# Resource Allocation in Flow-Constrained Areas with Stochastic Termination Times And Deterministic Movement

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*Abstract*— **In this paper we address a stochastic air traffic flow management problem. Our problem arises when airspace congestion is predicted, usually because of a weather disturbance, so that the number of flights passing through a volume of airspace (flow constrained area – FCA) must be reduced. We formulate an optimization model for the assignment of dispositions to flights whose preferred flight plans pass through an FCA. For each flight, the disposition can be either to depart as scheduled but via a secondary route, or to use the originally intended route but to depart with a controlled (adjusted) departure time and accompanying ground delay. We model the possibility that the capacity of the FCA increases at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that time. This paper extends our earlier work on this problem by allowing the FCA to move in a 2-D spatial plane with a constant speed rather than being stationary. The FCA can have any given constant speed and any given direction. We conduct experiments considering a range of such speeds and directions and draw conclusions regarding appropriate strategies.**

***Keywords: ATM; Air Traffic Manegemnt; FCA; Flow Constraint Area; Rerouting; Stochastic Programing; Ground Delay; Airborne Delay.***

## I. INTRODUCTION

A flow-constrained area (FCA) is a region of the national airspace system (NAS) where a capacity-demand imbalance is expected, due to some unexpected condition such as adverse weather, security concerns, special-use airspace, or others. FCAs might be drawn as polygons in a two-dimensional space, although in practice they are usually represented by a single straight line, functioning as a cordon.

When an FCA has been defined, it is then often the case that an airspace flow program (AFP) is invoked by the Federal Aviation Administration (FAA). An AFP is a traffic management initiative (TMI) issued by the FAA to resolve the anticipated capacity-demand imbalance associated with the FCA. It is the goal of this paper to develop a method by which, given the aggregate data described here, specific orders for individual flights can be developed for a single moving FCA that a) maximize the utilization of the constrained airspace, b) prevent the capacity of the FCA from being exceeded, and c) achieve a system-wide delay minimization objective. We will emphasize analyzing the effects of a moving FCA due to wind on our model results and will present a methodology to take into account such effects through our model. As reported in weather forecasts, a thunder storm can move up to 50 miles per hour. When the FCA is moving, therefore, a flight that departs a few hours after the beginning of the time horizon may intersect the FCA at a totally different time or location than what would have been calculated for a stationary FCA.

These assumptions are the basis for our motivation to conduct research to investigate the effect of a moving FCA on our model results.

## II. RELATED RESEARCH

The research in this paper and our earlier work on this problem builds on stochastic ground holding models. Several stochastic integer programming models have been developed to address the ground holding problem [1], [2], [3], [6], [7], [12]. While our model of the FCA capacity is conceptually similar to airport arrival capacity models, we also explicitly represent the possibility of reroutes, including their dynamic adjustment under stochastic changes in FCA capacity.

There is also a growing literature on airspace flow management problems. Our work also builds on earlier work by Nilim and his coauthors on the use of "hybrid" routes that hedge against airspace capacity changes. In [11], the rerouting of a single aircraft to avoid multiple storms and minimize the expected delay was examined. In this model, the weather uncertainty was treated as a two-state Markov chain, with the weather being stationary in location and either existing or not existing at each phase in time. A dynamic programming approach was used to solve the routing of the aircraft through a gridded airspace, and the aircraft was allowed to hedge by taking a path towards a storm with the possibility that the storm may resolve by the time the aircraft arrived. The focus of the work was on finding the optimal geometrical flight path of the aircraft, and not on allocation of time slots through the weather area. Follow-on work expanded to modeling multiple aircraft with multiple states of weather and attempted to consider capacity and separation constraints at the storms[9], [10].

Initial steps at a concept of operations that describes the terminology, process, and technologies required to increase the effectiveness of uncertain weather information and the use of a probabilistic decision tree to model the state space of the weather scenarios was provided in [1]. Making use of this framework is a model recently proposed that uses a decision-tree approach with two-stage stochastic linear programming with recourse to apportion flows of aircraft over multiple routing options in the presence of uncertain weather [4]. In the model, an initial decision is made to assign flights to various paths to hedge against imperfect knowledge of weather conditions, and the decision is later revised using deterministic weather information at staging nodes on these network paths that are close enough to the weather that the upcoming weather activity is assumed known with perfect knowledge. Since this is a linear programming model, only continuous proportions of traffic flow can be obtained at an aggregate level, and not decisions on which individual flights should be sent and when they should arrive at the weather. In [8], a stochastic integer programming model is developed based on the use of scenario trees to addressed combined ground delay-rerouting strategies in response to en route weather events. While this model is conceptually more general than ours, by developing a more structured approach we hope to develop a more scalable model.

