Predicting Controller Communication Time for Capacity Estimation

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***Abstract*—We consider the complexity and controller workload problem common to air traffic management. Expanding upon previous works that correlate controller communications to workload and complexity, a stochastic model is developed to determine the distribution of free-time available to an air traffic controller managing a sector. The resulting model aims to serve as a predictive tool for rapidly determining future workload/complexity of air traffic by considering free-time as a metric. Additionally, a model for estimating the distribution of the number of conflicts is established to be used in conjunction with the controller communication model.**

I. Introduction

The projected growth in air traffic demand over the next twenty years is likely to generate traffic that will exceed the control capacity of air traffic controllers. Both the United States Federal Aviation Administration (FAA), and EUROCONTROL, recognize the need to predict air traffic demands for enroute sectors, and plan for staffing requirements for tactical controller positions. Furthermore it is expected that deconflicting traffic, especially at the tactical level, will require advanced decision support tools to ensure robust levels of safety [1], [2]. Consequently there has been significant investment in the development of workload metrics to evaluate when and where capacity issues may lead to safety concerns. The approach presented here seeks to provide an objective estimate of the probability of sector overload. Therefore it will help provide a quantitative risk analysis to help respond more accurately to capacity issues. This study takes a statistical approach to communication capabilities based on analysis of airspace geometry and air traffic flow distributions. The work builds upon prior research on controller workload, complexity measures, and their relation to communication activity between pilots and controllers.

Controller workload is defined by Stein as "the amount of effort, both physical and psychological, expended in response to system demands (taskload) and also in accordance with the operators internal standard of performance" [3]. The effective controller capacity limit to properly handle air traffic directly depends on the actual controller workload [4]. Unfortunately, controller workload can only be measured subjectively and depends on individual controllers capacities and perception. Historically, determining a sector limit capacity has relied on a simple metric: the number of aircraft present in a sector. This value is established by the Monitor Alert Parameter (MAP). The MAP value is appropriate for considering nominal traffic patterns, however, whenever system dynamics are present (introduction of weather, or changing traffic patterns), MAP values no longer accurately represent sector capacity - and often times lead to congestion, or conversely, under-utilization of the airspace.

Prior work has shown that subjective controller workload can be evaluated through objective metrics, such as complexity measures [5], [6], [7] and radio communication times [8], [9]. For instance, complexity measures reflect that traffic patterns consisting of multiple crossings and altitude changes where aircraft are likely to conflict will tend to result in greater workloads for managing traffic, than aircraft on spatially separated trajectories passing through a sector. A number of factors used to evaluate complexity are based on airspace and air traffic geometry, they may include aircraft density, potential conflicts, number of hand-offs, heading and speed variation between aircraft, aircraft separation distances, and presence of weather [10], [7], [11], [12], [13], [14].

Similarly, radio communication time has been considered as an objective metric to evaluate controller workload. [15] conducted a series of experiments which concluded that realistic radio activities can be used to provide objective measures of workload. Additionally, another study demonstrated the high correlation ($r = .88$) between communication duration and controller workload, thereby effectively validating communication time as another workload measure [8]. Additional studies have also focused on the number and amount of communications [16], [17]. More recently, research has suggested that routine Air Traffic Control communication events provided a good estimate of controller workload [9]. While a detailed analysis of the different type of communication events provided accurate estimates, they also concluded that the total number and duration of communication events were significantly correlated with controller workload.

This article analyzes airspace flow configuration and arrival distribution probabilities to estimate sector overcapacity. The model developed estimates the probability distribution of communication times, which have been proven to reflect controller workload, thereby providing the theoretical construct of the study. This model is based on physical considerations, and therefore intrinsically includes factors used to derive complexity measures. One of the advantages of this method is that it yields a probability of error or defect; in this case, the probability that the time required to manage the airspace is beyond the allowed time interval. This is a standard criteria in operations research and may be easier to use than the subjective controller workload to assess potential air traffic safety issues.

The remainder of the paper deals with the formulation and development of a communications model and probability of proximity model to determine distributions of controller communication times. The formulation takes advantage of aggregate flow characteristics to generate analytical results that can be calculated in real-time. The major advancement of this paper is the introduction of a predictive communication distribution based on estimated traffic flow into a sector. Furthermore, the model can be adjusted in real-time for dynamic analysis of any enroute airspace.

In Section II, we provide a general description of the problem. In Section III, we describe the general mathematical process describing controller communication. In Section IV, an enroute sector is analyzed, resulting in a mathematical model describing the distribution properties for the minimum standard communications required to manage traffic. Next in Section V, a proximity/conflict model is presented to consider increasingly more critical communication requirements. Then, in Section VI a brief analysis of the model is presented, and in Section VII, our conclusions.

II. Problem Description

Consider a set of aircraft flows, $\mathscr{F}_1 \dots \mathscr{F}_n$ traversing a sector, as shown in Figure 1. The majority of flows consist of commonly utilized jet routes that exist in $\mathbb{R}^3$; such dominant aircraft flows may consist of planer tracks, or trajectories that include ascending or descending in altitude. Furthermore, tracks may consist of a series intersections, merge points, and splits. In general, each flow is defined according to its entrance and exit locations, altitude, and designated way points. The aircraft arrivals into each flow are shown to arrive according to a stochastic process in Ssection IV.

