Optimal Route Decision with Weather Risk Hedging using a Geometric Model

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***Abstract*— Adverse weather is the dominant cause of delays in the national airspace system (NAS). Since the future weather condition is only predictable with certain accuracy, managing traffic in the weather affected airspace is a challenging task. The current Air Traffic Management (ATM) utilizes a combination of strategic planning based on deterministic model and reactive tactical control, often resulting in inefficient system utilization. To address such limitation, there has been growing interest in air transportation community to develop a routing decision model based on probabilistic weather. In the probabilistic air traffic management (PATM), decisions are made based on the stochastic weather information in the expected total cost sense. In this paper, we propose a geometric model to generate optimal route choice to hedge against weather risk. The geometric recourse model (GRM) is a strategic PATM model that incorporates route hedging and en-route recourse to respond to weather change. Hedged routes are routes other than nominal or detour path, and aircraft is re-routed to fly direct to the destination, or *recourse*, when the weather restricted airspace become flyable. Aircraft takes either the first recourse or the second recourse. The first recourse occurs when weather clears before aircraft reaches it flying on the initial route. The second recourse occurs when weather when aircraft is in at the weather region. There are two variations of GRM: Single Recourse Model (SRM) with first recourse only and Dual Recourse Model (DRM) with both first and second recourse. When the weather clearance time follows a uniform distribution, SRM becomes convex with optimal solution is either at the upper bound or interior. Convexity gives optimality conditions in a close form and analytic form of interior solution is approximated with marginal error. We prove that DRM has an important property such that when the maximum storm duration time is less than the flight time to the tip of the storm on detour path, it is always optimal to take nominal route. Numerical study shows a substantial cost saving from using geometric recourse model, especially with DRM. It also indicates the need to consider ground holding in combination of route hedging. This electronic document is a "live" template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document.** *(Abstract)*

Keywords-ATM; PATM; stochastic optimization; geometric model; risk hedging

I. INTRODUCTION

There is a growing interest in air traffic management (ATM) strategies that incorporate uncertainty in the national airspace system (NAS). Research in "probabilistic air traffic management" (PATM) seeks to guide decisions on ground-holding or otherwise modifying aircraft four-dimensional trajectories (4DTs) in order to minimize the expected cost, or to hedge against "worst case" scenarios in the next generation air transportation system.

This research studies the problem of developing a minimum-cost aircraft routing strategy when some weather condition inhibits the use of nominal route for an indefinite period. In conventional Air Traffic Management (ATM), two options are commonly considered in this situation; the flight is either held at the origin airports until the nominal route becomes flyable or rerouted to avoid the weather region. The choice between these options is based upon a deterministic and conservative characterization of future weather, often resulting in underutilized airspace and unnecessary delay if the weather clears early.

In this paper, we propose a geometric model to find an optimal route when the weather clearance time is stochastic. The route decision takes into account the probability distribution of storm clearance times, the possibility of route hedging, and recourse opportunities. When facing uncertain weather, there are two potential risks to hedge against: persistence risk and clearance risk. Persistence risk is the risk when we take an "optimistic" route and weather persists, resulting in unplanned re-routing and delay. Clearance risk is the risk when we take a "pessimistic" route and weather clears sooner, resulting unnecessary flight time. To mitigate these risks, we need to consider intermediate routing options that may not be optimal under either persistence or clearance, but hedge against either possibility. In doing so, we must consider how the route might be adjusted if the storm clears during the course of flight. We assume that the flight plan can be amended in such event so that the plane can go direct to the destination.

In our model, the routing decision is made based on four parameters; nominal route between origin and destination airport, storm location, storm size, and maximum storm duration time. The *optimistic* route is the nominal one while *pessimistic* route goes around the storm. A *hedged* route is one that is between the optimistic and the pessimistic ones. We use the term *recourse* for a change in a routing that results from the storm clearing. We consider two recourse possibilities. First, the storm may clear before the aircraft reaches it, so that it can be rerouted directly to its destination. This is called *first recourse*. The storm may instead persist beyond the time when the aircraft reaches it—so that the plane must turn and fly around it-- but clear before the tip of the storm is reached. The aircraft may then be rerouted direct to the destination; we refer to this as *second recourse*.

In our model, which we term the geometric recourse model (GRM), a triangle is drawn where the base is the nominal path between the origin and destination airport, and the vertex is the tip of the storm, which we assume to be a straight line perpendicular to the nominal path. We seek routes that minimize expected total flight cost, which in some cases are hedged routes. We consider two variations of geometric recourse model: the single recourse model (SRM) and dual recourse model (DRM). The SRM allows first recourse only, while the DRM allows both first and second recourse. Both SRM and DRM involve reroutes away from the weather region, while DRM includes reroutes in that region as well. The SRM is more conservative, while the DRM is more flexible and results in additional cost saving.

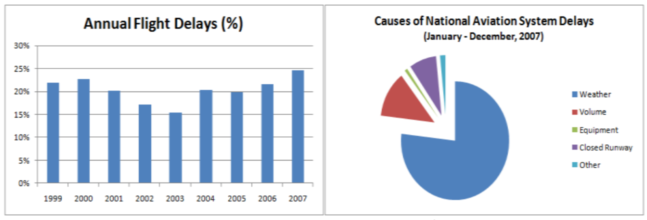
This paper introduces the concept of geometric recourse model and formulates nonlinear stochastic optimizations for the SRM and DRM. We assume that the storm clearance time follows a uniform distribution. With this assumption, we show that the SRM becomes convex, and find optimality conditions and the approximate analytic solution in closed form. We also find a condition that guarantees the nominal route to be optimal in DRM. Through numerical study, we compare the total expected flight cost and cost saving for optimal routes obtained from the SRM and DRM under a wide range of parameter values.