Recently, a Ration-by-Distance (RBD) method was proposed as an alternative to the Ration-by-Schedule (RBS) method currently used for Ground Delay Programs (GDPs) [5], that maximizes expected throughput into an airport and minimizes total delay if the GDP cancels earlier than anticipated. This approach considers probabilities of scenarios of GDP cancellation times and assigns a greater proportion of delays to shorter-haul flights such that when the GDP clears and all flights are allowed to depart unrestricted, the aircraft are in such positions that the expected total delay can be minimized. While this problem was applied to GDPs, the principles of a probabilistic clearing time where there is a sudden increase in capacity and making initial decisions such that the aircraft are positioned to take the most advantage of the clearing is similar to our problem.

## III. THE MODEL

### A. Model Inputs

Our base model inputs consist of information about the FCA, which is consistent with the information used in AFP planning:

- Location of the FCA
- Speed and direction of the FCA
- Nominal capacity of the FCA
- Reduced capacity of the FCA
- Start time of the AFP
- Planned end time of the AFP

From a list of scheduled flights and their flight plans, we determine the set of flights whose paths cross the FCA and which therefore would be subject to departure time and/or route controls under an AFP. We also require a set of alternate routes for each flight. The alternate route for each flight should be dependent on the geometry of the FCA and the origin-destination pair it serves. These most likely would be submitted by carriers in response to an AFP; for the purposes of this paper it is assumed they are submitted exogenously, although for testing purposes it was necessary to synthesize alternate routes.

### B. Controls

In order not to exceed the (reduced) FCA capacity, each flight will be assigned one of two dispositions in the initial plan reacting to the FCA:

1. *The flight is assigned to its primary route, with a controlled departure time that is no earlier than its scheduled departure time.* Given an estimate of en route time, this is tantamount to an appointment (i.e., a slot) at the FCA boundary. Some flights might be important enough that they are allowed to depart on time, the AFP notwithstanding. Other flights might be assigned some ground delay.
2. *The flight is assigned to its secondary route, and is assumed to depart at its scheduled departure time.*

Several assumptions underlie our model:

- We do not consider airborne holding as a metering mechanism to synchronize a flight on its primary route with its slot time at the FCA.
- We assume that any necessary number of flights can be assigned to their secondary routes without exceeding any capacity constraints in other parts of the airspace.
- We assume that, when the weather clears, the FCA capacity increases immediately, back to the nominal capacity.
- The random variable is the time at which the FCA capacity increases back to its nominal value. We assume that perfect knowledge of the realization of this random variable is not gained until the scenario actually occurs, and so no recourse can be taken until the scenario is realized.

### C. Scenarios and future responses

The outputs of this model are:

1. An initial plan that designates whether a flight is assigned to its primary route or secondary route; for those assigned to their primary route an amount of ground delay (possibly zero) is assigned. For those assigned to their secondary route a specific directional angle is assigned.
2. A recourse action for each flight under each possible early clearance time.

We model the time at which the weather clears (i.e. FCA capacity increases) as a discrete random variable, with some exogenous distribution. For any realization of the capacity increase time, the flights in question will be in some particular configuration as specified in the initial plan. Some will have departed, either on their primary or secondary routes, some will already have completed their journeys, and some will still be at their departure airports.

Flights that were originally assigned to their primary route and that have already taken off will be assumed to continue with that plan. For any such flight, the primary route is assumed to be best, so no recourse action is necessary.

We now consider flights originally assigned to their primary route that have not yet taken off. The only possible change in disposition for these flights involves potentially changing their controlled departure time, i.e. reducing their assigned ground delay.

All other flights not yet considered were originally assigned to their secondary routes, with departure times as originally scheduled. These secondary routes avoid the FCA somehow. Under the FCA capacity windfall, some of those flights may now have an opportunity to use the FCA. If a flight has not yet taken off, and it is decided that it can use the FCA, the lowest cost way to do this is to re-assign it back to their primary route, with some controlled departure time no earlier than their scheduled departure time. If, on the other hand, the flight has already taken off, then the only mechanism to allow it the use of the FCA is a hybrid route that includes that portion (and perhaps more) of the secondary route already flown, plus a deviation that traverses the FCA and presumably rejoins the primary route at some point after the FCA (see Fig. 1). A flight that is already en route via its secondary route may or may not prefer such a hybrid path, depending on the difference in cost (time, fuel, etc.) between doing that and continuing on its secondary route. There may be many possible hybrid routes, and perhaps only a limited set of those would be reasonable.

