Investigating String Stability of a Time-History Control Law for Airborne Spacing

Lesley A. Weitz
The MITRE Corporation
7515 Colshire Drive
McLean, Virginia 22102–7508
Email: lweitz@mitre.org

***Abstract*—Airborne spacing is a concept being developed as a part of Next-Generation Air Traffic Systems (NextGen) and Single European Sky ATM Research (SESAR). The objective of airborne spacing is to more precisely space aircraft relative to a target aircraft using avionics equipment that presents a speed command to the flight crew. In an approach-spacing application, it is expected that a string of aircraft will be formed, where each aircraft is spacing relative to the preceding aircraft. In the design of a speed-control algorithm, one must not only examine the performance and stability of one aircraft relative to another, but also the performance and stability of the entire string. This paper presents, a simplified, closed-form analysis of a time-history speed control law. Because the simplified analysis does not take into account all of the nonlinearities that may be included in a speed-control algorithm, simulation results are presented to illustrate how different parameters affect string behavior.**

I. Introduction

Researchers throughout the United States and Europe are investigating and developing airborne-spacing concepts as a part of Next-Generation Air Traffic Systems (NextGen) research and Single European Sky ATM Research (SESAR), respectively. A major component of these research efforts is the augmentation and, potentially, eventual replacement of the radar system in place today with a satellite-based system. Future air-traffic systems will employ Automatic Dependent Surveillance-Broadcast (ADS-B), which will broadcast aircraft identification and state information, such as global-positioning system (GPS) position and ground speed, for use by other aircraft and ground-based air traffic control. ADS-B enables flight crews to have improved situational awareness of neighboring traffic, and similarly, ground-based tools for controllers can be improved using the more frequently-updated, higher-precision state information from aircraft within their region of control. It is expected that ADS-B equipage will be mandated in certain airspace in the US by 2020.

The advent of ADS-B technology is leading to the development of cockpit-based avionics that take advantage of increased situational awareness available to the flight crew. Airborne spacing is just one of a number of concepts that are enabled by ADS-B. The goal of airborne spacing is to achieve precise inter-aircraft spacing as a means to increase the efficiency of a variety of operations currently managed by air traffic controllers. In airborne-spacing operations, spacing responsibility will be delegated to the flight crew. Avionics equipment will be designed to provide the flight crew with speed commands in order to maintain a desired spacing interval relative to one or more target aircraft. Approach spacing is an initial application of airborne spacing, where the operational goal is to increase the arrival capacity of busy airports by more precisely spacing aircraft at the final approach fix (FAF) or runway threshold.

The speed-control algorithm in the avionics calculates speed commands for the spacing aircraft to achieve a desired spacing interval; the commanded speed is a function of the state information from one or more target aircraft. Researchers at NASA Langley Research Center have developed a speed control law for terminal-area spacing, which uses estimated times of arrival (ETAs) at the runway threshold for both the spacing and target aircraft [1]–[4]. This is a trajectory-based speed control law, which assumes some knowledge of the planned arrival trajectories for the spacing and target aircraft in order to calculate the ETA. One primary advantage of NASA's trajectory-based control law is that aircraft on different lateral paths can still be spaced precisely at the runway threshold. EUROCONTROL has developed and analyzed a time-history control law, which is sometimes referred to as a constant-time-delay control law, for spacing aircraft along a common path. The time-history control law is designed to minimize the longitudinal spacing error between the spacing aircraft and the target aircraft's position τ seconds in the past, where τ is equal to the desired spacing interval [5]–[7]. A primary advantage of EUROCONTROL's time-history control law is that information about the planned trajectories for the spacing and target aircraft is not required. Both of these speed control laws have had extensive performance testing in simulation environments. Additionally, a variant of EUROCONTROL's time-history control law has been implemented and flight tested by United Parcel Service (UPS) [8].

In future approach-spacing applications, it is expected that a string of aircraft will be formed, where each aircraft implements speed changes in order to achieve the desired spacing relative to its immediately preceding target aircraft. In this case, the string stability of the selected speed-control algorithm must be investigated. String stability describes how spacing errors are propagated through the aircraft string as a result of disturbances or perturbations to the target aircraft speed. With a goal of developing airborne-spacing avionics for near-term national airspace system (NAS) improvements, a greater understanding of string-stability analysis and implications to actual operation are needed.

For near-term implementation, it is not expected that the airborne-spacing equipment will be coupled to the Flight Management System (FMS). Therefore, a commanded speed will be displayed to the flight crew to be manually flown. Analysis of overall system performance must also consider pilot response time to implement speed commands from the airborne-spacing equipment.

The objective of this paper is to present the string-stability analysis for a time-history control law. A closed-form string-stability analysis is shown for a simplified aircraft model which draws upon well-known results from string-stability literature. The closed-form analysis is intended to provide insight into string behavior for this control law. However, nonlinear effects that represent a more realistic system are difficult to analyze in a closed-form manner. Simulation results are used to investigate the effects of nonlinearities in the speed-control algorithm for different parameters. These results are used to illustrate how different parameters affect string behavior. Results are interpreted for the practical implementation of the airborne-spacing application.

The paper is organized as follows. In Section II, additional background on the time-history control law and string stability are presented. In Section III, a closed-form string-stability analysis is shown for a simplified aircraft model, and limitations to the simplified analysis are described. Simulation results are presented in Section IV for a variety of algorithm parameters, and lastly, conclusions are presented in Section V.