II. BACKGROUND

While traffic in the national airspace system has temporarily abated, its pre-recession level was approaching the capacity limit, with air travelers frequently experiencing flight delays and cancellations. Out of all causes of such delays, weather has been the most dominant one. According to the US Department of Transportation, air travelers experienced the worst flight delay in 2007 since year 2000, and weather accounted for more than 75% of these delays, as shown in Fig. 1.

In the event of adverse weather, one of the most widely used delay mitigation processes is the ground delay program (GDP). In a GDP, flights are held on the ground at the origin airport and assigned to new departure times based on available capacity at the destination airport. Although serving the purpose of handling the arrival capacity restrictions well, GDP are less well-suited for airspace capacity restrictions. Consequently, the Federal Aviation Administration (FAA) implemented the Airspace Flow Program (AFP) in June 2006. The purpose of AFP is to control the en-route traffic demand in regions of airspace that are capacity-constrained, most commonly as the result of severe weather.

Neither GDPs nor AFPs explicitly recognize that future weather is uncertain. As a result, when weather changes unexpectedly, a significant amount of reactive and tactical control is required, often resulting in inefficient system



Data Source: Bureau of Transportation Statistics

Figure 1. Annual Flight Delay and the Causes

utilization. The motivation of this research is to integrate probabilistic weather information into strategic planning to provide flexible and effective decision support in order to reduce losses from imperfect information about future weather.

III. LITERATURE REVIEWS

There have been numerous efforts to address weather-related disruptions in the air traffic management. Earlier traffic flow management models such as Bertsimas [1] and Goodhart [2], often have a deterministic setting. More recently, Nilim et al. [4] proposed a dynamic aircraft routing model with robust control. This paper adopted shortest-path algorithms in a grid structure, by discretizing time into stages when the routing decisions are made, and airspace as a two-dimensional grid. The weather condition in each potential storm region is assumed and modeled as a Markovian process with two states: 0 (No storm) and 1 (Storm). The transition matrix is estimated based on the historical weather forecasts. Optimization results show a promising improvement compared to flying around the storm without recourse. The method in their paper has a robust control algorithm that has a wide range of applications. In the air transportation system, however, the frequent routing adjustments entailed by this approach may place undue workload on controllers and pilots. Moreover, the Markovian assumption is of doubtful validity in the context of convective weather. Two of the goals in our study are to set up a model that has the flexibility to adopt a variety of probability distributions of storm clearance times, and to limit re-routing decision to a reasonable number.

Bertsimas et al. [3] proposed a two-stage optimization model based on a dynamic network flow approach. The authors set up a multi-aircraft optimization model minimizing the weather delay cost, based on a deterministic weather scenario. One important aspect of their study is that the cost function covers all phases of aircraft operation costs, such as fixed cost, ground holding cost, aircraft availability, and airborne cost. From the air traffic management perspective, it would be ideal to utilize both Ground Delay Program (GDP) and airborne rerouting to mitigate weather related disruptions, especially since ground delay is less costly than extra flight time. Here, we do not explicitly consider the ground delay option, but instead focus on the choice of routing for a given time of departure. The extension of the model to support choice among alternate departure times is discussed at the conclusion of this paper.

IV. GEOMETRIC RECOURSE MODEL (GRM)

A. *Geometric Recourse Model Concept*

Consider the problem of routing a single flight in the presence of a single storm. Given an origin and destination pair, assume there is a linear storm of known size blocking the direct route at a certain location. Using these five parameters- origin (O), destination (D), storm-route intersection (SL), and storm tip *ST*, construct a triangle *ODST*, where the nominal route is the base $\overline{OD}$ and storm size is the altitude $\overline{S_L S_T}$, as illustrated in Fig. 2. Note that while the storm has two tips, we choose the one nearer to SL, since this is the one that the aircraft would be routed around. Defining the unit of distance such that the aircraft cruises at a constant speed of 1, we refer to the base $\overline{OD}$ as the *nominal* route, the altitude $\overline{S_L S_T}$ as *front* of the storm and the vertex *ST* as the *tip* of the storm. Route $\overline{OS_T D}$, which goes around the storm, is called the *detour* route. Routes available to the aircraft include the nominal and detour routes, and those in between.

During the course of the flight, aircraft may be re-routed to fly direct to the destination when the storm clears; we refer to such route changes as *recourse*. Depending on the timing of storm clearance, there are three recourse possibilities as illustrated in Fig. 3: (a) recourse if the storm clears before the aircraft reaches it; (b) recourse at the storm front if the storm persists until after the aircraft reaches it, but clears as the aircraft flies along the storm front toward the tip; or (c) no recourse is possible because the storm persists until after the aircraft reaches the tip of the storm. We define the case (a) as the *first recourse*, the case (b) as the *second recourse*, and the case (c) as *no recourse*. Given the geometric setup, the objective is to find the route that minimizes expected total flight cost. Choosing a route is equivalent to choosing an angle between zero and the base angle $\angle S_T O S_L$. Although such a decision variable is intuitive, the resulting objective function involves complex trigonometric terms that make it difficult to analyze. Instead, we propose a ratio-based model in which complexity is reduced without loss of generality.

In the ratio-based model, the nominal path and weather parameters are expressed as ratios to the nominal path as illustrated in Fig. 4. We also introduce new decision variable *x*, which is the distance from the origin to the storm front. The ratio-based model is then formulated as follows.

1: length of nominal path between origin and destination.