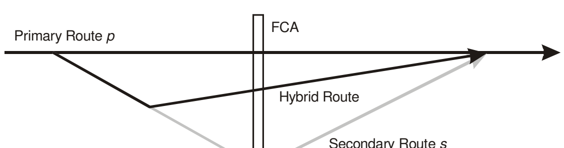


**Figure 1** Reverting from secondary route back to primary route through FCA.

For each possible value of the capacity windfall time, we determine the expected locations of all affected flights at that time, and also what would be the best change in disposition, if any, for each of those flights according to a system performance metric. With this information, we can compute the conditional cost associated with these adjusted flights based on the realization of the stochastic event. Ultimately, then, the goal of the optimization problem is to minimize the expected total cost, given these conditional costs and their associated probabilities.

### *D. Model Development*

We start by defining the discrete lattice on which time will be represented. We assume there is an index set $\{1,\ldots,T\}$ of size $T$ that demarcates equally spaced time slots, each of duration $\Delta t$. Each of these represents a possible appointment time window at the FCA. The nominal capacity of the FCA should be specified in terms of the maximum number of flights permissible during one of these time windows. The number of time slots $T$ then depends directly on $\Delta t$ and the total duration of an AFP, perhaps inflated to allow for ending times later than the original estimate. The reference time $t=1$ can be chosen as the earliest scheduled departure time of all of the affected flights.

The flights affected by the FCA can be determined from the filed flight plans for that day, minus known cancellations and re-routes at the time the AFP is invoked. These flights are indexed according to the set $\{1,\ldots,F\}$. In this, any specific reference to a time period $t$ and flight $f$ assumes that $t \in \{1,2,\ldots,T\}$ and $f \in \{1,\ldots,F\}$.

#### *1) Initial Plan*

There are two sets of assignment variables that are related to decisions about the dispositions of flights. One set represents the initial plan, which is the set of decisions provided by the model that will be enacted immediately once the model is run and the AFP is declared. The second set represents conditional decisions (recourse actions) based on the random variable representing the time at which the capacity windfall takes place, which we do not know at the time of the execution of this optimization problem, but that we condition for when determining the best initial plan.

For the initial plan, we define the following set of binary decision variables:

$$x_{f,t}^{p} = \begin{cases} 1, \text{ if flight } f \text{ uses its primary route and} \\ \text{has an appointment time } t \text{ at the FCA} \\ 0, \text{ otherwise} \end{cases}$$

$$x_{f}^{s} = \begin{cases} 1, \text{ if flight } f \text{ is assigned to its secondary} \\ \text{route} \\ 0, \text{ otherwise} \end{cases}$$

Every flight *f* needs to have an assigned disposition under the initial plan, thus:

$$\sum_{t} x_{f,t}^{p} + \sum_{r} x_{f,r}^{s} = 1 \quad \forall f \tag{1}$$

We require that any flight that is assigned to its primary route cannot be given an appointment slot at the FCA that is earlier than its scheduled departure time plus the expected en route time required to arrive at the FCA. If $E_f \Delta t$ represents the en route time (from its origin to the FCA) for flight *f*, and $D_f \Delta t$ is the scheduled departure time for flight *f*, then:

$$\sum_{t=1}^{D_f + E_f} x_{f,t}^{p} = 0 \quad \forall f \tag{2}$$

No similar constraint is applied to flights assigned to their secondary routes under the initial plan, because they are not metered at any point and hence are expected to depart at their originally scheduled departure time. There is no provision in the model for a flight to depart early, despite the fact that the secondary route takes more time than the primary route (since, subject to minor variations, airlines do not allow flights to take off before their scheduled departure times).

It might be the case that for a particular flight *f*, there is a latest slot time $l_f$ at the FCA that the carrier who owns that flight would be willing to accept. Slots later than $l_f$ can be prevented via the following constraint:

$$\sum_{t=l_f+1}^{T} x_{f,t}^{p} = 0 \qquad (3)$$

For any flight for which $l_f$ is not explicitly provided, $l_f$ is the time beyond which the secondary route will be chosen.

The initial constrained capacity (maximum number of flights) for time window *t* can now be defined as $C_t^0$ and the constraint to enforce it is:

$$\sum_{f} x_{f,t}^{p} \le C_t^0 \quad \forall t \qquad (4)$$

*2) Second Stage*

The variables and constraints defined so far represent the first stage of the stochastic program. It is assumed that these decisions will be enacted deterministically immediately after the FCA is declared. Next, we describe the second stage of the stochastic program – those variables that represent the conditional decisions we expect would be made if any of a number of possible capacity windfall times happens to come true in the future. We model the time slot at which this occurs as a discrete random variable with domain $\Omega$ and probability mass function

$$f_U(u) = \Pr\{U = u\} \quad \forall u \in \Omega$$

Under a capacity windfall, a flight that was originally assigned to its primary route with a controlled departure time might still be given the same general disposition, although its departure time could be moved earlier if that were beneficial to the system goal. We let

$$y_{f,t}^{p} \mid u = \begin{cases} 1, \text{ if at the time } U = u \text{ of the capacity windfall,} \\ \text{flight } f \text{ is assigned to its primary route with} \\ \text{appointment slot } t \text{ at the FCA} \\ 0, \text{ otherwise} \end{cases}$$

We will (shortly) introduce other variables for the other possible second stage flight dispositions, and we will require that all flights be assigned a disposition under every possible realization of the stochastic event *U*. For now, we proceed by obviating values of $y_{f,t}^{p} \mid u$ that would either be physically infeasible or politically imprudent. Later, structural constraints plus pressure from the objective function will lead to the best possible selection of second stage dispositions for all flights.

