Queueing Models for Operations in NextGen

Tasos Nikoleris, Mark Hansen
Department of Civil and Environmental Engineering
University of California at Berkeley
Berkeley, CA, USA
nikoleris@berkeley.edu

Abstract—This paper develops a queuing model for trajectory-based aircraft operations, expected to take place in the Next Generation Air Transportation System. Aircraft are assigned scheduled times of arrival at a server, which they meet with some normally distributed stochastic error. A recursive queuing model is formulated, and Clark's approximation method is employed to estimate each flight's expected queueing delay. The model is further developed to account for aircraft's runway occupancy time, and to track any aircraft's delay through a network of servers.

Keywords—queue; aircraft; NextGen; 4D operations

I. INTRODUCTION

The nation's air transportation system (NAS) will incur major transformations in the coming years, developing towards the so-called Next Generation Air Transportation System (NextGen). NextGen features a shift from the current static system of routes and sectors to one that is adaptive to weather, traffic, and user preferences. Users will exchange coordinates information and supply the Air Navigation Service Provider with greater amounts of information about future traffic demand. This will be used to anticipate and resolve conflicts well in advance, reducing the need for tactical air traffic control. It will also allow controlled times of arrival into busy terminals, weather-impacted airspace, and other bottlenecks. This transformation is expected to greatly reduce human operator workload and significantly increase airport and airspace capacity.

The motivation for this research is the fact that the ability to control and predict 4D aircraft trajectories (4DT) with high precision is a cornerstone of NextGen. 4DT capability, with time being the fourth dimension, is defined as the ability to precisely fly an assigned 3D trajectory while meeting specified timing constraints on arrival at waypoints [1]. This will allow high density flows that rely on controlled times of arrival for critical resources, including entry and exit to/from airspaces, taxiways, and runways [1]. Thus, in this research we assume that an aircraft’s flight path includes a series of waypoints (that can be either points in the airspace or the runway’s threshold) that the aircraft has to cross at a scheduled time. In other words, we assume that under 4DT operations aircraft will be metered at fixes.

However, even with the deployment of the very best 4D trajectory precision and navigation tools, adherence to 4D trajectories will not be perfect. Sources of imprecision include airframe-to-airframe variation in aerodynamic performance, limitations in wind prediction capability, variations in flight crew technique, and varying degrees of exactitude in navigational performance [1]. As the NAS evolves from its current state to a future condition where location precision is maximized, a spectrum of trajectory uncertainty will be manifested. It will range from low precision, corresponding to today's operations in the NAS, to high precision, brought on by full deployment of precision navigation and 4DT trajectory awareness tools. For a comparison of delays corresponding to the two ends of this precision spectrum, see [2]. While the models for such cases are well established, it is far more challenging to consider intermediate levels of stochasticity. Such cases are far more representative of the future NAS, in which trajectory adherence will be imperfect. Thus, the objective of this paper is to model aircraft operations in NextGen using queueing theory, in a way that accounts for levels of trajectory uncertainty in all intermediate phases of precision navigation deployment.

Existing analytical queueing models typically assume that the aircraft arrival process at an airport’s terminal airspace area is a non-homogeneous
Poisson process [3]. However, for trajectory-based operations in NextGen, the Poisson-arrivals assumption does not capture the concept of metered aircraft operations. Thus, a queueing model with arrivals that are scheduled to a server is proposed in this paper, to analyze flight delays in a high-precision trajectory-based operational environment, as currently being planned for NextGen.

Within transportation engineering context, queueing models with scheduled arrivals have been proposed to study port operations. Sabria and Daganzo [4] examine single server queueing systems where customers must be served in an order that is specified by a timetable, i.e. in a First-Scheduled-First-Served (FSFS) order. Each customer has a scheduled time of arrival at the server, where they actually arrive with some stochastic lateness (positive or negative). Exact transient solutions are obtained for the case when the lateness distribution is Gumbel, and service times are deterministic. In the present paper though, stochastic deviations from scheduled times of arrival are assumed to follow a Normal distribution, while the rule of FSFS service is maintained.

The rest of the paper is organized as follows: Section II presents the general form of our model and discusses the applicability of Clark’s approximation method to obtain estimates for the expected queueing delay of each airplane. In Section III the model is further developed to handle aircraft’s runway occupancy time as a separate random variable. Moreover, the model is extended to estimate delays when aircraft traverse two consecutive servers. That constitutes the analysis unit for a network of queues. Finally, Section IV summarizes our main findings and conclusions.