II. Background

A. Time-History Speed Control Law

The time-history speed control law presented here is considered for in-trail flight only, where the spacing and target aircraft are on a common lateral path. The spacing error is defined as the difference in time between when the target and spacing aircraft cross a common point on the lateral path and the desired spacing interval.

$$\text{spacing error}(p) \equiv (t_i - t_{i+1}) + \tau \qquad (1)$$

Here, p refers to a common point on the lateral path; t_i and t_{i+1} are the times when the target and spacing aircraft cross point p, respectively; and, τ is the desired (time-based) spacing interval.

A range error is used in place of the spacing error in the time-history control law. The range error is defined as the difference in the longitudinal position of the spacing aircraft at time t and the longitudinal position of the target aircraft at time $t-\tau$. The positions of the target and spacing aircraft are denoted as x_i and x_{i+1}, respectively.

$$\text{range error} \equiv e_i(t) = x_i(t-\tau) - x_{i+1}(t) \qquad (2)$$

The commanded speed is the sum of the range-error term and the ground speed of the target aircraft at time $t-\tau$. The speed command to the spacing aircraft, v^c_{i+1}, is shown below, where k is a constant control gain.

$$v^c_{i+1}(t) = v_i(t-\tau) + ke_i(t) \qquad (3)$$

Other implementations of a time-history control law have used gain scheduling to achieve more aggressive correction of range errors as the spacing aircraft gets closer to an endpoint [5]. Non-constant control gains and implications to string stability will be briefly addressed in Section III.

B. String Stability

The time-history speed control law is designed to drive range errors, and equivalently spacing errors, to zero, which is indicative of a locally-stable control law. However, string-stability analysis, which reveals overall system performance, must also be considered for the case when a string of aircraft has been coupled through the speed control law. String stability deals with how spacing errors are propagated through the aircraft string due to perturbations in a target aircraft's speed[1]. A string-stable control form means that spacing errors between adjacent aircraft do not grow or amplify along the string of aircraft. In string-stable systems, spacing errors are in fact dissipated along the string, and thus, aircraft far downstream from the disturbance will not detect the upstream disturbance.

The concept of string stability has been extensively studied for automated highway systems with vehicle strings of infinite length [9]–[11]. In the references listed here, a frequency-response approach is applied to determine whether the control laws in question yield a string-stable system. Using the spacing-error definitions and control inputs, the spacing-error transfer function that relates adjacent spacing errors is formed. The magnitude, or gain, of the spacing-error transfer function determines whether spacing errors will propagate along the aircraft string.

In the literature, constant-distance control laws, which aim to maintain a constant distance relative to the immediately preceding vehicle only, are shown to yield a string-unstable system [9]. String stability can be gained by spacing relative to both the immediately preceding vehicle and the first vehicle in the string [9], [12], [13]. In contrast, a constant-time-headway control law, or a cruise-control control law, has been shown to be weakly string stable [10][2].

Researchers at EUROCONTROL have investigated string-stability of the time-history control law in a simulation environment. Results show that an additional parameter in the control law, the spacing-anticipation time, τ_{sa}, can lead to a string-stable system [14]. The modified control law is shown below; note that the spacing-anticipation time is only applied to the target ground-speed term.

$$v^c_{i+1} = v_i\left(t - \tau + \tau_{sa}\right) + k\left[x_i(t-\tau) - x_{i+1}(t)\right] \qquad (4)$$

[1]String stability is evident on busy highways when drivers speed up and slow down to maintain a desired distance relative to the preceding vehicle; i.e., the vehicles have a sinusoidal-like speed profile. The sinusoidal speeds have a greater and greater effect along the string.

[2]In weakly-string-stable systems, the errors do not grow along the string, but errors will not dissipate either.

This modification will be revisited in Section IV.

In the next section, a closed-form string-stability analysis will be applied to the time-history control law in equation (3).

III. Closed-Form String-Stability Analysis

To facilitate the closed-form string-stability analysis, a simplified aircraft model is chosen. A single-degree-of-freedom, double-integrator model represents longitudinal aircraft motion.

$$\dot{x} = v; \qquad \dot{v} = u \tag{5}$$

Here, x is the inertial position, v is the aircraft ground speed, and u is the commanded acceleration. The spacing control law shown in equation (3) still commands a velocity, v^c, and u is designed to track the commanded velocity: $u = k_v\left(v^c - v\right)$.

Derivatives of the range error are taken to reveal the relationship between adjacent range errors.

$$e_i(t) = x_i(t-\tau) - x_{i+1}(t) \tag{6}$$

$$\dot{e}_i(t) = v_i(t-\tau) - v_{i+1}(t) \tag{7}$$

$$\begin{aligned}\ddot{e}_i(t) &= \dot{v}_i(t-\tau) - \dot{v}_{i+1}(t)\\ &= u_i(t-\tau) - u_{i+1}(t)\\ &= k_v\left[v_i^c(t-\tau) - v_i(t-\tau)\right] - k_v\left[v_{i+1}^c(t) - v_{i+1}(t)\right]\\ &= k_v\left[v_{i-1}(t-2\tau) + k e_{i-1}(t-\tau) - v_i(t-\tau)\right] -\\ &\quad - k_v\left[v_i(t-\tau) + k e_i(t) - v_{i+1}(t)\right]\end{aligned} \tag{8}$$

Using the following relationships for $\dot{e}_i(t-\tau)$ and $\dot{e}_i(t)$, equation (8) can be rearranged.

$$\dot{e}_{i-1}(t-\tau) = v_{i-1}(t-2\tau) - v_i(t-\tau) \tag{9}$$

$$\dot{e}_i(t) = v_i(t-\tau) - v_{i+1}(t) \tag{10}$$

The coupled error dynamics are shown below.