A communications model will be developed according to real-world operations. At a minimum, each aircraft is communicated with at least twice by the managing air traffic controller, once to acknowledge the aircraft as enters the sector, and again when the aircraft is leaving the airspace to communicate a frequency change as part of the hand-off to the next sector. Additionally, pilots will acknowledge the air traffic controller. Depending on weather conditions (including turbulence), additional messages may be passed to pilots. In response to such messages, pilots will typically request clearance to fly at another flight level or to propose a new route, to which controllers will permit or suggest another option. Another prevalent communication typically occurs when an airspace is congested, and there is the potential for conflict or proximity. Aircraft are considered to be in proximity when there is sufficent concern for air traffic controllers to check for conflict and issue resolution commands. When conflicts are present, air traffic controllers must determine safe routes of passage for all aircraft, and communicate these to each pilot. For this process to occur in a safe manner there must be sufficient time for the controller to gain situational awareness, determine a course for the aircraft, dictate any resolution commands, and then monitor progress of the aircraft to ensure implementation of the commands.

For each event (arrival, departure, conflict) there will be an associated stochastic model describing that process. Corresponding to each event is also a time cost. For example, each aircraft entering from $\mathscr{F}_i$ will be acknowledged by the controller through a spoken message. That message will then be followed by a pilots response. As such, the total communication time required for the interaction will be given by the random variable $\mathbf{T}_{i,a}$, where a denotes the arrival process and i is the flow. Similarly, each other event will have a corresponding time. For the case of conflict communications, this is in itself a random process. Each aircraft entering the sector will initially be conflict-free. However, due to traffic configurations a fraction of aircraft entering into the sector will require conflict resolution and communication. While behavior of air traffic controllers varies depending on the congestion within the sector, for the current model presented, the time required to generate a safe resolution and to communicate the action will be considered to be fixed.

The input to the problem will be a description of the total aircraft arrival and departure rate into the airspace, λ_a and λ_d, as well as a description of the traffic probability distributed among the flows, $[p_1, \dots, p_n]$, and any distinctive features of the flow trajectories (crossing angles, altitude changes, etc). The output will be a probability distribution for all required standard communications, i.e. the total communication time, $\mathbf{T}_s$, corresponding to the summation of each communication time required.

The total communication time, $\mathbf{T}_s$, will be considered over a 1 minute period. A time interval of 1 minute is selected because it serves as a fundamental building block of controlling aircraft. Communication with aircraft entering a sector can be delayed for a short time period, as long as no immediate conflicts are present. Following a 1 minute communication, aircraft typically traverse between 5-10NM, so conflicts are unlikely to be immediate. Additionally, because of the short time period, each window will only allow for a single aircraft to arrive at an entrance at a time, which will allow for modeling the arrival process as a binomial process, thereby allowing for the counting process to exist over a smaller space.

III. Total communication time and application

The model proposed considers the total communication time required by an air traffic controller to minimally manage traffic. Communication time of the controller is segmented according to task: acknowledgement for any aircraft entering the center, clearance and requests for any aircraft requiring altitude or other trajectory changes, and notice to departing aircraft of the frequency in the next airspace. There are also additional communications that may occur: courtesy statements, advisories (weather, traffic, etc), and read-backs of commands.

Each communication type is associated with an interval of time, during which no other tasks or events can occur (mental or vocal). The time intervals take into account all interchange between pilots and the controller, including pauses. Let the random variable representing the total time durations for each task be defined accordingly:

- Acknowledgement: $\mathbf{T}_a$
- Clearances and requests: $\mathbf{T}_c$
- Read-backs: $\mathbf{T}_r$
- Frequency changes: $\mathbf{T}_f$

Other forms of communication will be left out for now. Particularly, advisory statement will not be initially included in the communications, as this is typically a discrete mode of operation. For example, if weather, turbulence, or traffic congestion is present then said information will be repeatedly relayed to all aircraft by the controller. As such, if there exist environmental conditions that warrant an advisory, the additional time can be included as part of the total acknowledgement time, $\mathbf{T}_a$. Furthermore, lets define $\mathbf{T}_c$ to be sum of the minimum time required for a request and clearance communication to take place. This occurs when the airspace is relatively clear such that the probability of conflict is low.

From the definitions above, the random variable for total communication time, $\mathbf{T}_s$ within any 1 min time block is given by:

$$\mathbf{T}_s = \mathbf{T}_a + \mathbf{T}_c + \mathbf{T}_r + \mathbf{T}_f \tag{1}$$

Additionally, we will make use of the random variable for the number of events for each communication, to be labeled by $\mathbf{N}_a$, $\mathbf{N}_c$, $\mathbf{N}_r$, and rvN_f corresponding to the previously defined time duration blocks.

The value of $\mathbf{T}_s$ will refer to the minimum standard communication expected between pilots and the air traffic controller. This is a corollary of the assumption that request and clearance communications only occur for the simple conflict-free cases; the present model lacks a notion of conflict detection and resolution. As such, given that $\mathbf{T}_s$ is the minimum time required for resolving traffic, then it is asserted that $\mathbf{T}^{free} = 1 - \mathbf{T}_s$ is free-time available for resolving conflicts, and issuing clearances for more difficult configurations.