II. THE MODEL AND AN APPROXIMATE SOLUTION

A. Model Formulation

Our queueing system consists of a single server, which is a fix (either a point in the airspace or a runway’s threshold), and of airplanes that must cross this fix. Aircraft are assigned scheduled times of arrival at the fix, and they fly 4D trajectories to arrive at the fix just on time. However, due to imprecision in trajectory adherence, aircraft’s actual time of arrival at the fix has some stochastic deviation from its scheduled arrival time. The sources of imprecision might include airframe-to-airframe variation in aerodynamic performance, limitations in wind prediction capability, variations in flight crew technique, and varying degrees of exactitude in navigational performance [1]. In addition, consecutive aircraft must maintain a minimum headway $h$ for safety reasons, which can vary over pairs of arriving aircraft. Since air traffic controllers impose the exact values for $h$, we consider it as a deterministic variable in our model that reflects a particular air traffic control policy initiative. Moreover, we assume that $h$ is the binding constraint among all factors that may affect the minimum required separation between consecutive aircraft.

Following Sabria and Daganzo’s approach, each airplane $i$ has an arrival time at the server $A_i$ that consists of a deterministic and a stochastic portion. The deterministic component $a_i$ is the scheduled arrival time at the fix, while the stochastic component is denoted as $\tilde{A}_i$ and represents the lateness (positive or negative) with which the aircraft arrives at the fix, due to imprecision in trajectory adherence. Therefore, we have $A_i = a_i + \tilde{A}_i$.

The first key assumption in our model is that deviations $\tilde{A}_i$'s are small enough, such that serving aircraft on a FSFS order will not result in excessive delays. As an order of magnitude, NextGen planners foresee accuracies of seconds in aircraft meeting scheduled times of arrival [5]. Under a FSFS queue discipline, the actual time airplane $i$ departs from the server, $D_i$, would be $A_i$ if there were no queue at the server by the time it arrived, or the time the previous scheduled aircraft $i-1$ crossed the fix plus a minimum required separation headway $h_{i-1,i}$ between the two aircraft. The actual times that aircraft cross the fix under study would then be:

$$D_i = A_i$$
$$D_i = \max\left(A_i, D_{i-1} + h_{i-1,i}\right), \quad \forall i \geq 2$$

If there were no stochasticity in the system, the deterministic time of departure from the server would be:
\[ d_i = \max\left(a_i, d_{i-1} + h_{i-1,i}\right), \quad \forall i \geq 2 \]

Accounting for stochasticity, the departure time from the server of airplane \( i \) is:

\[ D_i = d_i + \tilde{D}_i \]

The distribution of the stochastic component \( \tilde{D}_i \) clearly depends on \( \tilde{A}_i \), which captures all stochastic effects that cause flight \( i \) to arrive at a time other than its scheduled one \( a_i \):

\[ \tilde{D}_i = \tilde{A}_i \]

(1a)

\[ \tilde{D}_i = \max\left(a_i + \tilde{A}_i, d_{i-1} + \tilde{D}_{i-1} + h_{i-1,i}\right) - d_i, \forall i \geq 2 \]  

(1b)

The second pivotal assumption is that the vector of stochastic errors \( \tilde{A}_i \) follows a multivariate normal distribution with zero means (without loss of generality), standard deviations \( \sigma_i \), and a covariance structure \( \Sigma: \tilde{A} \sim \text{Normal}(0, \Sigma) \). The normality assumption stems from the observation that the probability distribution for \( \tilde{A}_i \) is generated by convolving the individual distributions of low-correlated stochastic factors. It should be emphasized though, that \( \tilde{A}_i \)'s do not represent factors such as severe weather or en-route congestion that cause significant amounts of delays; lateness effects due to such factors have already been incorporated in the estimation of scheduled arrival times \( a_i \).

In practice, values for schedule deviation could be aggregated to represent classes of aircraft that have similar capabilities of adherence to 4D trajectories. For example, one could assume two different values for the standard deviation, \( \sigma_x \) and \( \sigma_y \), in order to roughly represent aircraft with and without Area Navigation (RNAV) and Required Navigation Performance (RNP) capabilities.