$$\ddot{e}_i(t) + k_v\dot{e}_i(t) + k_v k e_i(t) = k_v\dot{e}_{i-1}(t-\tau) + k_v k e_{i-1}(t-\tau) \tag{11}$$

The range-error transfer function is found by transforming equation (11) to the frequency domain.

$$H_i(s) \equiv \frac{E_i(s)}{E_{i-1}(s)} = \frac{\left(k_v s + k_v k\right) e^{-\tau s}}{s^2 + k_v s + k_v k} \tag{12}$$

It can easily be shown that the range-error transfer function $H_1(s)$, which relates the range error between the first two aircraft in the chain to the velocity of the first aircraft, is given by the expression below.

$$H_1(s) \equiv \frac{E_1(s)}{V_1(s)} = \frac{s e^{-s\tau}}{s^2 + k_v s + k_v k} \tag{13}$$

As a comparison, consider a constant-distance control law with the range error and speed control law defined below.

$$e_i(t) = x_i(t) - x_{i+1}(t) - d_i; \qquad v_{i+1}^c(t) = v_i(t) + k e_i(t) \tag{14}$$

Here, d_i is the constant desired distance between the ith and (i+1)th aircraft. The range-error transfer function has a similar form to equation (12).

$$H_i(s) \equiv \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_v s + k_v k}{s^2 + k_v s + k_v k} \tag{15}$$

Results in the literature show that the range-error transfer function in equation (15) yields a string-unstable system for all values of k, $k_v > 0$ [9].

Figure 1 shows the magnitudes of the frequency response for the time-history and constant-distance range-error transfer functions. Arbitrary gains were chosen to generate the plots: $k = k_v = \tau = 1$; the time-delay term in equation (12) is modeled using a fifth-order Padé approximation. The string-unstable nature of the control law is revealed by frequency-response magnitudes greater than one. For these control forms and gains, range errors and spacing errors will propagate along the string for disturbances with frequencies between 0 and 1.5 rad/sec. Furthermore, the results show that the two different control schemes yield identical frequency responses, which indicates that the time-history control law yields identical string behavior as a constant-distance control law.

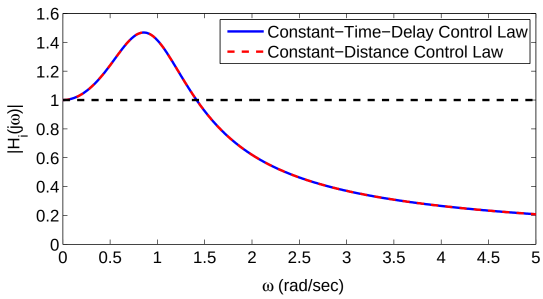

Fig. 1: Frequency response of the spacing-error transfer functions: time-history (blue solid line) and constant-distance (red dashed line) control laws.

The behavior of the aircraft string is confirmed by a simulation of a five aircraft string[3]; the aircraft dynamics are still represented by the simplified model in equation (5). In one case, the ground speed of the first aircraft is perturbed by a sinusoidal disturbance $d = 0.1\sin(\omega t)$, where $\omega = 0.5$ rad/sec. Figure 2(a) shows that the amplitude of the spacing errors, as defined in equation (1), increases along the string; thus, the disturbance yields string instabilities as predicted by Figure 1. In the other case, the frequency of the disturbance is changed to $\omega = 2$ rad/sec. This frequency does not yield string-unstable behavior as is evident by the decreasing spacing-error amplitudes along the string in Figure 2(b).

Additionally, the string-unstable behavior shown in Figure 2(b) results in increasing speeds along the string. Therefore, one would expect that aircraft further downstream of the disturbance would eventually reach speed limitations. This behavior leads to the definition of string stability for the airborne-spacing application: 1.) spacing errors are not propagated through the aircraft string, and 2.) control inputs do not grow along the aircraft string.

Previous analysis on the constant-distance control law indicated that more aggressive control gains along the string

[3]This is a simplified simulation, which aims to show trends in the aircraft behavior along the string. The aircraft are flying level with a desired speed $\dot{v} = 1$ distance unit/time unit (DU/TU).

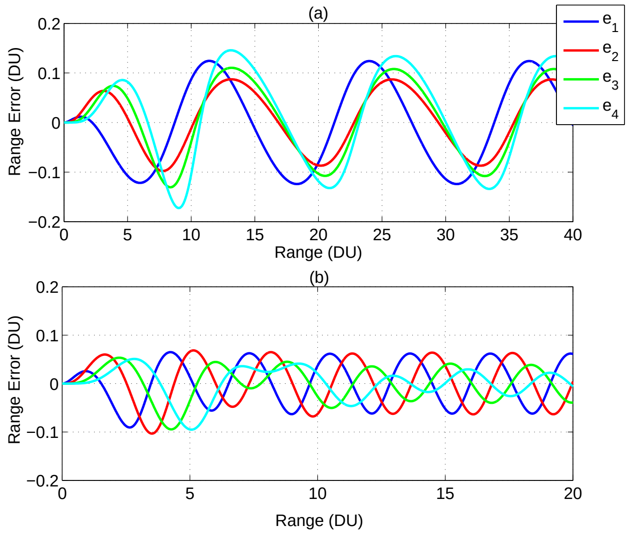

Fig. 2: Spacing errors between aircraft pairs when the first aircraft in the string is subject to a sinusoidal disturbance: (a) $\omega = 0.5$ rad/sec, and (b) $\omega = 2$ rad/sec.

prevent the growth of spacing errors along the string; e.g., the second aircraft in the string has more aggressive gains than the first aircraft [13]. Those results suggest that control gains can be designed, as a function of the distance to the FAF, that are initially more aggressive and become less aggressive closer to the FAF. This result is contradictory to the gain-scheduling approach that has previously been implemented for the time-history control law. The frequency-response function relating adjacent range errors for different control gains k_{i-1} and k_i, where $k_i > k_{i-1}$, is shown below.

$$H_i(s) \equiv \frac{E_i(s)}{E_{i-1}(s)} = \frac{(k_v s + k_v k_{i-1})\, e^{-\tau s}}{s^2 + k_v s + k_v k_i} \tag{16}$$

Without varying k_v, which is a gain related to the aircraft performance for tracking a commanded velocity, it is difficult to find gains k_{i-1} and k_i that yield a frequency-response function with magnitudes less than one for all frequencies.