First, it is impossible to assign a flight to a slot that would require it to depart before its scheduled departure time:

$$y_{f,t}^{p} \mid u = x_{f,t}^{p} \quad \forall f, u, \quad \forall t \in \{1, \ldots, D_f + E_f\} \qquad (5)$$

This constraint works with constraint (2) to achieve the required result.

Given the timing *U* of the capacity windfall, some flights may already have taken off. If they did so via their primary route (with a controlled departure time), then their second stage disposition should match that of the first stage:

$$y_{f,t}^{p} \mid u = x_{f,t}^{p} \quad \forall f, u, \quad \forall t \in \{1, \ldots, u + E_f\} \qquad (6)$$

A closer look at constraint (6) reveals that it also satisfies an important requirement for flights that have not yet taken off. For any particular flight *f* and given the capacity windfall time *u*, the collection of primary stage variables $\{x_{f,t}^{p}\}_{t=1}^{t=u+E_f}$ will either contain one at exactly one position or it will consist entirely of zeros. In the former case, this means that the flight has already taken off, and that situation has been dealt with. In the latter case, this is indicative of the fact that these slot times are infeasible. Thus, even for flights that have not yet taken off, constraints (2) and (6) insure that they will not be assigned, in the second stage, to their primary routes with slot times that they cannot achieve.

Looking at constraints (5) and (6), it is clear that they can be combined:

$$y_{f,t}^{p} \mid u = x_{f,t}^{p} \quad \forall f, u, \quad \forall t \in \{1, \ldots, \max(u, D_f) + E_f\} \qquad (7)$$

On the other hand, for flights that already took off via their secondary routes (and therefore at their scheduled departure times), the only possible second stage dispositions are secondary or hybrid routes, so assignments to primary routes for these flights must be prevented:

$$\sum_{t} y_{f,t}^{p} \mid u \le 1 - \sum_{r} x_{f,r}^{s} \quad \forall u, \forall f \ni D_f < u \qquad (8)$$

In addition, we will not allow a flight whose controlled departure time is being moved in the face of a capacity windfall to be worse off than it was before this event materialized:

$$y_{f,t}^{p} \mid u \le \sum_{q \ge t} x_{f,q}^{p} + \sum_{r} x_{f,r}^{s} \quad \forall u, f, t \qquad (9)$$

Notice that we want to allow for the possibility that flights originally assigned to their secondary routes can revert, under the appropriate circumstances and if the optimization decides this is best, to their primary route if they have not already taken off, which is why the variable $x_{f,r}^{s}$ appears in (9).

For flights that were originally assigned to the secondary route, the increased capacity at the FCA might allow some of these flights to pass through the FCA and thus improve their flight path by returning to the primary route at some point after the FCA or continuing directly to the destination. For a flight that has not yet departed, the same structure can apply, but the portions of the total flight path spent on the secondary and reverting routes have length zero. We define the second-stage decision variables for this choice as follows:

$$y_{f,t}^h \mid u = \begin{cases} 1, \text{ if flight } f \text{ was originally assigned to its} \\ \quad \text{secondary route, but under capacity} \\ \quad \text{clearing time } u \text{ has been assigned an} \\ \quad \text{FCA appointment slot } t \\ 0, \text{ otherwise} \end{cases}$$

This decision can only be reached for flights that were originally assigned to their secondary routes:

$$y_{f,t}^h \mid u \le x_f^s \quad \forall u, f, t \tag{10}$$

However, we note that the objective function will enforce this behavior implicitly. Such a flight will be on its secondary route, which may be altered to become a hybrid route that passes through the FCA. We need to impose constraints that insure that these flights are only assigned to FCA time slots they can feasibly reach. If a flight diverts from its secondary route to its hybrid route at time $t^d$ there will be an earliest time it can reach the FCA. Fig.1 illustrates the geometry used to compute the parameter used by our model:

$t_{f,t}^d$ is the time at which flight $f$ must alter its secondary route to become a hybrid route that arrives at the FCA at time $t$.