IV. Airspace Modeling: Arrivals and Departures

To accurately generate a model of standard communications, it is necessary that the traffic flow into the sector is appropriately described. From such a description, the arrival rate of communication events can be mathematically characterized. In this section, statistical information on aircraft arrivals into and departures out of a sector will be provided. It will be demonstrated that the event counting processes will follow a Poisson counting process, with the associated exponential inter-arrival times. For an arbitrary counting process, $\mathbf{N}$, defined by parameter λ [events/minute], the probability distribution is given by:

$$P(\mathbf{N}(t) - \mathbf{N}(t-s) = k) = \frac{(\lambda)^k}{k!} e^{-\lambda s} \tag{2}$$

with the associated cumulative distribution for the random variable $\mathbf{I}$ representing event inter-arrival times, is given by:

$$P(\mathbf{I} < a) = 1 - e^{\lambda a} \tag{3}$$

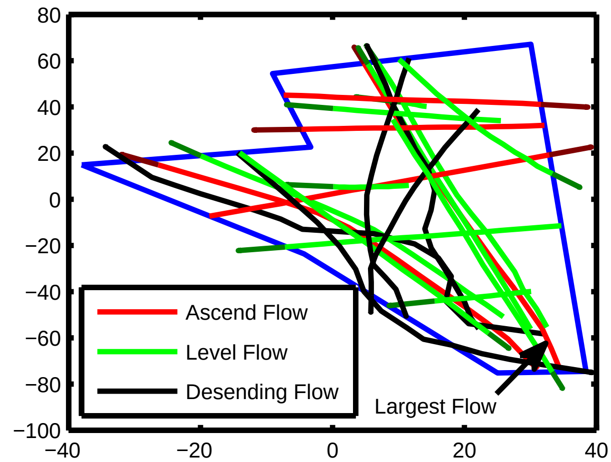

Fig. 1. Sector Map of ZTL36 [NM] with common flows indicated

To drive the modeling process and provide verification of the event model in (2), the airspace ZTL36 was selected for analysis as a sample case. The sector is generally a high-complexity sector with many flows, as it is dominated by north-bound ascending aircraft from KATL airport, and both east-bound and west-bound crossing traffic at the northern and southern ends of the sector. There are also other less prevalently traversed paths that occur spatially near dominating flows that lead to increased complexity. A figure of the sector geometry with clustered flights is shown in Fig. 1. A 15 minute time period from 20:00-20:15GMT over 42 days is selected for analysis, where the dates are sampled from June 12-August 22 of 2005. The selection of the time period is arbitrary, however, for the case at hand, a high traffic period was present during these hours. Aircraft counts within the sector over time are as shown in Fig. 2.

For the 15 minute period considered, the sampled cumulative distribution function of arrival times is shown in Fig. 3. Also, shown in Fig. 3 is the departure sampled cumulative distribution during the same time period. The inter-arrival times of aircraft arriving into and departing the sector fits an exponential distribution (3), with parameters $\lambda_a = 1.2532$ [aircraft/minute] and $\lambda_d = 1.0971$ [aircraft/min], respectively. This is a significant result as it validates modeling air traffic arriving into a sector as Poisson, and it allows for accurate representation and simple modeling of the system.

Note however that the arrival and departure distributions are not the same. Considering that the distribution of aircraft service times through the sector, that is the time to traverse the sector, range from 5-30 minutes, the departure rate can lag the arrival rate. If the arrival process is homogeneous, $\dot{\lambda}_a(t) = 0$, the arrivals and departures processes will have the same distribution. It appears that at least for sectors near major airports, it is unlikely arrival process will be homogeneous over 15-45 minute periods due to dynamic demand in the local area as a result of the airport.

