value from the resource will give the highest price. According to the first price sealed bidding principle, the player who gives the highest price will get the resource.

We get the differential eguation of $p^{1*}_{a_i}(t)$ about the derivative $v_{a_i,f_k}(t) = delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)$,

$$\frac{\partial p_{a_i}^{1*}(t)}{\partial v_{a_i,f_k}(t)} = \frac{n-1}{n * prob_{gdp}(t)} - \frac{1-n}{n}[v_{a_i,f_k}(t)]^{-n} \geq 0, \quad (10)$$

Obviously, $p_{a_i}^{1*}(t)$ is the increasing function about the variable $v_{a_i,f_k}(t)$. So the airlines whose flights suffer the more delay or rerouting cost will give the more prices about the auctioned resources, and will get more chance of wining.

When the biding resources are more than one, if the biding flights has the consistent utility for each resource unit in the same decision period, based on Eq(20)–Eq(23), similarly, we could get,

$$Ebid_{a_i,f_k}(p_{a_i}^{1*}(t)) = (GDP_{f_k}(t) - p_{a_i}^{1}(t)) * [\Phi(p_{a_i}^{1*}(t)) / delay_A(T)]^{n-m} \quad (11)$$

Due to there being m available resources, if only the biding price of the flight is above to any of (n-m) other bidders, not to any of (n-1) other bidders, the flight could win one capacity unit.

Likewise in (25), we get the equilibrium biding price under the condition that the biding resources are more than one, as follow,

$$p_{f_k,a_i}^{1*}(t) = \begin{cases} \frac{n-(m-1)-1}{n-(m-1)*prob_{gdp}(t)}[delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)] \\ -\frac{1}{n-(m-1)}[delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t)]^{1-n+(m-1)}, \\ if \;\; delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t) < reroute_{f_k}(t), \\ \frac{n-(m-1)-1}{n-(m-1)} reroute_{f_k}(t), \\ if \; reroute_{f_k}(t) < delay_{f_k}(t+\Delta t_{f_k}) - delay_{f_k}(t). \end{cases} \quad (12)$$

Where Let m denotes the auctioned certain capacity number.

Here the equilibrium value is just theory results. In practice, the behaviors of airlines in the bid games are hard to be assumed. However, the big and small of the equilibrium bid price is direct correlative to the additional operational cost. The equilibrium bid price of flight could reflect the true additional operational cost of flight in the assumption that every airline is rational and profit-oriented.

3. Conclusions

Given that the additional operational costs of flights are important components of the airline decision-making process, how the economic costs of flights under the different air traffic management tactics influence the airline decision behaviors have not analyzed in precious research. Our models allow for a test of the market mechanism effects on the airline decision behaviors in the context of air traffic management (ATM) that carefully optimizes the airspace system costly. The main contribution of this paper is to develop the auction method of the market mechanism. In theory the first pricing sealed auction could ensure the systemic benefit and equity and efficiency. The method makes an attempt to solve the airport congesting capacity allocation problem in the pre-tactics air traffic flow management.

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