β: ratio of storm size to the nominal path:

$$\beta = \frac{Storm\ Size}{Normial\ Path\ Length} > 0. \tag{1}$$

α: ratio of storm distance from origin along nominal route to the nominal route distance:

$$0 < \alpha = \frac{Storm\ Location}{Normial\ Path\ Length} < 1. \tag{2}$$

μ: random variable representing the storm clearance time on the chosen flight path, with probability density function $p(\mu)$.

x: ratio of distance to the storm along chosen route to nominal path length.

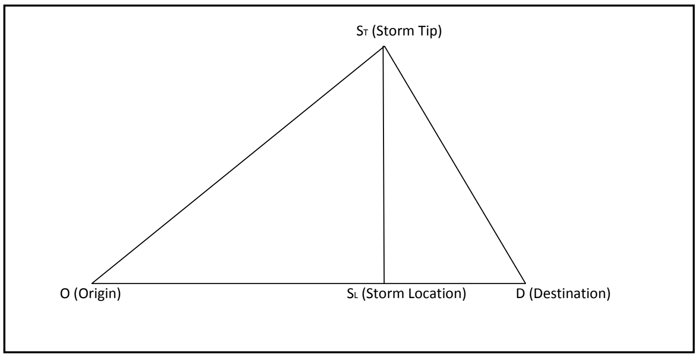


Figure 2. Geometric Model Concept

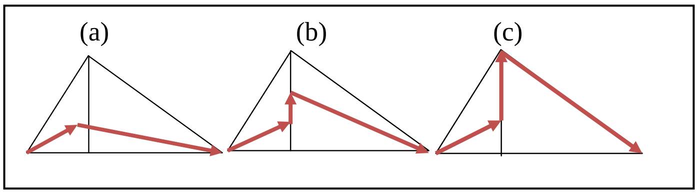


Figure 3. First, Second and No Recourse

(a) *First recourse*: Recourse before the storm front if the storm clears before the aircraft reaches the edge.

(b) *Second recourse*: Recourse in the storm region if the storm clears while the aircraft flies along the edge but before reaching the tip of the storm.

(c) *No Recourse:* If storm doesn't clear until the aircraft reaches its tip, then fly around the storm.

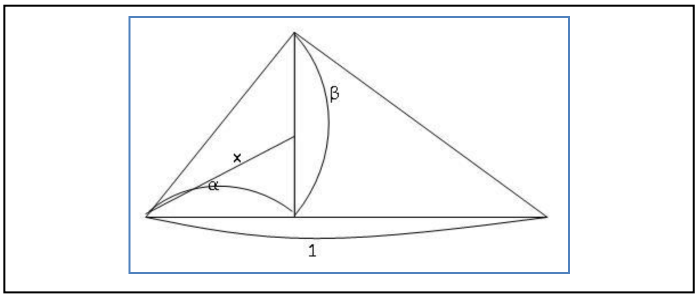


Figure 4. Ratio Based Geometric Model

We consider two variations of geometric recourse model – single recourse model (SRM) and dual recourse model (DRM). The dual recourse model is more flexible allowing both the first and the second recourse. The single recourse model is more conservative allowing only the first recourse. It is obvious that the single recourse model is an upper bound to the dual recourse model.

B. *Single Recourse Model (SRM)*

Single Recourse Model (SRM) is a geometric recourse model with first recourse only. The optimization model is formulated as follows.
Decision Variable: x
Objective Function:

$$min \int_0^x \left(\mu + \sqrt{1 + \mu^2 - 2\mu\frac{\alpha}{x}}\right) p(\mu)d\mu + \int_x^\infty \left(\mu + \sqrt{\left(x + \beta - \sqrt{x^2 - \alpha^2}\right)^2 + \sqrt{(1-\alpha)^2 + \beta^2}}\right) p(\mu)d\mu$$

s.t.

$$\alpha \le x \le \sqrt{\alpha^2+\beta^2}$$

, where

$$0<\alpha<1, \beta>0$$

In the objective function, the first integral is the expected total flight cost when first recourse is taken, and the second integral is the case when no recourse is possible. In the following chapter, we prove that SRM becomes convex and identify optimality conditions as well as approximated analytic solution when weather follows a uniform distribution.

C. Dual Recourse Model (DRM)

Dual recourse model allows recourse both before and at the storm region, providing most flexible environment. The optimization model formulation is as follows:

Decision Variable: x

Objective Function:

$$min \int_0^x \left(\mu+\sqrt{1+\mu^2-2\mu\frac{\alpha}{x}}\right)p(\mu)d\mu$$
$$+\int_x^{x+\beta-\sqrt{x^2-\alpha^2}}\Bigg(\mu$$
$$+\sqrt{\left(\mu-x+\sqrt{x^2-\alpha^2}\right)^2+(1-\alpha)^2}\Bigg)p(\mu)d\mu$$
$$+\int_{x+\beta-\sqrt{x^2-\alpha^2}}^{\infty}\Bigg(\mu$$
$$+\sqrt{\left(x+\beta-\sqrt{x^2-\alpha^2}\right)^2+\sqrt{(1-\alpha)^2+\beta^2}}\Bigg)p(\mu)d\mu$$

s.t.

$$\alpha \le x \le \sqrt{\alpha^2+\beta^2}$$

, where

$$0<\alpha<1, 0<\beta$$

In the objective function, the first integral is the expected total flight cost when first recourse is taken, the second integral is the case when the second recourse is taken, and the third integral is the case when no recourse was possible. DRM provides higher flexibility than SRM and an upper bound to SRM.

In this chapter, we introduced the concept and two variations of geometric recourse model (GRM). In the next chapter, we present analytic study to find optimality conditions and an approximation of optimal solution in a closed form. A discussion on several important properties of GRM is followed. Performances of two models are compared and discussed in the following numerical analysis chapter.