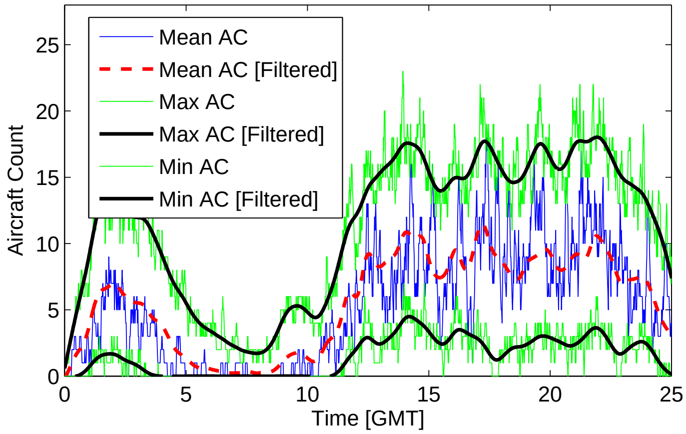

Fig. 2. Aircraft counts in ZTL36 over 24hr period

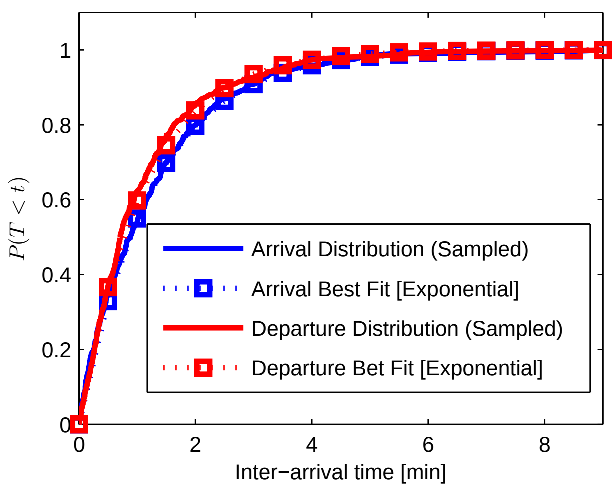

Fig. 3. Aircraft inter-arrival distribution into the sector between 20:00-20:15 GMT

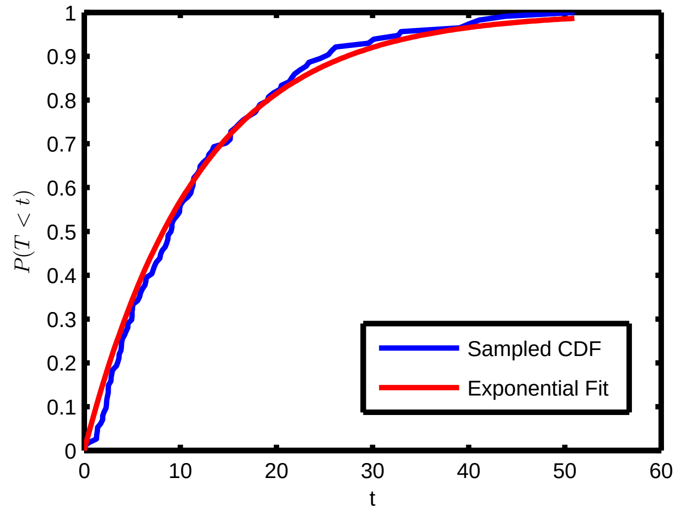

Fig. 4. Aircraft inter-arrival distribution into flow 5 between 20:00-20:15 [GMT]

From the above description of the traffic arrival and departure model, we will make the assumption that flows into the sector are also Poisson, and inter-arrival times are described through an exponential distribution. For example, inter-arrival times for the set of aircraft entering the sector that requesting altitude clearances can be modeled as exponential. This assumption is validated for all the major flows present. Figure 4, for example, validates that aircraft at level flight entering the sector along flow 5 are distributed according to an exponential distribution like in (3).

It is now possible to describe the distribution of communication time in a form similar to (1). The arrival process will be broken into multiple processes. Let the random variable $\mathbf{N}_a$ represent that total number of aircraft arrival events that occur within the 1 minute period. The total can be subdivided according to flow and required controller actions. The counting process for each flow i is grouped and divided into four sets:

$$\begin{aligned} S_1 &= \{i|\text{flow } i \text{ is at level flight and requiring read-back}\} \\ S_2 &= \{i|\text{flow } i \text{ is at level flight and doesn't requiring read-back}\} \\ S_3 &= \{i|\text{flow } i \text{ requires clearance and read-back}\} \\ S_4 &= \{i|\text{flow } i \text{ requires clearance and without read-back}\} \end{aligned} \tag{4}$$

Then,

$$\mathbf{N}_a = \mathbf{N}_1 + \mathbf{N}_2 + \mathbf{N}_3 + \mathbf{N}_4 \tag{5}$$

where, each random variable $\mathbf{N}_k$ is the associated counting process to the set S_k listed in (4).

The λ_k parameter characterizing the counting process $\mathbf{N}_k$ in (5) can be calculated using flow parameters, and the probability distribution of read-back being required. If $\lambda_{i,a}$ is the mean arrival rate for flow i, and the probability of aircraft arriving into the sector requiring read-back is $p_{i,a,r}$, then the rate of aircraft from flow i requiring read-back is $\lambda_{i,r} = p_{i,a,r}\lambda_{i,a}$. Similarly, this can be calculated for all other flows. For each set in (4) the arrival parameter is given by:

$$\begin{aligned} \lambda_1 &= \sum_{i \in S_1} p_{i,a,r}\lambda_{i,a} \\ \lambda_2 &= \sum_{i \in S_2} (1 - p_{i,a,r})\lambda_{i,a} \\ \lambda_3 &= \sum_{i \in S_3} p_{i,a,r}\lambda_{i,a} \\ \lambda_4 &= \sum_{i \in S_4} (1 - p_{i,a,r})\lambda_{i,a} \end{aligned} \tag{6}$$

The compound Poisson process for each the above arrival models is in fact the original aircraft arrival model.