**B. Solution with Clark’s Approximation Method**

In (1), for \( i = 2 \) both terms of the \( \max \) operator are normally distributed. The \( \max \) operation on normal random variables, in contrast to the \( \text{add} \) operation, does not yield a normal random variable. A well-known result due to Clark [6] derives analytical formulas for the mean and variance of the maximum of two normally distributed random variables. Let \( X \) and \( Y \) be normally distributed random variables, \( X \sim \text{N}(\mu_x, \sigma_x) \) and \( Y \sim \text{N}(\mu_y, \sigma_y) \), \( \rho \) represent the correlation coefficient between \( X \) and \( Y \), and \( Z \) be the maximum of \( X \) and \( Y \), \( Z \triangleq \max(X,Y) \). The mean \( \mu_Z \) and variance \( \sigma_Z^2 \) of \( Z \) are then:

\[ \mu_Z = \mu_x \Phi(\alpha) + \mu_y \Phi(-\alpha) + \gamma \varphi(\alpha) \]

\[ \sigma_Z^2 = (\sigma_x^2 + \mu_x^2)\Phi(\alpha) + (\sigma_y^2 + \mu_y^2)\Phi(-\alpha) + (\mu_x + \mu_y) \gamma \varphi(\alpha) - \mu_z^2 \]

where

\[ \gamma \triangleq \left(\sigma_x^2 + \sigma_y^2 - 2\rho \sigma_x \sigma_y\right)^{1/2} \]

\[ \alpha \triangleq \left(\frac{\mu_x - \mu_y}{\gamma}\right) \]

\[ \varphi(x) \triangleq \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2}\right) \]

\[ \Phi(y) \triangleq \int_{-\infty}^{y} \varphi(x) dx \]

The coefficient of linear correlation between \( Z \) and a third normal random variable \( W \), \( r[Z,W] \), can also be estimated, given that we know the coefficients of linear correlation between \( X \) and \( W \) \( (\rho_{X,W}) \), and between \( Y \) and \( W \) \( (\rho_{Y,W}) \):

\[ r[Z,W] = \left(\sigma_x \rho_{X,W} \Phi(\alpha) + \sigma_y \rho_{Y,W} \Phi(-\alpha)\right) / \sigma_Z \]

The above formulas give the exact mean and variance of \( Z \). The approximation is introduced by assuming that \( Z \) follows a normal distribution with mean \( \mu_Z \) and variance \( \sigma_Z^2 \).

In the context of our problem with scheduled aircraft arrivals, Clark's method can be used for all \( i \geq 2 \) to approximate \( D_i \)'s as normal random variables, and estimate their mean \( E(D_i) \) and variance \( Var(D_i) \) in a recursive manner:

\[ E(D_i) = a_i \Phi(\alpha_i) + \left[ E(D_{i-1}) + h_{i-1,i}\right] \Phi(-\alpha_i) + \gamma \varphi(\alpha_i) \]

(2)
\[ Var(D_i) = (\sigma_i^2 + a_i^2) \Phi(\alpha_i) + \\
+ \left[ Var(D_i) + \left[ E(D_i) + h_{i-1,i} \right]^2 \right] \Phi(-\alpha_i) \]  
(3)

\[ + \left[ a_i + E(D_{i-1}) + h_{i-2,i-1} \right] \varphi(\alpha_i) - \left[ E(D_i) \right]^2 \]

\[ r \left[ A_{i,1}, D_i \right] = [\sigma_i \cdot \rho_i \cdot \Phi(\alpha_i) + \\
+ \sqrt{Var(D_{i-1})} \cdot \rho_2 \cdot \Phi(-\alpha_i)] / \sqrt{Var(D_i)} \]  
(4)

where

\[ \gamma_i = \left( \sigma_i^2 + Var(D_{i-1}) - 2 \cdot \rho_i \cdot \sigma_i \cdot \sqrt{Var(D_{i-1})} \right)^{1/2} \]  
(5)

\[ \alpha_i = \left( a_i - E(D_{i-1}) - h_{i-1,i} \right) / \gamma_i \]  
(6)

and at each iteration \( i \)

\[ \rho = r \left[ A_{i,1}, D_i \right], \quad \rho_i = r \left[ A_{i,1}, A_i \right], \quad \rho_2 = r \left[ A_{i,1}, D_{i-1} \right]. \]

Note that \( r \left[ A_{i,1}, D_{i-1} \right] \) and \( r \left[ A_{i+1}, D_{i-1} \right] \) are obtained through equation (4) in previous iterations. Effectively, the method is implemented by estimating at each step \( k \) \( r \left[ A_{i,k}, D_k \right] \) for all \( i > k \).