The following constraint prevents a flight from diverting to its hybrid route before the weather is actually cleared.

$$y_{f,t}^h \big| u = 0 \quad \forall f, u, \quad \forall t \big| t_{f,t}^d \le u \tag{11}$$

The final option possible is that a flight carries out its originally planned secondary route:

$$y_f^s \mid u = \begin{cases} 1, \text{ if flight } f \text{ was originally assigned to its} \\ \quad \text{secondary route, and if, under AFP stop} \\ \quad \text{time } u\text{, that decision remains unchanged} \\ 0 \text{ otherwise} \end{cases}$$

Practically speaking, it would never make sense to assign a flight to its secondary route under the recourse if it had not also been given the same assignment in the initial plan. It might seem, therefore, that the following constraint is necessary:

$$y_f^s \mid u \le x_f^s \quad \forall u, f \tag{12}$$

However, it can be seen that the objective function enforces this behavior implicitly. If it were cost-effective to assign a flight to its secondary route under the recourse, it would also be cost-effective to do so under the initial plan.

Constraints (10) and (12) can be combined into a single constraint:

$$y_{f,t}^h \mid u + y_f^s \mid u \le x_f^s \quad \forall u, f, t \tag{13}$$

It would be possible, given the constraints developed so far, to assign a flight to a hybrid route that essentially reverts to the primary route immediately. In other words, this would be an assignment that is tantamount to taking off on the primary route at the scheduled departure time, which is a more logical way to interpret this outcome. Therefore we introduce the following constraint to enforce this behavior:

$$y_{f,D_f+E_f}^h \mid u = 0 \quad \forall f, u \tag{14}$$

For each time scenario $u$, every flight $f$ must be assigned to one of these dispositions. Furthermore, if the disposition involves being scheduled into a slot appointment at the FCA, no more than one slot can be assigned to a given flight. Given that the decision variables are required to be binary, the following constraint addresses both of these concerns

$$\sum_t y_{f,t}^p \mid u + \sum_t y_{f,t}^h \mid u + y_f^s \mid u = 1 \quad \forall u, f \tag{15}$$

For any value $U = u$, there will be a new capacity profile $C^u(t)$ that agrees with $C^0(t)$ up to time $t = u$, but represents an increase in capacity beyond that point. For example, if $C^0(t)$ had been a constant vector, then $C^u(t)$ could be a step function that makes a jump at time $t = u$. On the other hand, if $C^0(t)$ had been a periodic 0-1 function, then $C^u(t)$ might just have an increased duty cycle after time $t = u$. A wide variety of profiles for $C^u(t)$ are possible; the only real requirements are that it agree with $C^0(t)$ prior to time $t = u$, and that after that time, it supports a higher rate of flow than was possible under the initial plan. The capacity constraint under the scenario $U = u$ can now be written as:

$$\sum_f y_{f,t}^p \mid u + \sum_f y_{f,t}^h \mid u \le C_t^u \quad \forall u, t \tag{16}$$

The last constraint prevents flights, for which the FCA will have moved out of their primary path by the time they get there (i.e $c_f^s = 0$ ) from being a candidate to get an appointment slot at the FCA

$$\sum_t x_{f,t}^p \le M c_f^s \quad \forall f \tag{17}$$

where $M$ is a constant number that makes $M c_f^s > 1$ for all flights $f$. By this constraint such flights will be forced to be assigned to their secondary route, which will cause no delay.

*3) Objective Function*

Since our model involves the specification of decisions that are conditioned random events, the objective function will be an expected value. To emphasize the paradigm of creating a plan (our initial plan) together with contingency plans (our recourse actions), we represent the objective function as the sum of the deterministic cost of the initial plan minus the expected savings from recourse actions.

Therefore the objective function can thus be represented as:

$$Min\left[C(X) - \sum_u P_u S(Y_u)\right] \tag{18}$$

Or more precisely:

$$Min \quad Z = z^1 + z^2 - \sum_u P_u \left(z_u^3 + z_u^4\right) \tag{19}$$

where

$$z^1 = \sum_f \sum_t c_{f,t}^p x_{f,t}^p \tag{20}$$

$$z^2 = \sum_f c_f^s x_f^s \tag{21}$$

$$z_u^3 = z^1 - \sum_f \sum_t c^p_{f,t}\, y^p_{f,t} \big|u + \sum_f \sum_t c^s_f\, s^p_{f,t} \big|u \qquad (22)$$

$$z_u^4 = \sum_f \sum_t sv^h_{f,t}\, y^h_{f,t} \big|u \qquad (23)$$

where

$c^p_{f,t}$ is the cost of assigning flight *f* to its primary route so that it arrives at the FCA at time *t*.
$c^s_f$ is the cost of assigning flight *f* to its secondary route.
$sv^h_{f,t}$ is the savings incurred if flight *f* starts out on its secondary route but reverts to a hybrid route that arrives at the FCA at time *t*.
$s^p_{f,t}$ is a dummy binary variable that works as an indicator. It takes the value of one when a flight initially assigned to its secondary route is assigned back to its primary route under the revised plan.
So;

$$s^p_{f,t} = Min\left(x^s_f, y^p_{f,t}\right) \qquad \forall f,t \qquad (24)$$

## IV. The Parameters

In the following section we will show how the movement of the FCA will affect our previous flight path geometries and we will provide the related calculations for each case. One can obtain the same functions for the case of a stationary FCA simply by setting $v_a$ and $v_c$ to zero.