V. ANALYTIC STUDY

A. Uniform Weather Distribution

We assume that the weather (storm) clearance time follows a uniform distribution ranging between 0 and *T*, or μ~Uniform $[0,T]$. Forecast on convective activities in the airspace is included in several weather forecast products published by National Oceanic and Atmospheric Administration (NOAA)'s Storm Prediction Center (SPC). One of the widely used forecast both in practice and in research is the convective outlook watch. According to SPC, they publish roughly 1,000 watches each year to address possible severe weather condition in the next few hours, and each convective activity is associated with a probability. Uniform distribution can utilize the single probability provided in the forecast, and be easily updated with new information as a new watch or warning is published.

With the uniform distribution assumption, we now have an additional parameter T, which is the latest possible time that storm will remain, or maximum storm duration time. Note that in the ratio-based optimization model, T is the actual maximum storm duration time divided by nominal path length. With the introduction of T, it is clear that we have a trivial solution $x^*=\alpha,\ \ if\ T\le\alpha$.

B. Single Recourse Model (SRM)

1) Convex Optimization and Optimiality Conditions

Given μ~Uniform$[0,T]$, let $f_s(x)$be the expected total cost function of SRM.

Theorem 1. $f_s(x)$ is a continuous convex function over $x\in\left[\alpha,\sqrt{\alpha^2+\beta^2}\right]$ and $f_s'(\alpha)<0$.[1]

Theorem 1 gives optimality condition of SRM as follows.

$$x^*=\begin{cases}\in\left(\alpha,\sqrt{\alpha^2+\beta^2}\right), & f_s'\left(\sqrt{\alpha^2+\beta^2}\right)>0\\ \sqrt{\alpha^2+\beta^2}, & f_s'\left(\sqrt{\alpha^2+\beta^2}\right)\le 0\end{cases} \qquad (3)$$

In other words, if the gradient at the upper bound is positive, then there exists an interior solution. Otherwise, the upper bound of x is the optimal solution. Interior solution is equivalent to taking a route inside the triangle and represents the case when hedging is valuable against weather risk.

Rearranging parameters yields that the first condition in Eq. 2 $f_s'\left(\sqrt{\alpha^2+\beta^2}\right)>0$ is equivalent to $T<g_s(\alpha,\beta)$. In Fig. 5, the condition $T<g_s(\alpha,\beta)$ is shown as a contour map in the α-β plane. In the contour map, each contour line corresponds to a value of $g_s(\alpha,\beta)$[2]. Using this map, one can determine whether there is an interior solution or not, once the weather parameters are known, For example, if *α*=0.6 and *β*=0.4, there is an interior solution when *T*=1, and no interior solution when *T*=2. Note that as *T* gets larger, it is less and less likely to have an interior solution, which matches our intuition. The contour map is a valuable decision reference to determine whether hedging is worth considering or not without solving optimization.

[1] See Appendix A.1. for proof
[2] See Appendix A.2. for complete formula

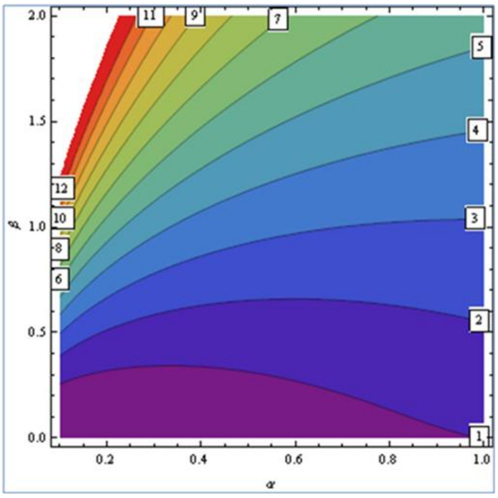


Figure 5. Condition of Interior Solution of SRM
Once the weather parameters are known, one can refer to this plot to decide whether it is worthwhile to find an optimal angle below the maximum angle, or it would be best to simply fly around the storm. Number in the white squares is the value of $g_s(\alpha,\beta)$. For example, with α=0.4 and β=0.5, it is optimal to fly around the storm if T=3. On the other hand, if T=2, then there is an interior solution. Note that as T gets larger, it is less likely to have an interior solution.

2) Analytic Solution Approximation using Taylor Series

The optimization problem doesn't have an analytic solution. We apply Taylor series expansion to approximate our objective function as a polynomial of degree 2 in x, around the middle point of its domain $\frac{\alpha+\sqrt{\alpha^2+\beta^2}}{2}$.

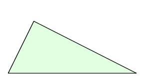
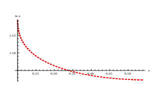
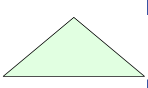
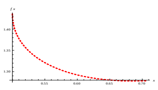
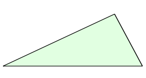
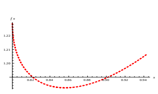
The Taylor series approximation, which we call $f_{ts}(x)$, is quite complex, but we're only interested in whether the minimizing x falls inside the domain or not. In other words,

$$x^* = \begin{cases} \alpha & ,if\ T \le \alpha \\ \approx -\frac{Coefficient(f_{ts},1)}{2Coefficient(f_{ts},2)} & ,if\ \alpha < T < g_s(\alpha,\beta) \\ \sqrt{\alpha^2+\beta^2} & ,if\ T \ge g_s(\alpha,\beta) \end{cases}$$

, where $Coefficient(f,n)$ denotes the coefficient of x^n of polynomial function f.

We tested our model with different weather parameters and approximation error was less than 1% in most cases. A sample results are shown in the Table I.