Moreover, \( r \left[ A_{i+1}, A_i \right] \) is considered as input from covariance matrix \( \Sigma \). Equations (2)–(6) are easy to program and they are computationally efficient. Finally, for a stream of \( N \) flights scheduled for to arrive at a fix, the total expected delay is defined as:

\[ E[W_N] = \sum_{i=1}^{N} E(D_i) - a_i \]

This completes the formulation of our queueing model. In summary, the model requires as inputs a schedule of arrival times \( a_i \), a capacity profile expressed in terms of time separation headways \( h_{i-1,i} \), and a covariance matrix of trajectory adherence errors \( \Sigma \). These, coupled with the assumption for relatively small \( \sigma_i \)’s, enable the estimation of expected flight delays through Clark’s approximation method.

C. Approximation Error

Although the maximum \( Z \) of two normal random variables \( X \) and \( Y \) is not normally distributed, our model is based on approximating \( Z \) with a normal random variable. In particular, in estimating \( D_i = \max(A_i, D_{i-1} + h_{i-1,i}) \) it is assumed that \( D_{i-1} \) is normally distributed. That enables the estimation of the mean and variance of \( D_i \), which is then also approximated as a normal random variable. However, each pair-wise operation introduces some error that is propagated and might affect the accuracy of our estimates.

To test the accuracy of Clark’s Approximation Method in the context of our analysis several operational scenarios were considered. The estimates from the analytical queueing model were then compared against the average of \( 10^4 \) Monte Carlo simulation runs, which is considered as ground truth.

Each operational scenario was formulated as follows: a total of 120 aircraft must cross a fix, and the minimum required separation between any two successive aircraft is set to \( h_{i-1,i} = 30, 60, \) or 90 seconds. Each aircraft is assigned a scheduled time of arrival at the server \( a_i = d_{i-1,i} + h_{i-1,i} + b \), where \( b \) denotes a buffer time inserted. Aircraft arrive at the server with some imprecision that follows a normal distribution and has a standard deviation \( \sigma \). Zero covariance was assumed across the aircraft arrival times at the server \( A_i \). A total of 90 scenarios were examined:

- 10 different sequences of \( h_{i-1,i} \) (each sequence has an equal mix of 30, 60, and 90 seconds)
- \( b = 0, 10, \) and 20 seconds (held constant within each sequence)
- \( \sigma = 10 \) seconds (uniform across all aircraft), 30 seconds (uniform across all aircraft), and an equal mix of both.

Two metrics for the approximation method accuracy were considered:

- Percentage Error in Total Delay % (PE):

\[ \frac{E[W_N]^{\text{app}} - E[W_N]^{\text{sim}}}{E[W_N]^{\text{sim}}} \cdot 100 \]

- Flight Departure Time Mean Absolute Deviation (MAD):

\[ \frac{\sum_{i=1}^{N} |D_i^{\text{app}} - D_i^{\text{sim}}|}{N} \]
The first metric evaluates the accuracy of the approximation method in estimating the expected total aircraft delay against assigned scheduled times of arrival. The second metric provides a measure of the error in predicted outcomes for individual flights.

The results are presented in Table 1. Each entry in the table represents the average value across the ten scenarios of different \( h_{i-1,i} \) sequences. In all but one case, the Total Delay PE metric indicates that the approximation method is within -4% accuracy in estimating the total delay in the system, as compared to simulation. Also, the MAD metric indicates that the approximation method estimates the expected delay of each aircraft with accuracy better than 1 second, on average. The accuracy of the method slightly decreases when the fleet contains aircraft with different navigation capabilities. This must be due to heterogeneity in the variance of the normal distributions for \( A_i \) that enters in the \( \max \) operator in each step of the recursion. In summary, these experimental results indicate that our proposed model accurately predicts operational consequences of metered operations with good but imperfect 4DT adherence, as might be expected in NextGen.

### III. Model Extensions

#### A. Runway Occupancy Time (ROT)

So far we have considered one generic minimum separation requirement \( h_{i-1,i} \) between two successive arriving aircraft \( i-1 \) and \( i \). In this section we distinguish between airborne separation requirement and the single runway occupancy rule. While the first constraint imposes minimum safety headways between any pair of leading and trailing aircraft when airborne, the second constraint requires that no more than one aircraft may occupy the runway at any time moment.