The first set of equations show how the primary route cost functions are recalculated. We assume that the intersection of the FCA and a flight primary path (i.e. the point that the flight will enter the FCA) will move with a constant speed either toward or away from the flight. In these equations, $v_f$ is the speed of the aircraft along its path, $v_a$ is the projection of the FCA speed vector on the flight's primary path, $v_c$ is the orthogonal component of that velocity, *t* is the time of arriving at the FCA, and $t_d$ and $t_s$ are the actual and scheduled departure times.

$$v_f\left(t - t_d\right) + v_a\left(t - t_d\right) = a - v_a\left(t_d - t_s\right) \qquad \forall t$$

$$t_d = t + \frac{v_a}{v_f}\left(t - t_s\right) - \frac{a}{v_f} \qquad (25)$$

$$c^p_{f,t} = t_d - t_s = \left(1 + \frac{v_a}{v_f}\right)\left(t - t_s\right) - \frac{a}{v_f} \quad \text{and } c^p_{f,t} \ge 0$$

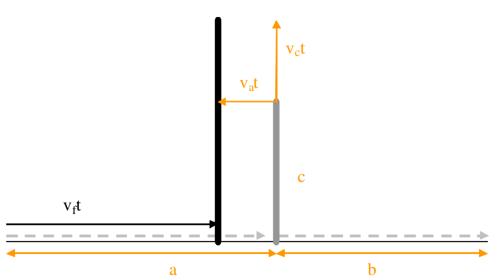


**Figure 2**

Next we show how the secondary cost functions are recalculated. Let $\alpha$ and $\beta$ be the required directional angles of the reroute to avoid a moving and stationary FCA, respectively. The gray dashed lines represent the flight path in the case of a stationary FCA and the black lines represent those of a moving FCA.

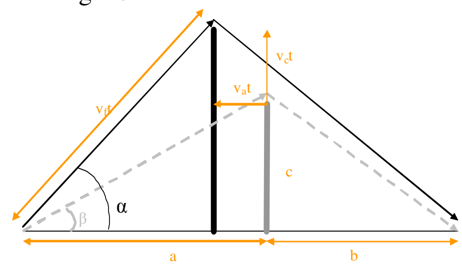


**Figure 3**

$$\left.\begin{aligned} v_f\left(t - t^d_f\right)\cos\alpha &= a - v_a\left(t - t^d_f\right) \\ v_f\left(t - t^d_f\right)\sin\alpha &= c + v_c\left(t - t^d_f\right) \end{aligned}\right\} \Rightarrow$$

$$t - t^d_f = \frac{a}{v_f \cos\alpha + v_a} \Rightarrow \frac{a v_f \sin\alpha}{v_f \cos\alpha + v_a} = c + \frac{a v_c}{v_f \cos\alpha + v_a}$$

$$\Rightarrow a\sin\alpha - c\cos\alpha = \frac{c v_a + a v_c}{v_f}$$

$$d = \sqrt{c^2 + a^2}, \quad \cos\beta = \frac{a}{d}, \quad \sin\beta = \frac{c}{d}$$

$$\Rightarrow \frac{a}{d}\sin\alpha - \frac{c}{d}\cos\alpha = \frac{c v_a + a v_c}{d v_f} = \cos\beta\sin\alpha - \sin\beta\cos\alpha \Rightarrow$$

$$\sin\left(\alpha - \beta\right) = \frac{c v_a + a v_c}{d v_f} \qquad (26)$$

With the new directional angle α, we can calculate the cost of airborne delay of the reroute;

$$d' = \frac{a}{v_f \cos(\alpha) + v_a}$$

$$c^s_f = d' + \sqrt{d'^2 + (a+b)^2 - 2\cos(\alpha) d'(a+b)} - a - b \qquad (27)$$

Finally, once we have found the adjusting angle for the secondary route compromising the FCA movement, we need to calculate the interrelated changes to our hybrid route cost saving function as well. To do so we assume that the weather clears after *i* minutes of the flight's departure. With the speed of $v_f$, our flight would traverse a distance $\overrightarrow{AD} = i$ times $v_f$ along its secondary route. As shown in Fig. 4, by knowing $\overrightarrow{AD}$ we can compute its counterpart angle $\mu$.