TABLE I. ANALYTIC SOLUTION APPROXIMIATION AND ERRORS

	Geometry	E(Total Cost)	Int. sol	x*	x*ts	error
α=0.2 β=0.5 T=2.5			N	0.53	0.53	0%
α=0.5 β=0.5 T=1.1			Y	0.66	0.64	2%
α=0.8 β=0.5 T=1.1			Y	0.85	0.85	0.3%

C. Dual Recourse Model (DRM)

Dual Recourse Model (DRM) is neither always convex nor concave and its properties are best addressed in our numerical analysis in the following chapter. However, DRM has an important property that when the maximum storm duration time is relatively short, taking nominal path is always optimal.

Theorem 2. In the dual recourse model, $x^* = \alpha\ if\ 0 < T \le \sqrt{\alpha^2+\beta^2}$.[3]

In other words, if the maximum storm duration time is less than the time to fly to the tip of the storm on detour path, then it is always optimal to fly on the nominal path. Note that theorem 2 holds regardless of the weather probability distribution, and it yields critical cost saving opportunity without even considering route hedging.

One of our primary interests is to measure the improvement from adopting DRM or SRM. Since DRM is an upper bound to SRM, DRM guarantees less cost than SRM. Likewise SRM guarantees less cost than taking detour path. In the next chapter, we discuss various performance metrics based on numerical analysis.

VI. NUMERICAL STUDY

In numerical study, a 3-D grid structure is created in α-β-T plane with each grid being a cube with side of 0.05, or 5% of the nominal path. We also set reasonable limit to storm size and maximum storm duration time as 2 and 4 respectively. Therefore, $\alpha \in [0.05,0.95], \beta \in [0.05,2], T \in [0.05,4]$. Note than when α=0 or α=1, the storm is located either at the origin or the destination airport, in which case ground delay program works best.

In numerical analysis, we consider three cases as detailed below. The available options for each scenario are summarized in Table II.

- *Baseline*: take detour path and whenever storm clears before aircraft reaches the storm tip, fly direct to the destination
- *SRM*: utilize SRM with first recourse option only
- *DRM*: utilize DRM with both first and second recourse options.

It is clear that the baseline case is an upper bound of SRM, which is then an upper bound of DRM. For each (α,β,T), we find solutions for these three cases to generate performance metrics.

In the following section, we compare optimal cost and cost saving of those three cases. Note that if $T \le \alpha$, all three cases yield the same solution $x^* = \alpha$, and we exclude these trivial cases from our analysis.

TABLE II. OPERATION OPTIONS FOR BASELINE, SRM AND DRM

Model	Hedging	First Recourse	Second Recourse
Baseline	X	O	X
SRM	O	O	X
DRM	O	O	O

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[3] See Appendix A.3. for proof

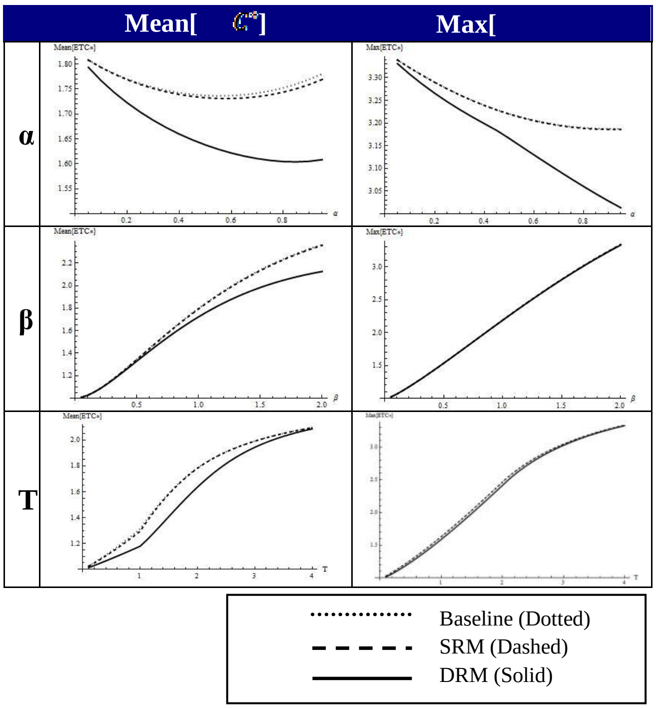
A. *Minimum Expected Total Cost (ETC*)*

To study the optimal cost *(ETC*)* with respect to each weather parameter α, β and T, we find the minimum, average and maximum of all ETC*s, when one of the parameters is fixed at a certain value. For example, to analyze ETC* with respect to α, first set α=0.05 and collect all ETC*s of baseline, SRM and DRM respectively and find minimum, average and maximum of ETC*s for three cases. Continue for α=0.1, 0.15, … 0.95. We repeat the same process for β and T, and the result is summarized in Table III.

$\alpha - Mean[ETC^*]$ plot shows how the average optimal cost changes with respect to α. We can see that on average, $ETC^*$of baseline and SRM is nearly the same and tends to reach its minimum when α is near 0.6. In other words, there is little difference in the average performance between baseline and SRM, while both of them performs best when α is around 0.6. As expected, DRM always performs better than baseline and SRM, and ETC^* gradually decreases as α increase until α is almost 1. We also observe for storm located near the destination, DRM performs much better than SRM or baseline. $\alpha - Max[ETC^*]$ plot shows the worst-case performances. We can see that all three cases show little difference when storm is very near the origin and optimal cost gradually decreases as storm moves toward the destination, especially for DRM.

In the β plots, we find that ETC^* increases as β increases, while average performance of DRM is better than the baseline or SRM. It matches our intuition since as the storm gets larger, second recourse option in DRM will pay off, although in the worst case when no recourse is possible, all three models will perform the same.