Whereas some general system behavior can be derived by understanding the simplified, linear system presented here, the nonlinear nature of a higher-fidelity aircraft representation and nonlinearities in the control-law implementation make the overall system behavior more difficult to analyze in the closed-form manner as shown. Simulation results presented in the next section assume a nonlinear, point-mass model for the aircraft, and nonlinearities in the speed-control algorithm are included.

IV. Simulation Results

In this section, the assumptions for the simulation, including the aircraft model and approach trajectory, are presented. Simulation results for Gaussian-distributed initial spacing errors are presented assuming limitations on the speed commands and realistic ADS-B update rates. Selected simulation results are shown to illustrate the effects of different algorithm parameters on string stability.

A. Simulation Assumptions

1) Aircraft Model: An aircraft point-mass model is used for the simulation analysis. Because the airborne-spacing algorithm is used for longitudinal aircraft spacing, the simulation is designed for straight-line, longitudinal dynamics only. The longitudinal model, simplified from a commonly-used point-mass model, is shown below [15].

$$\dot{x} = V \cos\gamma \tag{17}$$

$$\dot{h} = V \sin\gamma \tag{18}$$

$$\dot{V} = \frac{T - D}{m} - g \sin\gamma \tag{19}$$

Here, x is the horizontal position, h is the altitude of the aircraft, and V is the true airspeed (TAS); m is the aircraft mass, g is the gravitational constant, and D is the drag force, which is dependent upon aircraft configuration and TAS. The control inputs to the aircraft are thrust, T, and flight-path angle, γ. To model non-instantaneous changes in the flight-path angle, a first-order model is assumed to track the commanded flight-path angle, γ_c: $\dot{\gamma} = k_\gamma(\gamma_c - \gamma)$.

Guidance control laws are designed to track the vertical profile and the commanded speed determined by the time-history control law. The FMS is assumed to be in *VNAV-path* mode, which uses pitch, or flight-path angle for the point-mass model presented here, to control the aircraft to the vertical path, and thrust is used to control longitudinal speed [16].

EUROCONTROL's Base of Aircraft Data (BADA) is used to calculate the drag coefficients, stall speeds, and thrust limitations for different flight phases [17].

2) Approach Trajectory: Scenarios are simulated for an approach trajectory starting 30 nautical miles (NM) from the FAF. The initial altitude and indicated airspeed (IAS) constraints are 12,000 feet (ft) and 230 knots (kts), respectively; the final altitude and IAS constraints are 4,000 ft and 180 kts, respectively.

For a Boeing 767-300 and a range of aircraft weights, an idle descent from the initial altitude to the start of the deceleration is assumed. The deceleration segment of the trajectory assumes a 0.30 energy-share factor to decelerate to the final speed constraint. Figure 3 shows the altitude and IAS as a function of range to the FAF for the trajectory used in the analysis.

B. Gaussian-Distributed Initial Spacing Errors

A ten-aircraft string was simulated to illustrate the behavior of the speed control law. The initial spacing errors of the second through tenth aircraft are normally distributed with a variance of 3 seconds. It is assumed that the ADS-B messages are updated every second based upon expectations for actual operations. Therefore, the spacing aircraft has a record of the target aircraft's states each second. Linear interpolation is used to determine the target aircraft's states at more frequent

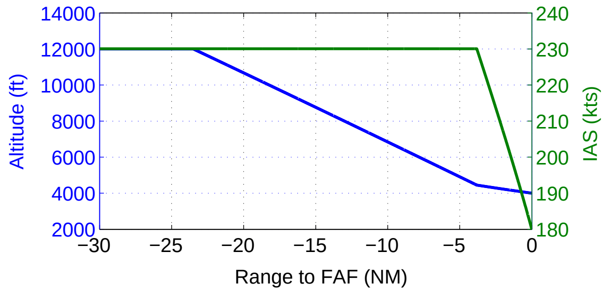

Fig. 3: Altitude (blue line) and IAS (green line) of the reference trajectory as a function of range to the FAF.

intervals. The speed control law is updated every 0.1 seconds. A commanded ground speed calculated by equation (3) is converted to an IAS, which is then rounded to the nearest integer value to represent the value that would be displayed to the flight crew. Speed changes less than two knots are not implemented, and speed commands are limited to $\pm 15\%$ of the reference IAS to keep speeds within the performance envelope of the aircraft. In this case, it is assumed that speed commands are instantaneously implemented and are constant between control-law updates. All of these parameters are nonlinearities in the speed-control algorithm, which make the integration of the nonlinear aircraft model and the more-realistic control-law implementation difficult to analyze in a closed-form manner.