Similar to the formulation in section II.A, our queueing system consists of a single fix, which is the runway’s threshold. Aircraft are assigned scheduled times of arrival at the threshold, which they must cross in the order specified by the schedule. We define as \( O_i \) the time period from the moment aircraft \( i \) crosses the runway threshold to the moment it has completely exited the runway. Moreover, let \( h_{i-1,i} \) denote the required airborne separation at the moment when the leading aircraft \( i \) traverses the runway threshold. Letting \( A_i \) and \( D_i \) be the actual times of arrival and departure, respectively, from the server, we have:

\[
D_i = A_i \quad (7a)
\]

\[
D_i = \max \left( A_i, D_{i-1} + h_{i-1,i}, D_{i-1} + O_{i-1} \right), \forall i \geq 2 \quad (7b)
\]

Therefore, the time when each aircraft traverses the runway threshold is determined by three factors:

- The time it would arrive at the fix in the absence of queue, \( A_i \)
- The time the previous aircraft crossed the fix plus the minimum required headway, \( D_{i-1} + h_{i-1,i} \)
- The time the previous aircraft exited the runway, \( D_{i-1} + O_{i-1} \)

As an additional assumption we approximate the probability distribution of \( O_i \) as normal and with uniform parameters across all landing aircraft: \( O_i \sim \text{Normal}(\mu_o, \sigma_o) \) for all \( i \). In this way, we can use the Clark’s approximation method, as described in section II.B, to estimate the mean and variance of \( D_i \).

That is performed in two steps; first we define as \( L_i \triangleq \max \left( D_{i-1} + h_{i-1,i}, D_{i-1} + O_{i-1} \right) \). It can be shown that the coefficient of linear correlation between the two terms in the \( \max \) operator is

<table>
<thead>
<tr>
<th>Buffer = 0 (sec)</th>
<th>Buffer = 10 (sec)</th>
<th>Buffer = 20 (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Delay PE</td>
<td>Total Delay PE</td>
</tr>
<tr>
<td></td>
<td>MAD (sec)</td>
<td>MAD (sec)</td>
</tr>
<tr>
<td><strong>σ = 10 (sec)</strong></td>
<td>-0.62%</td>
<td>-3.26%</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>σ = 30 (sec)</strong></td>
<td>-0.49%</td>
<td>-1.69%</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td><strong>Mixed</strong></td>
<td>-1.52%</td>
<td>-5.74%</td>
</tr>
<tr>
<td></td>
<td>0.89</td>
<td>0.65</td>
</tr>
</tbody>
</table>

**TABLE I. RESULTS OF APPROXIMATION ACCURACY TESTS**
r[D_{i,2} + O_{i,1} + D_{i,1}] = \left( \frac{\text{Var}[D_{i,2}] \cdot \text{Var}[D_{i,1} + O_{i,1}]}{\text{Var}[D_{i,1} + O_{i,1}]} \right)^{1/2}.

Applying (2)–(6) we compute \( E[L_i] \) and \( \text{Var}[L_i] \). Subsequently, we use those estimates in the second step to estimate \( D_i = \max(A_i, L_i) \), employing again Clark’s approximation formulas (2)–(6). Note that \( A_i \) is independent of \( L_i \) and, as in section II.A, it is assumed normally distributed around a scheduled time of arrival \( a_i \) with standard deviation \( \sigma_i \).

It should be emphasized that the analysis presented above is based on assuming a normal distribution for ROT. A model with non-normal distribution is the subject of ongoing research.

B. Network of Queues

For the transition from a single server analysis to a network of queues, the critical step is to define the process by which customers that depart from a given server are directed to downstream ones. Classical queueing theory typically assigns a probability that customer \( i \) will be directed to server \( j \) [7]. In NextGen, however, digital communications between the airplane cockpit and air traffic controllers will enable information sharing in real time of each aircraft’s planned 4D flight trajectory [1]. On those grounds, we assume complete knowledge of each aircraft’s trajectory and therefore the particular fix (whether a runway threshold or a waypoint in airspace) where it is bound to, at any time moment. Furthermore, we assume no queue spillovers from any server \( j \) to upstream ones. In other words, the departure time of aircraft \( i \) from server \( j \) is not impeded by queueing effects taking place at a downstream server.