TABLE III. MEAN AND MAXIMUM OF WITH RESPECT TO WEATHER PARAMETERS



In summary, there is little difference between SRM and baseline case, while DRM sometimes performs substantially better. On average, expected total cost increases with decreasing rate as the storm size and the maximum duration time increases. On the other hand, expected total cost is convex with respect to the storm location, as it decreases up to a minimum point then increases. We observe that DRM reduces weather risk further with a storm located near the destination airport. This is also true when storm size is large. There is a range of T where DRM achieves substantially less expected total cost, although such advantage disappears as T becomes very large.

B. *Cost Saving*

Let's define cost saving of SRM and DRM as follows.

$$S(SRM) = 1 - \frac{ETC^*[SRM]}{ETC^*[Baseline]} \quad (4)$$

$$S(DRM) = 1 - \frac{ETC^*[DRM]}{ETC^*[SRM]} \quad (5)$$

$, where\ ETC^*[\cdot]$ is the minimum expected total cost of the selected model.

The cumulative distribution functions of S(SRM) and S(DRM) are shown in Fig. 6. With SRM, nearly 90% of cases have less than 1% saving compared to the baseline case, and more than 99% cases has less than 5% saving with the largest saving close to 6%. With DRM, more than 32% has savings larger than 5% with largest saving close to 30%. In fact, about 20% shows significant saving larger than 10%.

In Table IV, the average, minimum and maximum cost savings are plotted with respect to each parameter. Maximum and average savings of both α-S(SRM) and α-S(DRM) are monotonic increasing functions. The convex shape of α-S(SRM) plot suggests that SRM works best with storms very near the destination, while the concave shape of α-S(DRM) plot suggests that DRM works well with wide range of storms as well as those very near the destination. We also observe the average cost saving of SRM is negligible.

In β-S(SRM) plot, cost saving increases until β reaches near 0.5 then stays flat afterward. It is intuitive that without second recourse option, SRM is limited to hedge the risk of larger

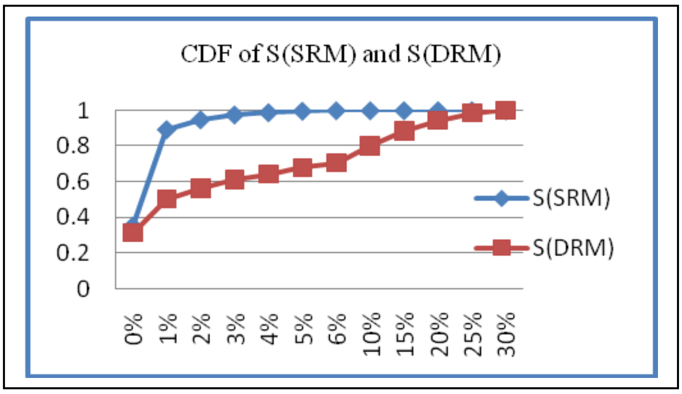
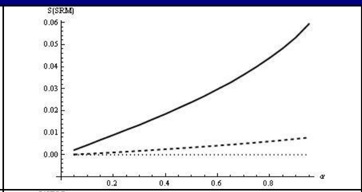
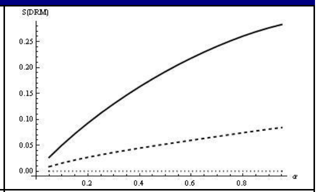
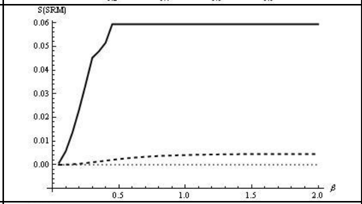
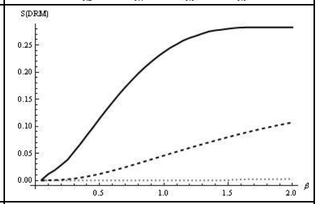
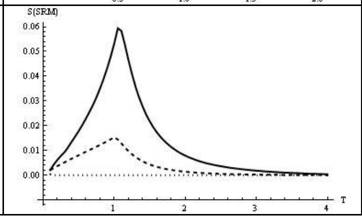
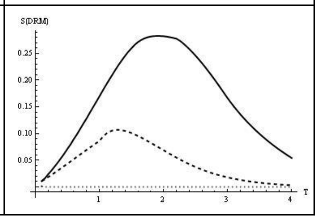


Figure 6. Cumulative Distribution Function of Cost Saving of SRM (S(SRM)) and Cost Saving of DRM (S(DRM))

storm. Compared to β-S(SRM), β-S(DRM) plot shows wide range of β that DRM saves meaningful cost, which coincides with our finding in the previous section. More importantly, the average cost saving maintains increasing trend even after maximum saving plateaus, which suggests that DRM is effective in reducing the risk of large-sized storms.

TABLE IV. MEAN AND MAXIMUM OF S(SRM) AND S(DRM) WITH RESPECT TO WEATHER PARAMETERS

	S(SRM)	S(DRM)
α		
β		
T		

Minimum (Dotted)
Average (Dashed)
Maximum (Solid)

In both T-S(SRM) and T-S(DRM) plots, there are ranges of T showing the peak savings. The largest saving of SRM is close to 6% when T is between 1 and 1.2, and it is close to 30% when T is between 1.9 and 2.1 for DRM. There is an interesting observation when it comes to average cost saving. For SRM, maximum average cost saving of T-S(SRM) is much higher than those of α-S(SRM) and β-S(SRM). Although it appears that the average cost saving of SRM is negligible in α-S(SRM) plot, there are cases when it becomes meaningful when T is in a certain range. We make similar observation for DRM as well. Such observations suggest that performance of these geometric recourse models is more dependent to the maximum storm duration than the location or the size of storm. It also suggests to consider ground delay in combination with route hedging, which essentially reduces the maximum storm duration time. Another important observation is the large gap between the average and maximum cost saving in general, which indicates that there exist certain combinations of α, β and T that these models show true advantage.