The counting process for departure events can be expressed similarly. Defining the sets:

$$\begin{aligned} S_5 &= \{i|\text{flow } i \text{ departing requiring read-back}\} \\ S_6 &= \{i|\text{flow } i \text{ departing not requiring read-back}\} \end{aligned} \tag{7}$$

with corresponding counting process, $\mathbf{N}_5$ and $\mathbf{N}_6$ defined by parameters:

$$\begin{aligned} \lambda_5 &= \sum_{i \in S_5} p_{i,d,r}\lambda_{i,d} \\ \lambda_6 &= \sum_{i \in S_6} (1-p_{i,d,r})\lambda_{i,d} \end{aligned} \tag{8}$$

where $\lambda_{i,d}$ is the departure rate for flow i, and $p_{i,d,r}$ is the probability departing aircraft from flow i require read-back.

Each Poisson process associated each λ_k is associated with a constant communication time, $T_1,\ldots,T_6$. For example, all aircraft arriving into the sector requiring clearance and read-back will require T_1 seconds of communication between the pilot and the controller. The random variable for total communication time $\mathbf{C}_k$ required for each set S_k is

$$\mathbf{C}_k = \mathbf{N}_k T_k \qquad k = 1,\ldots,6 \tag{9}$$

and the probability distribution for each $\mathbf{C}_k$ is:

$$f_k(c) = P(C_k = c) = \begin{cases} \dfrac{\lambda_k^{c/T_k}}{(c/T_k)!} e^{-\lambda_k} & c/T_k \in \mathbb{Z}^{0,+} \\ 0 & else \end{cases} \tag{10}$$

The random variable for the total minimum standard communication time required of a controller in the minute is:

$$\mathbf{T}_T = \sum_{k=1}^{6} \mathbf{C}_k \tag{11}$$

The final result for the distribution of $\mathbf{T}_t$ can be determined analytically through the convolution:

$$P(\mathbf{T}_t = c) = f_1(c) \otimes f_2(c) \otimes \ldots f_6(c) \tag{12}$$

V. Conflict Model

The previous section detailed the minimum standard communication time expected for an air traffic controller. However, for the model to be complete, it should include communications relating to conflict/proximity resolution. A model for proximity will now be detailed. The proximity model will estimate the probability that any new aircraft entering the airspace within the next minute will require mental acknowledgement of a potential conflict, determination if action should be taken, actualization of any required control through communication, and finally verification that the proper maneuvers were implemented by the pilots. Extending results of the arrival distribution into the sector, a Markov state model will be developed. The Markov model will be based on a binomial approximation for the aircrat arrival process into the sector. The final result is a distribution of the required commincation and thought time for an air traffic controller to resolve any potential conflicts.

Restating the result in Section IV, the arrival flows into an enroute airspace can be characterized by exponential inter-arrival distributions. For the sector considered, the largest arrival rate, or maximum flow rate, $\lambda^* = max_i\{\lambda_{i,a}\}$, is given by $\lambda^* = 1/8.2903$ [aircraft/minute] corresponding to aircraft flow 4, ascending from KATL from the south to the north, as indicated in Fig. 1.

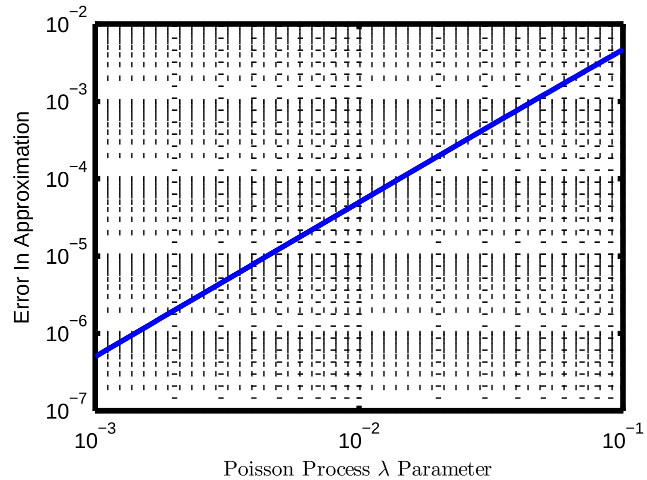

Fig. 5. Error in binomial approximation of $\tilde{p}_i^a$

Because arrival rates to any flow i satisfy the condition $\lambda_{i,a} \ll 1$, it is possible to approximate aircraft arrivals in any 1 minute time period by a binomial random variable. Let $\bar{a}_i$ be the number of aircraft arriving in the next minute according to the Poisson process in (2). By modeling the arrival process in any minute as a binomial random variable, introducing a_i, we are then restricted to the cases $a_i \in \{0,1\}$; the binomial variable a_i approximates the Poisson variable $\bar{a}_i$. For a standard Poisson process, it is possible that $\bar{a}_i > 1$, however, in this scenario the likelihood is small. According to a binomial approximation, the probability of an arrival along flow i, p_i^a, and the probability that no aircraft arrives, $\tilde{p}_i^a$, is established by taking the following approximation:

$$\begin{aligned} p_i^a = P(a_i = 1) = P(\tilde{a}_i = 1) &= \lambda_{i,a} e^{-\lambda a,i} \\ \tilde{p}_i^a = P(a_i = 0) &= 1 - p_i^a \end{aligned} \tag{13}$$

The binomial approximation in (13) is acceptable when $P(\tilde{a}_i \geq 2) \sim 0$, which is true for small $\lambda_{i,a}$. The result of this approximation is an error in $\tilde{p}_i^a$. The overestimation error in $\tilde{p}_i^a$ is given by $P(\tilde{a}_i \geq 2) = 1 - \lambda_{i,a}e^{-\lambda_{i,a}} - e^{-\lambda_{i,a}}$. The error is small for the range of $\lambda_{i,a}$ considered $(0\ldots 1/8]$ as shown in Fig. 5.