Figure 4(a) and 4(b) show the commanded IAS and TAS, respectively, as a function of range to the FAF. Note that figure 4(a) shows a large number of speed changes over the duration of the flight, and in particular, during the deceleration segment of the trajectory. This number of speed changes would not be acceptable to a flight crew that is manually implementing speed commands. However, frequent speed changes lead to the spacing-error performance shown in Figure 5, where spacing errors indicate that the aircraft are converging to their desired intervals. The spacing errors, as defined in equation (1), are calculated every 0.5 NM along the path. The initial spacing errors are within ± 5 seconds; these spacing errors are reduced to within ± 0.1 seconds prior to the deceleration segment; and, the spacing errors at the FAF are within ± 0.15 seconds.

C. String-Stability Simulations

To test the string stability of the nonlinear aircraft model integrated with the nonlinear control algorithm, the second aircraft is initialized with a non-zero spacing error relative to the first aircraft, and the third through tenth aircraft have zero spacing errors with respect to their targets. These initial conditions result in a step speed command to the second vehicle, which is then passed through the string via the control law. Step inputs can be modeled by a series of sine functions with different amplitudes; therefore, the system will be excited by a wide range of frequencies in order to induce any string-unstable behavior.

1) Ideal Conditions: To fully understand the effects of nonlinearities in the control algorithm, the ten-aircraft string

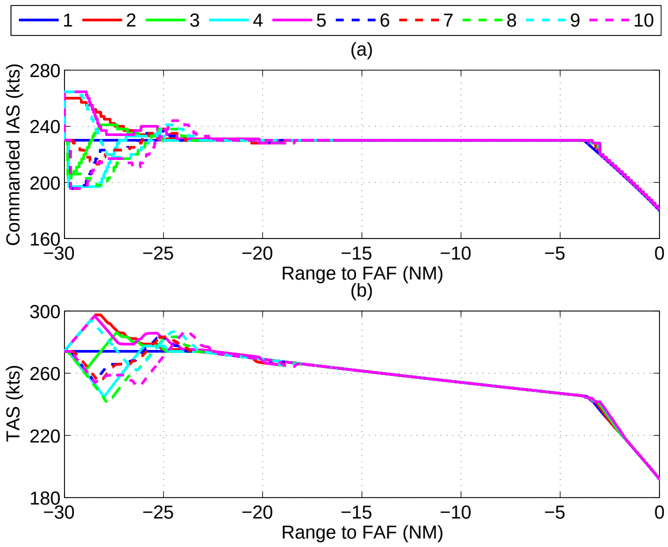

Fig. 4: Commanded IAS (a) and TAS (b) for Gaussian-distributed initial spacing errors. The speed-control algorithm includes speed limitations, assumes a realistic ADS-B update rate, and assumes frequent control-law updates.

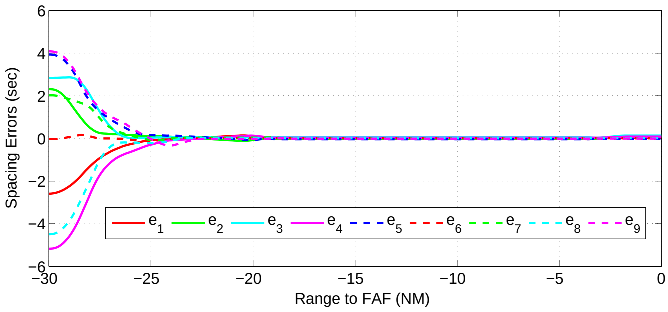

Fig. 5: Spacing errors as a function of range to FAF for Gaussian-distributed initial spacing errors.

is first simulated for ideal conditions: there are no speed-command limitations and speed commands are not quantized. Figure 6 shows the commanded IAS and TAS between -30 and -24 NM to the FAF. As predicted by the closed-form analysis, string instabilities are evident by the increasing speed commands along the string; the spacing errors (not shown) also grow along the string as evidence of the string instabilities. Whereas there is only a two-knot difference in the maximum speed commands between the second and tenth aircraft, this control algorithm would not support an infinite string of aircraft where speed limitations would constrain commanded speeds farther downstream of the disturbance. Additionally, the time for the target aircraft to achieve the desired IAS results in the commanded speeds along the string peaking closer and closer to the FAF.

Despite the disturbance, the speed control law is able to reduce the disturbance effects to precisely space aircraft at the FAF. Without the nonlinearities, the spacing errors at the FAF are within ± 0.02 seconds with the exception of the spacing error between the first and second aircraft, which is -0.3 seconds. This is believed to be a result of errors in tracking the commanded speed during the deceleration segment.

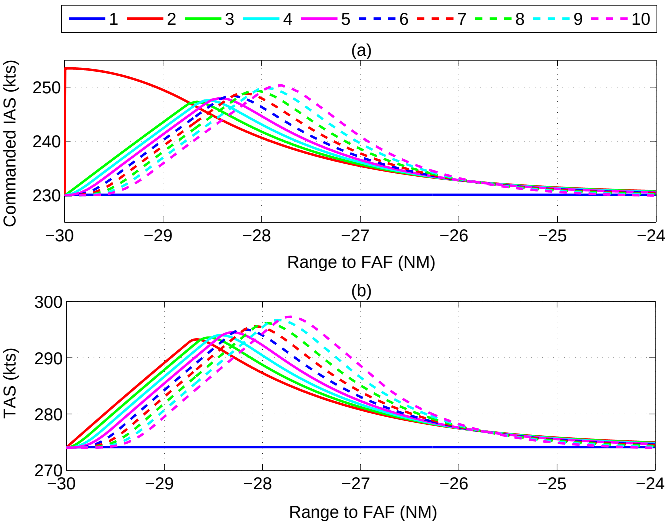

Fig. 6: Commanded IAS (a) and TAS (b) for a two-second initial spacing error between the first and second aircraft. Speed limitations are not imposed in the algorithm.