With complete knowledge of flight itineraries, the network problem can be reduced to analyzing two servers in series. Let \( D_{i,j} \) denote the time moment aircraft \( i \) departs from upstream Fix 1, \( D_{i,2} \) the moment when the same aircraft departs from downstream Fix 2, and \( F \) the set of flights that traverse both fixes. Also, let \( T_i \) be the unimpeded (from queueing effects) travel time of aircraft \( i \) between the two fixes. Consistent with our approach, we assume that \( T_i \)'s are normally distributed around \( t_i \)'s with covariance structure \( \Sigma \): \( \mathbf{T} \sim \text{Normal}(\mathbf{t}, \Sigma) \). The departure time of aircraft \( i \) from downstream Fix 2 can be expressed as:

\[
D_{i,2} = D_{i,1} + T_i \quad (8a)
\]

\[
D_{i,2} = \max\{D_{i,1} + T_i, D_{i,1} + h_{i,j}\}, \quad \forall i \geq 2 \quad (8b)
\]

Our goal is to estimate \( E[D_{i,2}] \) by employing (2)–(6). The main difficulty arises in (4), estimating the coefficient of linear correlation \( r[D_{i,1} + T_i, D_{i,1,2}] \). That is addressed through a series of steps, described in the following algorithm:

**Step 0:** Estimate \( E[D_{i,1}] \) for all aircraft departing from Fix 1 through (2)–(6). For each aircraft’s departure time \( D_{i,1} \) estimate its coefficient of linear correlation with all preceding aircraft \( k < i \):

\[
r[D_{j,i}, D_{k,i}] = \frac{\sqrt{\text{Var}[D_{j,i}] \cdot \text{Var}[D_{k,i}]}}{\sqrt{\text{Var}[D_{j,i}]}} \cdot \left(1 - \Phi(\rho_k)\right)
\]

**Step 1:** For the first aircraft departing from Fix 2 set

\[
r[D_{j,1} + T_j, D_{k,1}] = r[D_{j,1}, D_{k,1}] \text{ for all } i \in F
\]

**Step k:** For all \( i \in F \) and \( i \geq k \), compute

\[
r[D_{j,i} + T_j, D_{k,i}] = \sqrt{\text{Var}[D_{j,i} + T_j]} \cdot \Phi(\rho_1) + \sqrt{\text{Var}[D_{k,i}]} \cdot \Phi(\rho_2) / \sqrt{\text{Var}[D_{k,i}]}
\]

where \( \rho_1 = r[D_{j,1} + T_j, D_{k,1}] \)

and \( \rho_2 = r[D_{j,1} + T_j, D_{k,1}] \).

To estimate \( \rho_1 \), first it can be easily shown that for any pair \((i, k)\):

\[
\text{Cov}[D_{j,i} + T_j, D_{k,i} + T_k] = \text{Cov}[D_{j,1}, D_{k,1}] + \text{Cov}[T_j, T_k].
\]

Thus, \( \text{Cov}[D_{j,1}, D_{k,1}] \) is computed in **Step 0**, while \( \text{Cov}[T_j, T_k] \) is given as input in \( \Sigma \). Finally, \( \rho_2 \) is computed in step \( k - 1 \).
The reader will recognize that we have outlined a computational procedure for providing estimates of mean departure times from the downstream Fix 2. Future research will expand this elementary structure by including more than two fixes, and attempt to validate the approximation model estimates against simulation.

IV. SUMMARY AND CONCLUSIONS

In this paper a queueing model for trajectory-based aircraft operations is presented. Flights are assigned scheduled times of arrival at a fix, which they must cross in the order of the schedule. Aircraft meet these times with some stochastic error that is assumed to follow a normal distribution. A recursive queueing model was formulated, and Clark's approximation method was implemented to analytically approximate the mean and variance of individual aircraft delays. The model was further elaborated to include aircraft’s ROT as a separate random variable. Moreover, the model was extended to the case of two servers, which provides the basis for analyzing a network of queues.

All formulations provide analytical estimates of the expected queueing delay, without requiring any simulation. That, especially for a network of queues, can facilitate the exploration of a wide range of demand and capacity scenarios. Moreover, aircraft precision is a model parameter, thus enabling a sensitivity analysis of the effect of adherence to 4DT’s on delays in NextGen’s operational environment. Finally, Clark’s approximation method can work as a flexible platform for capturing correlated random variables, as demonstrated in the cases of ROT and two consecutive servers. Furthermore, our approximation tests in section II.C indicate that the method can provide with accurate estimates in the context of queueing models with scheduled arrivals.

ACKNOWLEDGMENT

This research effort was sponsored by NASA under Award # NNX07AP16A.

REFERENCES