The binomial approximation of aircraft arrivals will serve to generate a proximity model to calculate the probability new potential conflicts arise in the 1 minute period considered. For a new proximity event to occur, at least one aircraft must arrive in the sector during this time period. A proximity event will be defined to occur when a new aircraft will be within some proximity of another aircraft entering the sector, or an aircraft already in the sector.

The work presented below will calculate the probability a new aircraft from flow i will be within some proximity of an aircraft from flow j.

First, define the term $t_{i,j}^a$ to be the time it takes an aircraft from flow i to arrive at the intersection point (or location of shortest distance) with an aircraft from flow j; $t^a_{j,i}$ is be defined similarly .

Starting at $t=0$, lets assume an aircraft, A^+_i, from flow i arrives into the sector at time $E_i \le 1$ minute , and aircraft A^+_j from flows j, arrives at time E_j. The pair A^+_i and A^+_j will be considered in proximity of each other if they cross at the intersection within a time period of $s_{i,j}$. That is, A^+_i and A^+_j are in proximity if $|E_i + t^a_{i,j} - (E_j + t^a_{j,i})| < s_{i,j}$. Examples of calculations for $s_{i,j}$ are provided in [18] for crossing aircraft at level-flight.

To calculate the probability of proximity, two cases will be considered, when two new aircraft come in proximity of each other, and when an arriving aircraft with come into proximity with an aircraft already in the sector. For case **1)** A^+_i and A^+_j arrive within the 1 minute period ($0 < E_i, E_j \le 1$), and the aircraft come in proximity of each other, corresponding to the probability $p^{p,+}_{i,j}$ **2)** A^+_i arrives within 1 minute ($E_i \le 1$), and comes with in proximity of an aircraft A^k_j from flow j, where $A^k_j \neq A^+_j$, corresponding to the probability $p^{p,o}_{i,j}$.

Considering case **1**, an aircraft from flow i and flow j arrive within 1 minute, and A^+_i is in proximity of A^+_j, the corresponding probability of proximity for the two new arrivals, $p^{p,+}_{i,j}$, is represented by the following probability equation:

$$\begin{aligned} p^{p,+}_{i,j} &= P(-s_{i,j} < E_i + t^a_{i,j} - (E_j + t^a_{j,i}) < s_{i,j} | E_i, E_j \le 1) \\ &= P(-s_{i,j} - (t^a_{i,j} - t^a_{j,i}) < E_i - E_j < s_{i,j} - (t^a_{i,j} - t^a_{j,i}) | E_i, E_j \le 1) \end{aligned} \tag{14}$$

Because aircraft arrivals are Poisson, and only one aircraft arrives within the 1 minute period, the time at which the aircraft arrives is known to be uniformly distributed [19]. That is, for a single arrival within δT time period:

$$P(E_i \le t | N(\delta T) = 1) = 1/\delta T \tag{15}$$

Equation (14) can be evaluated by inserting the probability distribution function for E_i and E_j from (15), which are uniform, $f_{E_i} = 1$ and $f_{E_j} = 1$, and assigning the bound of integration to be:

$$\begin{aligned} a &= -s_{i,j} - (t^a_{i,j} - t^a_{j,i}) \\ b &= s_{i,j} - (t^a_{i,j} - t^a_{j,i}) \end{aligned} \tag{16}$$

then,

$$\begin{aligned} p^{p,+}_{i,j} &= \int_0^1 P(a + E_j < E_i < b + E_j | E_j = \tau_j) f_{E_j}(\tau_j) d\tau_j \\ &= \int_0^1 \int_{\max(0, a+E_j)}^{\min(1, b+E_j)} f_{E_i} d\tau_i f_{E_j}(\tau_j) d\tau_j \\ &= \int_0^1 \int_{\max(0, a+E_j)}^{\min(1, b+E_j)} d\tau_i d\tau_j \\ &= \int_0^1 [\min(1, b+E_j) - \max(0, a+E_j)] d\tau_j \end{aligned} \tag{17}$$

For case **2** in which A^+_i comes in proximity of A^k_j, there are two possible options: **2a)** An aircraft A^+_j arrives within the first minute. Or **2b)** No aircraft A^+_j arrives within the 1 minute period.

For case **2a**, let the probability of two new aircraft arrivals, where aircraft A^+_i intersects with aircraft A^k_j, be defined to be $p^{p,+,+,k}_{i,j}$. Calculating $p^{p,+,+,k}_{i,j}$ requires determining the length of a time-window, $E^w_{i,j}$, during which if aircraft A^k_j from flow j enters, it will conflict with the aircraft A^+_i. For A^+_i arriving at E_i, the length of the time-window is given by:

$$E^w_{i,j} = \max(0, 2s_{i,j} + \min(0, (t_j - 1) - (t_i - E_1 + s_{i,j}))) \tag{18}$$

The probability of A^+_i coming within proximity of an aircraft from flow j, not being A^+_j is calculated by integrating over the probability that an aircraft arrives from flow j within a time period of $T^w_{i,j}$. For calculation, we can make use of the fact that $\lambda_{a,j}(t)$ is not constant; we have access to $\lambda_{a,j}(-t_j)$; yet $\lambda_{a,j}(t)$ can be approximated as constant over the time-range $\pm s_{i,j}$.