$$\cos\mu = \frac{a + b - i v_f \cos\alpha}{\sqrt{\left(i v_f\right)^2 + \left(a+b\right)^2 - 2 i v_f \left(a+b\right)\cos\alpha}} \qquad (28)$$

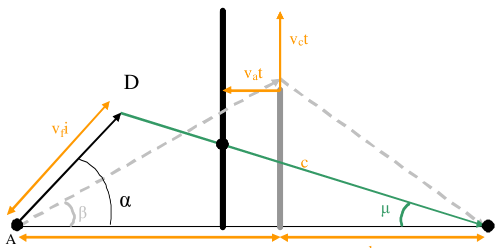


**Figure 4**

Now that we have $\mu$, we can build in our governing equation to calculate the time $t$, at which the flight arrives at FCA if it reverts from its secondary route after $i$ minutes of its departure.

$$v_f i\cos\alpha + v_f\left(t-i\right)\cos\mu = a - v_a t \Rightarrow$$
$$t = \frac{a + i v_f\left(\cos\mu - \cos\alpha\right)}{v_f\cos\mu + v_a} \quad (29)$$

This is only true if the flight has not yet reached the end of the FCA (i.e. point C). Therefore the following constraints should apply to maintain the feasibility of the above equations.

$$v_f i\sin\alpha - v_f\left(t-i\right)\sin\mu <= c + v_c t \quad (30)$$

and finally the saving incurred on the hybrid route:

$$sv_{f,t}^{h} = c_f^{s} - v_f i_t - \sqrt{\left(v_f i_t\right)^2 + \left(a+b\right)^2 - 2 v_f i_t\left(a+b\right)\cos\alpha} \quad (31)$$

## V. Computational Experiments

### A. Decision Impacts

To evaluate the impact of the FCA movement on our model we ran a set of experiments, where we varied the direction of the FCA movement. In this way we were able to mimic more realistic environments where the flights' paths (primary and reroute) are affected by the movement of the FCA as well as its presence. The cases vary relative to the speed, direction and severity of the weather activity and recourse actions are allowed and planned respectively. A recourse action is taken if the weather clears earlier than expected. In the ground delay case, this means a flight is released at a time earlier than its planned departure time. In the reroute case, this means a flight adjusts its original planned reroute to a more direct route. The key novel contribution of our model is its ability to take into account recourse actions when generating its initial plan.

We now describe the problem data. Flights, their routes, and alternate routes were generated artificially based on the airspace geometry given in Fig. 1. There were $F$=200 flights with random departure times ($t_s$=0,…,60). There were $T$=200 time slots; each slot had a width of $\Delta t$ =2 minutes. There were three possible early clearance times: $U \in \{30,50,70\}$ each occurring with probability 0.3 and 0.1 is the probability that the FCA does not clear until the end time of the AFP. The following alternate cases were considered. The ratio of airborne delay cost to ground delay cost was assumed to be 2.0.

Case 1: This case considers a stationary FCA and runs the model to find the best initial plan which will serve as a base for the purpose of evaluation of the other cases.

Case 2-9: in these cases the FCA has eight different directions with the same velocity approximately equal to 5% of the average flight speed. The reduced throughput of the FCA is one flight every 4 minutes and increased throughput is 2 flights per minute.

Case 10-17: these cases are similar to cases 2-9 except that the FCA velocity is approximately equal to 10% of the average flight speed.

Case 18: this case is similar to case 1 except that the reduced throughput of the FCA is one flight every 8 minutes and increased throughput is one flight per minute.

Case 19-26: these cases are similar to cases 10-17 except that the reduced throughput of the FCA is one flight every 8 minutes and increased throughput is one flight per minute.

The table below provides the results of an experiment under which all 26 cases were executed. First of all it should be clarified that for simplicity all the 200 flights are assumed to fly in the same direction but with different origin-destination distances, different scheduled departure times and different directional angles for their reroutes. Valuable insights should be obtainable even with this simplification, and more realistic scenarios can always be studied with the exact same formulation.

The first thing to notice is that the movement of the FCA can significantly change the total cost as well as the assignment of the dispositions to all flights affected by the presence of the FCA. The second interesting result is the consistent pattern with which the objective function value increases. In the result table we have sorted the similar cases (similar in terms of the FCA velocity and throughput) in an increasing order of the objective function value. In all three sections of the results table, perhaps not surprisingly, the maximum cost saving occurs when the FCA moves laterally (downward in Fig. 2 and Fig.3), in which case it either gets out of the way of the primary paths of the affected flights quickly or lowers the maximum length of the reroutes. The total cost is reduced by 38% and 64% with the low and high speed FCA, respectively.