2) Speed-Command Limits: Figure 7 shows the commanded IAS and TAS when speed changes less than two knots are not implemented, speed commands are rounded to the nearest integer, and commands are limited to $\pm 15\%$ of the reference IAS. Here, a six-second initial spacing error is applied to the second aircraft, which results in speed commands that were saturated at the upper speed limit. This behavior then results in slow-down commands to the trailing aircraft, thus leading to an inefficient operation. Note that only the ninth and tenth aircraft reach the lower speed limit. The string-unstable behavior is also obvious in the growth of spacing errors along the string as shown in Figure 8.

3) Spacing-Error Deadbands: Applying deadbands to the estimated spacing error is one approach to limit the number of speed changes, where estimated spacing errors within a chosen threshold are set equal to zero. Estimated spacing errors are calculated by dividing the range error by the spacing aircraft's ground speed. Figures 9 and 10 show the effects of using deadbands on the estimated spacing errors with a threshold of ± 2 seconds. Spacing-error deadbands prevent the growth of speed commands along the string: speed commands do not reach the saturation limit and slow-down commands are not exhibited. An obvious drawback is that estimated spacing errors within the threshold are not corrected as shown in Figure 10.

4) Less-Frequent Speed Command Implementation: Previous simulation results have assumed frequent control-law updates every 0.1 seconds. No delays have been assumed in the implementation of these speed commands; however, flight crews would be unable to manually adjust the IAS at this rate. Figure 11 shows the commanded IAS and TAS when the control law is updated every 15 seconds, which is a more-realistic frequency for flight crews within 30 NM of the FAF. Less frequent updates are evidenced by the larger speed changes. Also note that the tenth aircraft does not achieve the desired interval until 15 NM to the FAF.

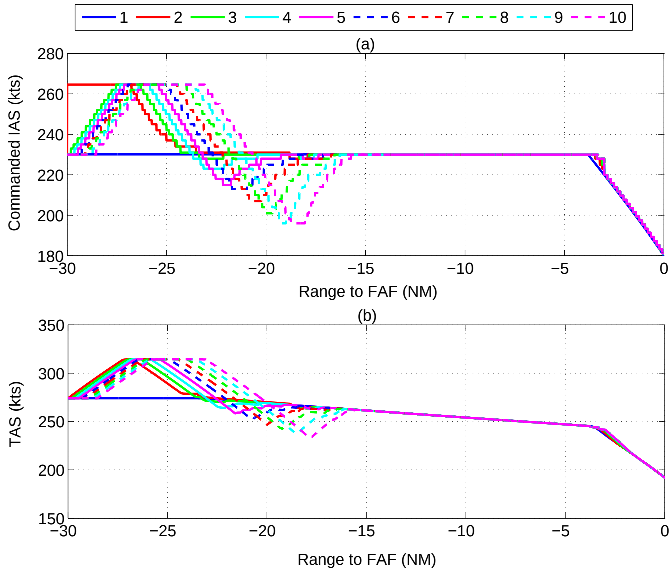

Fig. 7: Commanded IAS (a) and TAS (b) for a six-second initial spacing error between the first and second aircraft. Speed changes are limited to changes greater than $\pm$ 2 kts, speed commands are limited to $\pm 15\%$ of the reference speed and are rounded to the nearest integer.

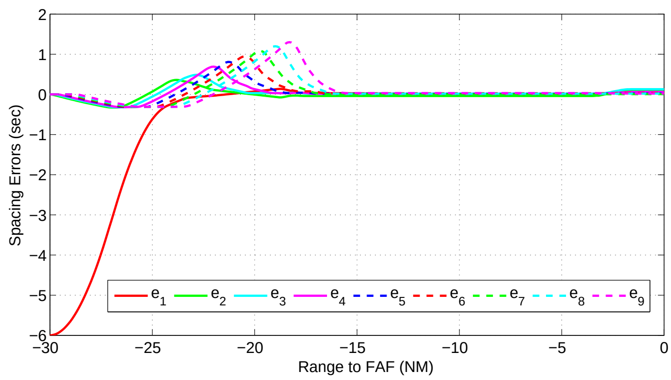

Fig. 8: Spacing errors as a function of range to FAF for a six-second initial spacing error between the first and second aircraft. Speed limitations are assumed.

5) Spacing Anticipation: The effects of using a spacing-anticipation parameter, as described by equation (4), are shown in Figure 12 for τ_{sa} = 10 seconds. The second aircraft again had a six-second initial spacing error. Results in Figure 12 show that using a spacing-anticipation parameter yields

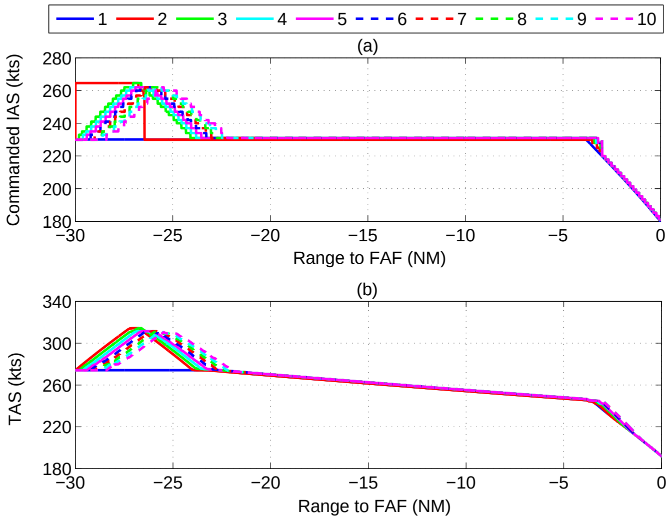

Fig. 9: Commanded IAS (a) and TAS (b) for a six-second initial spacing error between the first and second aircraft. The control law includes speed limitations and deadbands on the estimated spacing error.