The probability A^+_i comes within proximity of an aircraft, A^k_j from flow j, where $A^+_{j,1} \neq A^k_j$ is given by:

$$\begin{aligned} p^{p,+,+,o}_{i,j} &= \int_0^1 (1 - P(\nexists A^k_j \in [0, E^w_{i,j}])) d\tau_i \\ &= \int_0^1 e^{-\lambda_{a,j} T^w_{i,j}(\tau_i)} d\tau_i \end{aligned} \tag{19}$$

Equations (18) and (19) hold for **2b** in which A^+_i arrives within 1 minute, and no aircraft arrives from flow j, corresponding to the probability $p^{p,+,o,o}$.

The final result for case **2**, where aircraft A^+_i comes in proximity of A^k_j is:

$$p^{p,o}_{i,j} = p^a_i (1 - p^a_j) p^{p,+,o,o} + p^a_i p^a_j p^{p,+,+,o} \tag{20}$$

From (17) and (20) we have a complete description of the probability of aircraft A^+_i being in proximity with a new aircraft from flow j, or an aircraft already in the sector. This separation leads to the definition of the proximity state vector z^p_i for flow i:

$$z^p_i = \left[g^+_{i,1}, \ldots, g^+_{i,n}, g^o_{i,1}, \ldots g^o_{i,n} \right]^T \tag{21}$$

where $g^+_{i,j}$ corresponds to the state in which a recent aircraft from flow i will be in proximity with a recent arrival from flow j, and $g^o_{i,j}$ corresponds to state when a recent aircraft from flow i will be in proximity of an aircraft from flow j already in the sector.

From z^p_i, and a defined controller logic, the number of resolution commands and required time will be developed; the controller logic is based on sequential conflict resolution. First, let the set R^o represent the set of flows for which a recent arrival will come in proximity of an aircraft already in the sector. All new aircraft in this set will require a resolution command. The set of flows is given by:

$$R^o = \{ i | \sum_{j=1}^{n} g^o_{i,j} \ge 1 \} \tag{22}$$

New aircraft, arriving within the 1 minute period, may come in proximity of other new aircraft, but not with aircraft already within the sector. This set, R^+ containing the associated flows is defined as follows:

$$R^+ = \{i | g^+_{i,j} = 1, i, j \notin R^o\} \tag{23}$$

Base on a sequential conflict resolution algorithm, for the minimum controller workload that issues the minimum number of resolution commands, all aircraft from R^o are required resolution commands. Also, if we assume aircraft only come within proximity in a pairwise fashion, then only a fraction of aircraft in R^+ require resolution commands. The maximum number of resolution commands required, $\mathbf{N}_R$ is:

$$\begin{aligned} W &= card(R^o) \\ U &= \lceil card(R^+)/2 \rceil \\ \mathbf{N}_r &= W + U \end{aligned} \tag{24}$$

The value $\mathbf{N}_r$ is in fact a random variable dependent on the probability of proximity of various aircraft flows. The random variable, $\mathbf{T}_{res}$ corresponding to the maximum communication and thought time required for detecting and resolving aircraft within proximity is then given by:

$$\mathbf{T}_r = \mathbf{N}_r t_r \tag{25}$$

where t_r is the time required to resolve each proximity case.

VI. Completed Model

The completed communication model, considering both minimum standard communication time, $\mathbf{T}_s$, and the time to resolve any potential conflict, $\mathbf{T}_r$, is given by $\mathbf{T}_T$, where:

$$\mathbf{T}_T = \mathbf{T}_s + \mathbf{T}_r \tag{26}$$

Of particular interest is the probability air traffic controllers are unable to manage traffic within the allotted time, $P(\mathbf{T}_T > 1 minute)$. However, it is not necessarily a problem if $\mathbf{T}_T > 1$, as long as the next minute requires little communication. From this insight, a measure of the probability of unsafe operating conditions can be defined over a 5 minute period. Define $p_{\mathbf{T}_{t>60}} = P(\mathbf{T}_T > 1)$. For independent increments, the probability of 5 consecutive high workload periods is given by:

$$P(\text{High workload 5 minutes}) = (p_{\mathbf{T}_{t>60}})^5 \tag{27}$$

If the probability in (27) is sizable such that it will occur with some ε probability, it is possibility that air traffic controllers will be unable to manage the traffic. As such, traffic management measures must be taken to re-route traffic to other sectors in order to ensure safety.

VII. Case Study

The case study presented below considers only the minimum required standard communications model. For the final paper, conflict communications will also be included. The conflict model is more difficult to analyze due to requiring clustering algorithms to identify flows, and identify conflict or proximity areas.