VII. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we propose a geometric model to generate optimal route to hedge against weather risk in the airspace. The Geometric Recourse Model (GRM) enables weather risk hedging by allowing aircraft to take a route other than the nominal or the detour path. In addition, aircraft is re-routed to destination or recourse as soon as the weather restricted airspace becomes flyable again. There are two recourse possibilities depending on the timing of weather clearance. The first recourse option is the case when the weather clears before aircraft reaches its front, and the second recourse option is the case when weather persists when aircraft reaches it then clears while the aircraft is still in the weather region.

We consider two geometric recourse models; Single Recourse Model (SRM) with the first recourse option only and Dual Recourse Model (DRM) with both the first and the second recourse option. In SRM, the optimization model becomes convex when the weather probability follows a uniform distribution, and optimal solution occurs either in the interior or at the upper bound. Convexity gives optimality conditions closed forms and optimal solution is approximated with Taylor series with marginal error. DRM has an important property that it is always optimal take the nominal path when the maximum storm duration time is less than the time to fly to the tip of the storm.

In numerical study, a 3D grid structure is created in α-β-T plane, where α is the location, β is the size and T is the maximum duration time of the storm, and both models are solve for each tuple in the grid. Analyzing the optimal costs and cost savings leads us to conclusions as follows.

- SRM works best with storms very near the destination and relatively in small size.
- DRM works well with wide range of storm location and larger storms.
- Cost saving distributions show that nearly 90% of cases we tested have less than 1% saving with SRM with largest possible saving close to 6%. On the other hand, almost 30% of all cases have larger than 10% saving with DRM with the largest saving reaching 30%.
- Both models show peak cost savings for T in a certain range. The maximum average cost saving is also higher for those T values, compared to the maximum average saving with respect to α and β. These observations suggest that the performance of our models is more sensitive to the maximum storm duration time than other two parameters, which gives us a strong motivation to consider ground delay in combination with route hedging, especially with storms expected to last longer.

As an immediate follow-up study, we plan to research the value of hedging with various probability distributions. We also plan to study ground-airborne hybrid model, where ground delay is another decision factor in addition to route choice.

ACKNOWLEDGMENT

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Appendix A

A.1.

Theorem 1. Given μ~Uniform[0, T], SRM becomes convex with negative gradient at the lower bound.

Proof.

Let $f_s(x)$ be the expected total cost function of SRM, where $x \in \left[\alpha, \sqrt{\alpha^2+\beta^2}\right]$.

Given μ~Uniform[0, T],

$$f_s(x) = \int_0^x \left(\mu + \sqrt{1+\mu^2-2\mu\frac{\alpha}{x}}\right)\frac{1}{T}d\mu + \int_x^T \left(\mu + \sqrt{\left(x+\beta-\sqrt{x^2-\alpha^2}\right)^2 + \sqrt{(1-\alpha)^2+\beta^2}}\right)\frac{1}{T}d\mu$$

, which gives

$$lim_{x\to\alpha} f_s(x) = f(\alpha).$$

We also have

$$f'_s(\alpha) = lim_{h\to 0}\frac{f_s(\alpha+h)-f_s(\alpha)}{h} = \infty\frac{Sign\left[(-1+\alpha)\left(-1+\sqrt{(-1+\alpha)^2+\alpha}\right)\right]}{Sign\,[T]} < 0.$$

Now, let $f_{s1}(x) = \left(\mu + \sqrt{1+\mu^2-2\mu\frac{\alpha}{x}}\right)\frac{1}{T}$ and $f_{s2}(x) = \left(\mu + \sqrt{\left(x+\beta-\sqrt{x^2-\alpha^2}\right)^2 + \sqrt{(1-\alpha)^2+\beta^2}}\right)\frac{1}{T}$.

It is clear that both $f_{s1}(x)$ and $f_{s2}(x)$ are monotonic increasing function in their respective domain. Therefore, integrals of $f_{s1}(x)$ and $f_{s2}(x)$ are convex, which yields that $f_s(x)$ is convex. (Q.E.D)

A.2.