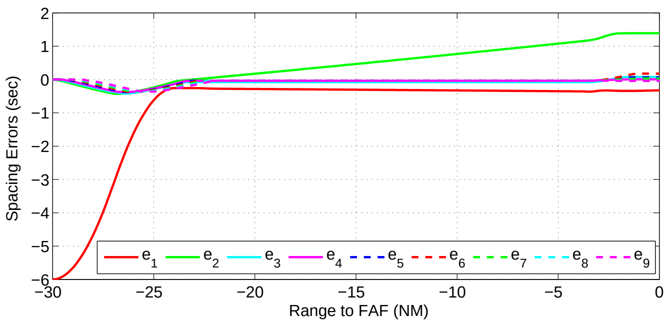

Fig. 10: Spacing errors as a function of range to FAF for a six-second initial spacing error between the first and second aircraft. Speed limitations and deadbands on the estimated spacing error are assumed.

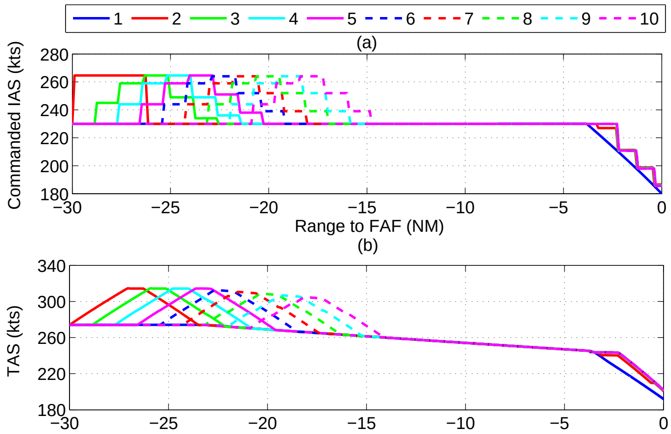

Fig. 11: Commanded IAS (a) and TAS (b) for a six-second initial spacing error between the first and second aircraft. The control law is updated every 15 seconds (compared to the 0.1-second update rate in previous results).

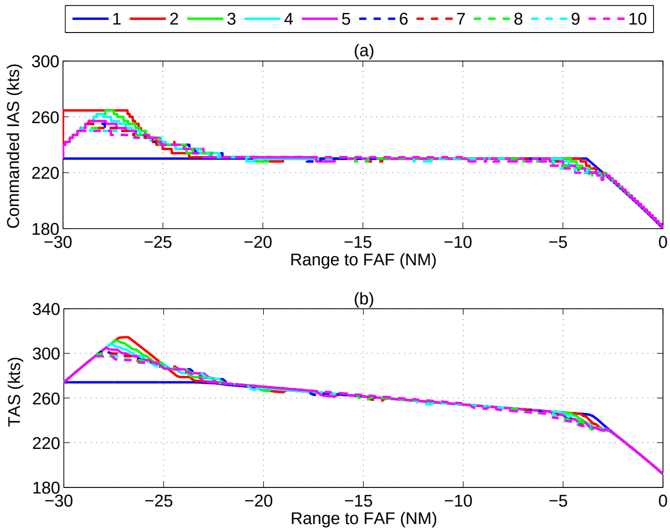

Fig. 12: Commanded IAS (a) and TAS (b) for a six-second initial spacing error between the first and second aircraft. The control law uses a spacing-anticipation parameter τ_{sa} = 10 seconds.

a string-stable system confirming the results presented in reference [14]. However, note that the spacing-anticipation parameter leads to undesired effects during the deceleration; the trailing aircraft begin to decelerate earlier and earlier in order to match the anticipated speed of their targets. This ultimately leads to poorer spacing at the FAF, as shown in Figure 13 with spacing errors between -0.5 and -0.2 seconds, although spacing errors of this magnitude are not operationally significant.

D. Discussion of Results

The spacing errors and speed commands for the idealized conditions (no speed limitations) indicate that the time-history control law leads to string instabilities. Adding speed limitations to the control algorithm can lead to saturated speed commands and inefficient operations. Less-frequent speed commands do not improve performance in that maximum speed commands are not decreased along the string. Using a spacing-anticipation parameter to anticipate changes in the target aircraft's speed does yield a string-stable system; however, there are drawbacks in the performance when the target aircraft is decelerating.

These results show that there are several parameters that affect the overall system behavior to disturbances. When designing a speed-control algorithm for airborne spacing, significant attention should be paid to speed limitations and parameters that could improve performance in certain conditions. For example, the spacing-anticipation parameter does improve string performance; however, performance is less ideal when

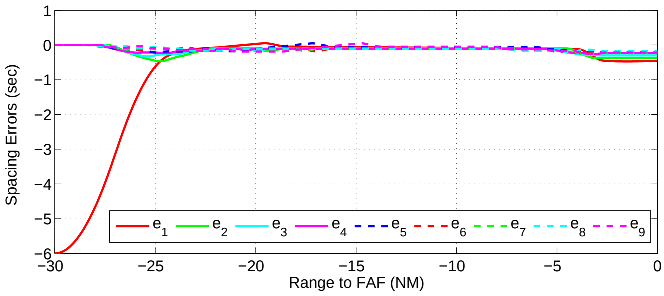

Fig. 13: Spacing errors as a function of range to FAF for a six-second initial spacing error between the first and second aircraft. The control law uses a spacing-anticipation parameter τ_{sa} = 10 seconds.

the target aircraft is decelerating.