Consider the sector ZTL36 as previously described between the time window 20:00-20:15 [GMT]. According to historical data, the aircraft arrival rate into the sector is $\lambda_a = 1.2532$ [aircraft/minute], while the aircraft departure rate is $\lambda_d = 1.0971$ [aircraft/min]. The following aircraft arrival rates and aircraft departure rates are calculated according to the sets defined in (4) and (7):

$$\begin{aligned} \lambda_1 &= p_{a,l} p_{a,l,r} \lambda_a \\ \lambda_2 &= p_{a,l}(1 - p_{a,l,r})\lambda_a \\ \lambda_3 &= (1 - p_{a,l}) p_{a,c,r} \lambda_a \\ \lambda_4 &= (1 - p_{a,c})(1 - p_{a,c,r})\lambda_a \\ \lambda_5 &= p_{d,r}\lambda_d \\ \lambda_6 &= (1 - p_{d,r})\lambda_d \end{aligned} \tag{28}$$

where $p_{a,l}$ is the probability an aircraft is at level-flight; $p_{a,l,r}$ and $p_{a,c,r}$ are the fraction of arrivals at level-flight and arrival requiring clearance that will have read-back; and $p_{d,r}$ is the probability an aircraft departing the sector will require read-back. The probability values are assigned to be: $p_{a,l} = .6$, $p_{a,l,r} = p_{a,c,r} = p_{d,r} = .05$

The time costs in seconds for each communication are defined as follows:

$t_1 = 7$	$t_2 = 4$	Arrivals at level-flight
$t_3 = 10$	$t_4 = 6$	Arrivals requiring clearance
$t_5 = 5$	$t_6 = 3$	Departure

The distribution of communication times for the case considered is shown in Fig. 6. In the case of a weather advisory to all aircraft, the effect is to increase the initial communication times for all entering aircraft. Using $t'_k = t_k + 2$ for $k = 1, 2, 3, 4$. The effect of a weather advisory on the communications distribution is also shown in Fig. 6.

This work can be expanding to include the probability air traffic controllers will see difficult workload conditions in any period, while still within some safely region. Assuming 45 seconds as a threshold for reasonable minimum stand communications, Fig.7 shows probability a pair of aircraft arrival and departure rates will lead to high levels of workload.

VIII. Conclusion

A new method for predicting controller workload according communication time has been presented. The results is a stochastic process model based on aircraft arrivals, departs, and interactions, which considers the underlying counting process. From the stochastic model, it is possible to predict in real-time a metric of the projected workload an air traffic controller will expect. Furthermore, because of the model considers random distributions, it is possible to establish confidence bounds on the predicted workload. As such, traffic flows can be adjusted accordingly.

Furthermore, a major result of the is paper, is the introduction of the sensitivity of the controller workload. By establishing a workload distribution, it is possible to determine the probability the system will overwhelm a controller.

For the proposed communication-time model, verification is still required. This future work will be completed in accordance with ZTL ARTCC.

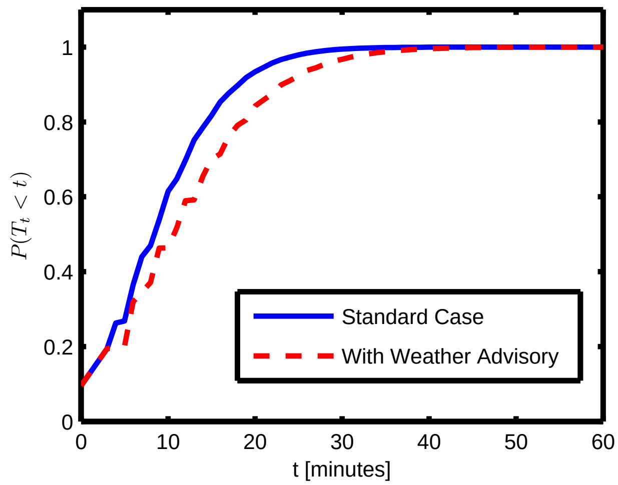

Fig. 6. Example distribution for minimum expected communication time over 1 minute

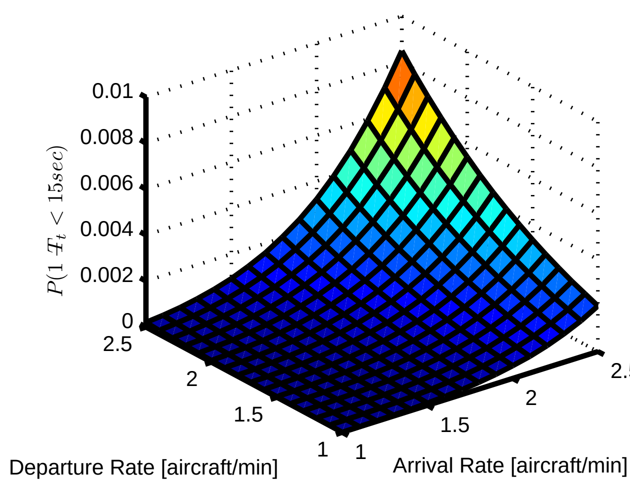

Fig. 7. Probability of standard communication time requiring more than 45 second.

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