One can observe that the effect of the longitudinal movement of the FCA, where it moves either toward or away from the oncoming traffic, is less significant than the lateral motion. The total cost is increased by 3% (8% for the high speed FCA) when the FCA is moving away from the traffic. When it is moving toward the traffic the total cost is increased by 19% (23% for the high speed FCA).

The second and the third columns are the total costs for ground delays and airborne delays of the first stage. The fourth and the fifth columns are the numbers of flights assigned to primary and secondary paths. The sixth column is the objective function value, which is the minimum expected total cost. The seventh and the eighth columns are the horizontal and the vertical component of the FCA velocity vector. The last column visualizes the FCA direction. The units of all costs are "numbers of time slots," which can be converted readily to minutes, and presumably to dollars if the analyst has data on economic time values.

**TABLE 1:Experimental results on the effects of a moving FCA**

| Case | c(xp=1) | c(xs=1) | n(xp=1) | n(xs=1) | Obj | Va | Vc | Wind |
|---|---|---|---|---|---|---|---|---|
| 1 | 80 | 459 | 67 | 133 | 379 | 0.00 | 0.00 | ● |
| 2 | 66 | 225 | 60 | 140 | 236 | 0.00 | -0.05 | ↓ |
| 3 | 97 | 285 | 65 | 135 | 310 | -0.035 | -0.035 | ↘ |
| 4 | 103 | 300 | 60 | 140 | 336 | 0.035 | -0.035 | ↙ |
| 5 | 124 | 494 | 71 | 129 | 391 | -0.05 | 0.00 | → |
| 6 | 108 | 437 | 61 | 139 | 452 | 0.05 | 0.00 | ← |
| 7 | 130 | 689 | 70 | 130 | 543 | -0.035 | 0.035 | ↗ |
| 8 | 117 | 670 | 67 | 133 | 572 | 0.035 | 0.035 | ↖ |
| 9 | 105 | 783 | 70 | 130 | 585 | 0.00 | 0.05 | ↑ |
| 10 | 65 | 93 | 58 | 142 | 135 | 0.00 | -0.10 | ↓ |
| 11 | 98 | 161 | 63 | 137 | 185 | -0.07 | -0.07 | ↘ |
| 12 | 84 | 173 | 56 | 144 | 234 | 0.07 | -0.07 | ↙ |
| 13 | 123 | 528 | 74 | 126 | 409 | -0.10 | 0.00 | → |
| 14 | 102 | 451 | 61 | 139 | 466 | 0.10 | 0.00 | ← |
| 15 | 150 | 1020 | 75 | 125 | 655 | -0.07 | 0.07 | ↗ |
| 16 | 130 | 899 | 66 | 134 | 752 | 0.07 | 0.07 | ↖ |
| 17 | 135 | 1192 | 71 | 129 | 799 | 0.00 | 0.10 | ↑ |
| 18 | 65 | 717 | 36 | 164 | 605 | 0.00 | 0.00 | ● |
| 19 | 55 | 190 | 32 | 168 | 247 | 0.00 | -0.10 | ↓ |
| 20 | 85 | 331 | 36 | 164 | 327 | -0.07 | -0.07 | ↘ |
| 21 | 94 | 305 | 32 | 168 | 389 | 0.07 | -0.07 | ↙ |
| 22 | 115 | 892 | 39 | 161 | 635 | -0.10 | 0.00 | → |
| 23 | 90 | 712 | 34 | 166 | 711 | 0.10 | 0.00 | ← |
| 24 | 133 | 1554 | 41 | 159 | 972 | -0.07 | 0.07 | ↗ |
| 25 | 143 | 1333 | 34 | 166 | 1082 | 0.07 | 0.07 | ↖ |
| 26 | 107 | 1812 | 38 | 162 | 1178 | 0.00 | 0.10 | ↑ |

## VI. CONCLUSIONS AND FUTURE WORK

In this paper, we have defined the basics of a stochastic optimization model for simultaneously making ground delay and reroute decisions in response to moving en route airspace congestion. We have also given the results of computational experiments that test the impact of the speed and direction of the movement of the flow-constrained area on decisions as well as the outcome. We believe that the model can serve as a basis for solving practical TFM problems using commercial IP solvers. Further, the results show that the models have the potential to substantially improve TFM decision making.

Our model can be re-run if, and as often as, real-time information suggest that the data supporting a previous execution of the model have changed significantly, for example, if carriers cancel some additional flights, or if the probabilistic weather forecast changes. The model can be forced to preserve earlier decisions by additional constraints fixing those decisions for flights currently in the air.

We anticipate the need to provide more refinements and extensions to this model to better address practical problem solving.


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