Whereas it is difficult to prove string stability for all possible conditions, a practical implementation of airborne-spacing may be to limit string lengths. By limiting string lengths, the growth of spacing errors along the string will not result in speed saturation and inefficient or unsafe operations. In the design of an algorithm, the control gains can also be evaluated, and future research may show that proper gain scheduling can yield positive results for different types or magnitudes of disturbances.

The simulation results presented here also give insight into the initialization criteria for airborne-spacing operations. If a string-unstable control law is used, a large spacing error between two strings of aircraft will cause spacing errors to be propagated through the trailing string upon initializing airborne spacing. Better system performance may result by keeping the two strings uncoupled. Initialization criteria defined using the type of analysis presented here can serve as guidelines for air traffic controllers tasked with setting up airborne-spacing operations.

V. Conclusions

An airborne-spacing concept, which is a NextGen/SESAR application, has been described with a specific focus on string stability. A time-history speed control law was presented, and a simplified, closed-form string-stability analysis indicated that the time-history control law is not string stable. Whereas the simplified, closed-form analysis aided in understanding the behavior of an aircraft string, nonlinearities in the control algorithm were difficult to analyze in a closed-form manner. Simulation results were used to implement the speed-control algorithm with nonlinearities, such as speed limitations, quantized speed commands, and different speed command implementation rates.

Simulation results indicated that the nonlinearities in the control algorithm have a notable impact on the string behavior. Limiting speed changes may improve string performance depending upon the initial spacing errors. Imposing speed limitations prevented increasing speed commands along the string, but led to inefficient performance. The use of a spacing-anticipation parameter, as proposed by researchers at EUROCONTROL, led to a string-stable system, but there were drawbacks when the target aircraft was decelerating. The simulation results revealed several of the parameters that affect string performance. Additionally, these parameters should be evaluated for different flight conditions and types of disturbances in order to design an algorithm that yields overall good performance. However, a likely practical result of string-stability analysis is that aircraft strings should be limited to a certain number of aircraft to prevent disturbances from affecting a large number of aircraft. Research into the optimal number of aircraft in a string to maintain desired and efficient string performance will continue.

References

[1] B. Barmore, “Airborne precision spacing: A trajectory-based approach to improve terminal area operations,” in *Proceedings of the 25th Digital Avionics and Systems Conference*, Portland, Oregon, 2006.

[2] K. Krishnamurthy, B. Barmore, F. Bussink, L. Weitz, and L. Dahlene, “Fast-time evaluations of an airborne merging and spacing concept for terminal arrival operations,” in *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, San Francisco, California, 2005.

[3] T. S. Abbott, “Speed control law for precision terminal area in-trail self spacing,” NASA Langley Research Center, Hampton, VA, Tech. Rep., July 2002, NASA/TM-2002-211742.

[4] B. E. Barmore, T. S. Abbott, W. R. Capron, and B. T. Baxley, “Simulation results for airborne precision spacing along continuous descent arrivals,” in *Proceedings of the AIAA Aviation Technology, Integration, and Operations Conference*, 2008.

[5] “Cospace 2005 - ASAS sequencing and merging: Flight deck user requirements,” Eurocontrol Experimental Center, Bretigny-sur-Orge, France, Tech. Rep., January 2006.

[6] E. Hoffman, D. Ivanescu, C. Shaw, and K. Zeghal, “Analysis of constant time delay airborne spacing concepts between aircraft of mixed types in varying wind conditions,” in *Proceedings of the 5th USA/Europe Air Traffic Management R&D Seminar*, Budapest, Hungary, 2003.

[7] D. Ivanescu, C. Shaw, E. Hoffman, and K. Zeghal, “Towards performance requirements for airborne spacing - a sensitivity analysis of spacing accuracy,” in *Proceedings of the AIAA Aviation Technology, Integration, and Operations Conference*, 2006.

[8] B. E. Barmore, R. S. Bone, and W. J. Penhallegon, “Flight-deck merging and spacing operations,” *Air Traffic Control Quarterly*, vol. 17, no. 1, pp. 5–37, 2009.

[9] D. Swaroop and J. K. Hedrick, “Constant spacing strategies for platooning in automated highway systems,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 121, no. 3, pp. 462–470, 1999.

[10] D. Swaroop and K. R. Rajagopal, “A review of constant time headway policy for automatic vehicle following,” in *Proceedings of the 2001 IEEE Intelligent Transportation Systems Conference*, Oakland, CA, 2001.

[11] P. Seiler, A. Pant, and K. Hedrick, “Disturbance propagation in vehicle strings,” *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1835–1841, 2004.

[12] G. L. Slater, “Dynamics of self-spacing in a stream of in-trail aircraft,” in *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 2002.

[13] L. A. Weitz, “Decentralized, cooperative control for multivehicle systems: Design and stability analysis,” Ph.D. dissertation, Texas A&M University, College Station, Texas, 2009.

[14] D. Ivanescu, C. Shaw, K. Zeghal, and E. Hoffman, “Propagation of airborne spacing errors in merging traffic streams,” in *Proceedings of the 7th USA/Europe Air Traffic Management R&D Seminar*, 2007.

[15] W. Glover and J. Lygeros, “A multi-aircraft model for conflict detection and resolution algorithm evaluation,” IST-2001-32460 of European Commission, Tech. Rep., September 2004.

[16] R. Walter, *Digital Avionics Handbook: Elements, Software and Functions*, 2nd ed. CRC Press, 2007, ch. 20.

[17] “User manual for the base of aircraft data (BADA) revision 3.7,” Tech. Rep., March 2009, Eurocontrol Experimental Center Technical/Scientific Report No. 2009-003.