$$\begin{aligned}
g_s(\alpha,\beta) = {}&((\alpha^2+\beta^2)^2\big(\alpha^5+\alpha^4(-2+\beta)+\beta^3+\beta^5+\alpha^3(1-3\beta+2\beta^2)+\alpha^2\beta(3-2\beta+2\beta^2)\\
&+\alpha\beta(-1+\beta-3\beta^2+\beta^3)\big)\\
&-\sqrt{\alpha^2+\beta^2}\big(\alpha^8+\alpha^7(-2+\beta)+3\alpha^5(-1+\beta)^2\beta+3\alpha^3(-1+\beta)^2\beta^3+\beta^6+\beta^8+\alpha^6(1-3\beta+4\beta^2)\\
&+\alpha^2\beta^3(1+3\beta-3\beta^2+4\beta^3)+\alpha^4\beta(-1+3\beta-6\beta^2+6\beta^3)+\alpha\beta^3(-1-2\beta^3+\beta^4)\big)\\
&-\sqrt{1-2\alpha+\alpha^2+\beta^2}\big(\alpha^8+\alpha^7(-1+\beta)+3\alpha^3(-1+\beta)\beta^4+\beta^8+2\alpha^6\beta(-1+2\beta)\\
&+2\alpha^2\beta^5(-1+2\beta)+2\alpha^4\beta^3(-2+3\beta)+\alpha\beta^5(-1-\beta+\beta^2)+\alpha^5\beta(1-3\beta+3\beta^2)\big)\\
&+\sqrt{-2\alpha^3+\alpha^4+\beta^2-2\alpha\beta^2+\beta^4+\alpha^2(1+2\beta^2)}\big(\alpha^7+\alpha^6(-1+\beta)+\beta^7+\alpha^3\beta^3(-4+3\beta)\\
&+\alpha^5\beta(-2+3\beta)+\alpha^4\beta(1-2\beta+3\beta^2)+\alpha^2\beta^3(1-\beta+3\beta^2)+\alpha\beta^3(-1-2\beta^2+\beta^3)\big)\\
&+\alpha^2\beta\Bigg(\alpha^4-\alpha^3\left(2+\sqrt{\alpha^2+\beta^2}+\sqrt{1-2\alpha+\alpha^2+\beta^2}\right)\\
&+\beta^2\left(1+\beta^2+\sqrt{\alpha^2-2\alpha^3+\alpha^4+\beta^2-2\alpha\beta^2+2\alpha^2\beta^2+\beta^4}\right)\\
&+\alpha^2\left(1+2\beta^2+2\sqrt{\alpha^2+\beta^2}+\sqrt{1-2\alpha+\alpha^2+\beta^2}+\sqrt{\alpha^2-2\alpha^3+\alpha^4+\beta^2-2\alpha\beta^2+2\alpha^2\beta^2+\beta^4}\right)\\
&-\alpha\Bigg(\sqrt{\alpha^2+\beta^2}+\sqrt{\alpha^2-2\alpha^3+\alpha^4+\beta^2-2\alpha\beta^2+2\alpha^2\beta^2+\beta^4}\\
&+\beta^2\left(2+\sqrt{\alpha^2+\beta^2}+\sqrt{1-2\alpha+\alpha^2+\beta^2}\right)\Bigg)\Bigg)\left(Log\left[-\alpha+\sqrt{\alpha^2+\beta^2}\right]\right.\\
&\left.-Log\left[-\alpha+\alpha^2+\beta^2+\sqrt{(\alpha^2+\beta^2)(1-2\alpha+\alpha^2+\beta^2)}\right]\right))/(\alpha^2+\beta^2)\sqrt{1-2\alpha+\alpha^2+\beta^2}(-\alpha+\alpha^2\\
&+\beta^2+\sqrt{(\alpha^2+\beta^2)(1-2\alpha+\alpha^2+\beta^2)})(\alpha^3+\alpha^2(\beta-\sqrt{\alpha^2+\beta^2})+\alpha\beta(\beta-\sqrt{\alpha^2+\beta^2})+\beta^2(\beta\\
&-\sqrt{\alpha^2+\beta^2}))
\end{aligned}$$

A.3.

Theorem 2. In DRM, $x^* = \alpha\ if\ 0 < T \leq \sqrt{\alpha^2+\beta^2}$.

Proof.

It is trivial that

$$x^* = \alpha, \text{ when } T \leq \alpha. \tag{1}$$

If $\alpha < T \leq \sqrt{\alpha^2 + \beta^2}$, the objective function $f_d(x)$ is as follows.

$$f_d(x) = \int_0^x \left(\mu + \sqrt{\mu^2 + 1 - 2\mu\frac{\alpha}{x}}\right) p(\mu)\ d\mu + \int_x^T \left(\mu + \sqrt{(1-\alpha)^2 + \left(\mu - x + \sqrt{x^2 - \alpha^2}\right)^2}\right) p(\mu)\ d\mu \qquad (2)$$

Then,

$$f_d(x) - f_d(\alpha) =$$
$$\int_0^\alpha \left(\sqrt{\mu^2 + 1 - 2\mu\frac{\alpha}{x}} - \sqrt{\mu^2 + 1 - 2\mu}\right) p(\mu)\ d\mu + \int_\alpha^x \left(\sqrt{\mu^2 + 1 - 2\mu\frac{\alpha}{x}} - \sqrt{(1-\alpha)^2 + (\mu-\alpha)^2}\right) p(\mu)\ d\mu +$$
$$\int_x^T \left(\sqrt{(1-\alpha)^2 + \left(\mu - x + \sqrt{x^2 - \alpha^2}\right)^2} - \sqrt{(1-\alpha)^2 + (\mu-\alpha)^2}\right) p(\mu)\ d\mu. \qquad (3)$$

To show $f_d(x) - f_d(\alpha) > 0, \forall x \in (\alpha, T]$, we show that each integrand in (3) is positive.

It is trivial that $\sqrt{\mu^2 + 1 - 2\mu\frac{\alpha}{x}} - \sqrt{\mu^2 + 1 - 2\mu}$. (4)

Since $\left(\mu^2 + 1 - 2\mu\frac{\alpha}{x}\right) - ((1-\alpha)^2 + (\mu-\alpha)^2) > 0$, where $\alpha < \mu \leq x$, we have

$$\sqrt{\mu^2 + 1 - 2\mu\frac{\alpha}{x}} - \sqrt{(1-\alpha)^2 + (\mu-\alpha)^2} > 0 \qquad (5)$$

Similarly, $\sqrt{(1-\alpha)^2 + \left(\mu - x + \sqrt{x^2 - \alpha^2}\right)^2} - \sqrt{(1-\alpha)^2 + (\mu-\alpha)^2} > 0$, where $x < \mu \leq T$. (6)

From (4), (5) and (6), we have $f_d(x) - f_d(\alpha) > 0, \forall x \in (\alpha, T]$. Therefore,

$$x^* = \alpha \text{ if } 0 < T \leq \sqrt{\alpha^2 + \beta^2}. \qquad \text{(Q.E.D)}$$